Introduction to Flight
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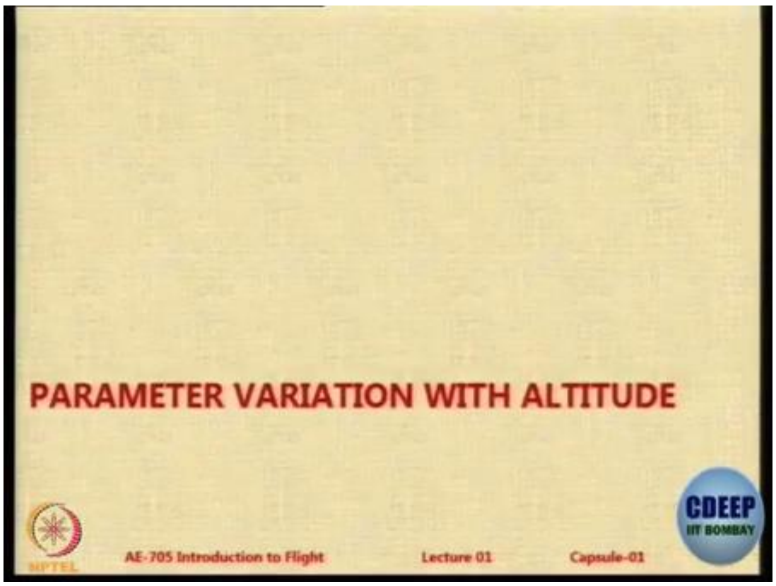
Lecture No. 01.3
Pressure, Temperature, Density and Viscosity Variation with Altitude in ISA

So, let us see the variation of these four parameters, this is something that you will need in calculation.



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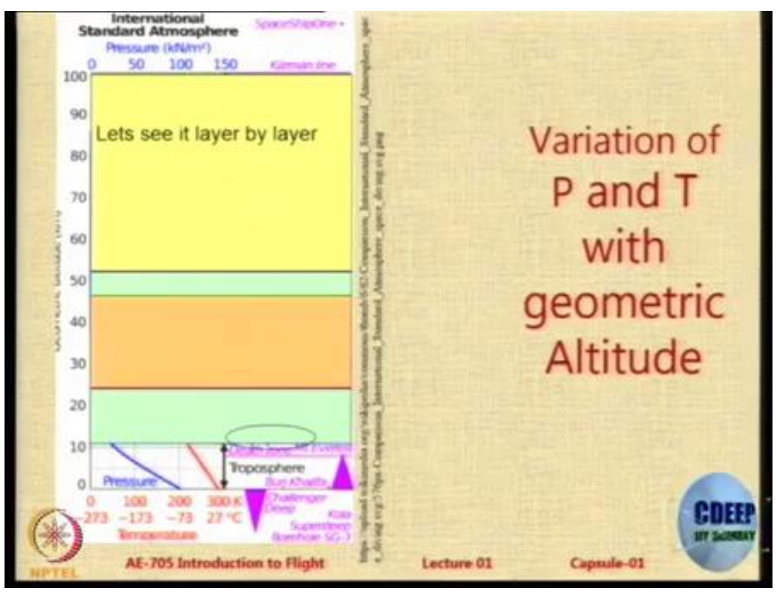

So, let us see what it is okay. So, let us look at 'P' and 'T' with geometric altitude. So, I have just hidden and I want you to go layer by layer. So, we will just remove one one layer. So, first we look at the troposphere the the troposphere, okay, which is the bottom most portion. We notice the red line shows that the *temperature* is reducing from around 288 K, it reduces linearly upto a value of I think 212, Kelvin linearly.

The pressure does not change linearly but it reduces, okay. It is a non-linear variation because dp has got that rho and rho is changing with altitude. If we remove one more layer and we come to the troposphere and the troposphere and tropopause, we find that in tropopause, the temperature is remaining constant, okay. However, pressure is still reducing, but the rate of change of reduction is reduced slightly.

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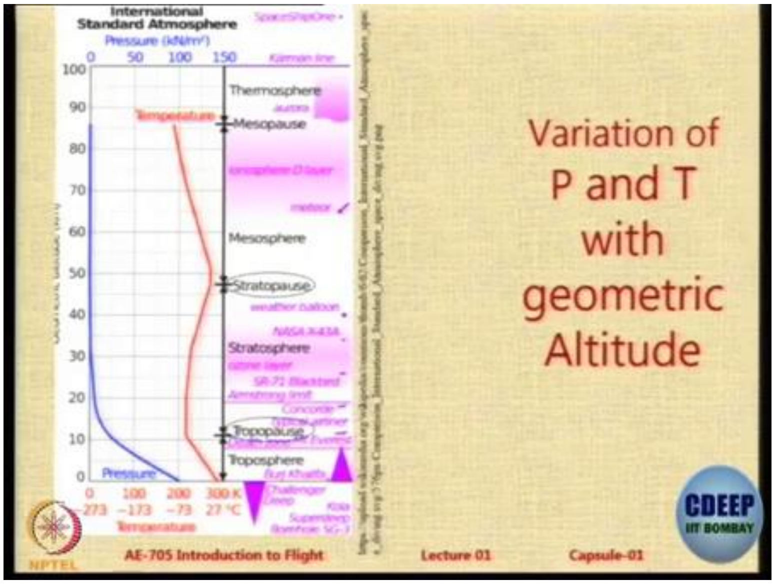

It is still reducing but at a lower rate. If we go one step further, now we find the temperature is increasing. However, pressure is still reducing, but at a very very low rate, hardly any change, but there is change. Then we go to stratopause, which is a very small zone, again temperature remains constant, pressure still reduces. And above this we have what is called as the mesosphere, mesopause, thermosphere, but we are not bothered about that too much, okay.

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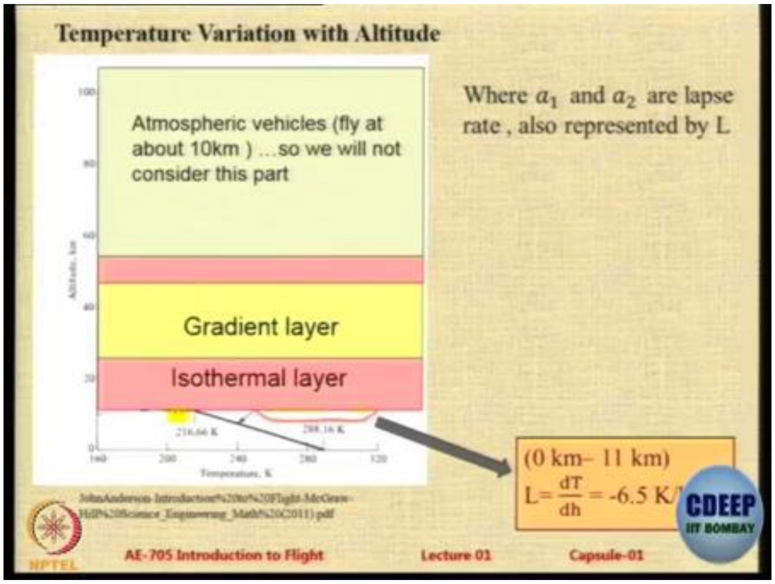

Because we will not take that part. When you do spacecraft dynamics or you do introduction to spacecrafts, we will look at that right now. So, what is happening is the bottom layer is the gradient layer, where temperature is reducing linearly. So, therefore, the reduction in temperature up to 11 kilometer, this is something that you should try to remember. This is called as basic aerospace engineering knowledge.

From sea level to 88.16 degree Kelvin or 15 degree centigrade up to around 11 kilometers it reduces linearly at the rate of six and a half degrees per kilometer, temperature and with that you can get the values of other parameters. In the isothermal layer which is from 11 to 25 kilometers generally, temperature remains constant, okay. In the next layer from 25 to 47, it increases by 3 degrees Kelvin per kilometer. Above that up to 53 kilometers again it remains constant.

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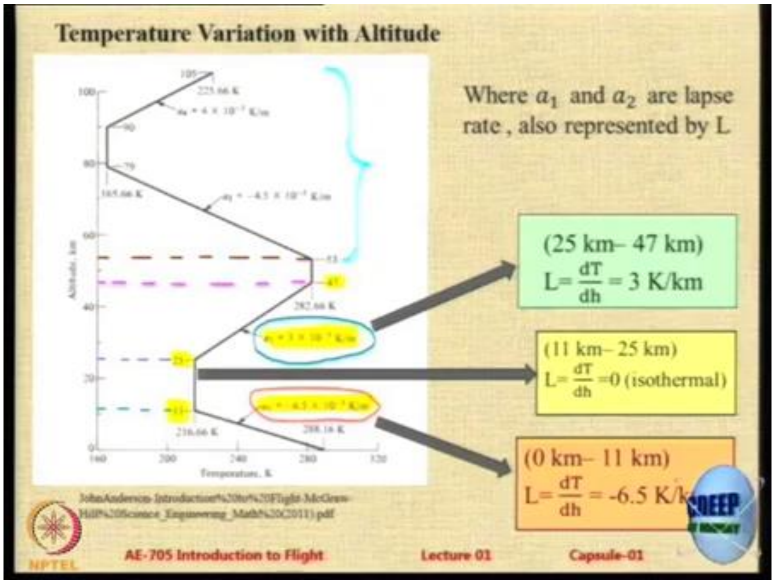

Beyond that we do not care because we will hardly look at at aircrafts which fly beyond 50 kilometers. Okay so, this is not important for us right now but I will just reveal that again there is a reduction, again there is a constant, again there is a increase, up to 105. So, you are not expected to remember all the values but atleast I would expect you to remember two things that up to 11 it reduces, it remains constant to 25 and abound beyond 25 up to 47 it becomes 3 degrees per, after that I do not expect you to remember, okay.

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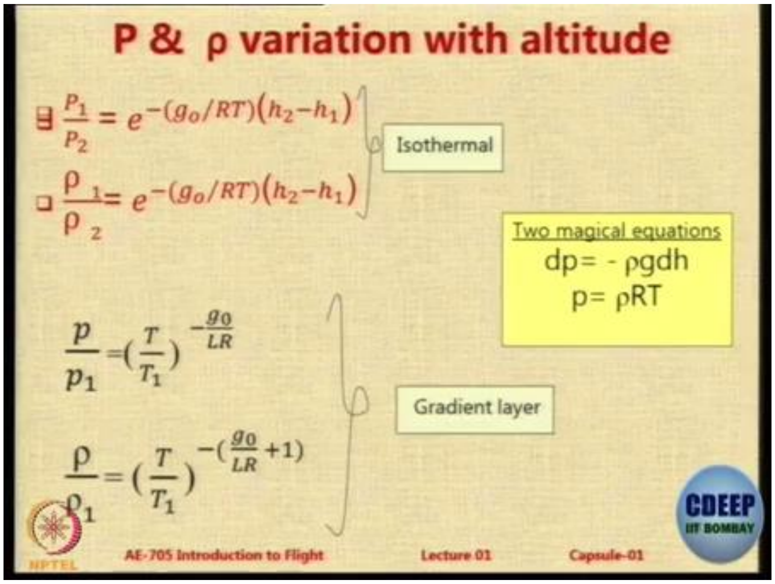

Alright, so, this is something, yes?

We will see it. We have that. See there is one special section on measurement of pressure and velocity, there I will talk about instruments used to measure. Today also I will talk about it, okay. Now, you have some homework to do or self-study. It is very easy for you to actually derive these expressions looking at standard textbooks. Many of you who are from aerospace background may have done this as part of your GATE preparations or as part of your regular undergraduate studies.

Those who have not done you can look up any textbook and do it. It is very straight forward because temperature is varying at some particular rate. You can get the relationship between the pressure ratios and the density ratios. So, g zero is 9.876 the the gravity, R is the universal gas constants, 287 Joules per kg degree Kelvin, T is a temperature in Kelvin and h2-h1 is the temperature, height difference. So, from some height to some height the pressure difference, the pressure ratios is this.

So, if you want to remember I will tell you what to remember. This value will become

$$\frac{P_1}{P_2} = e^{-(g_0/RT)(h_2-h_1)}$$

$$\frac{\rho_1}{\rho_2} = e^{-(g_0/RT)(h_2-h_1)}$$

That is what you can remember. Yeah?

No it can be anything. See here the value of h, h2 and h1, both of them are from the mean sea level. This formula is actually meant for the geometric altitude, okay. So, h2 minus h1 is a difference, it does not matter as long as this was the same reference does not matter, delta h. So if you want, if you know the pressure at 1 kilometer and you want at 2 kilometer, you can use t2 by t1 power 5 point 258 and get the value of t2 by t1. Got it, okay. But this is only true in the isothermal region.

That is the region on the top from 11 to yeah, this one is what is applicable in the gradient layer. So, I am sorry the value which I gave you 5.25 it is for this region actually, g0 by LR and g0 by LR plus 1, so yeah so the the rho by rho1 is going to be t by t1 to the power minus of 5 point 258 and p by p1 will be t by t1 to the power 4 point 25. It is very simple g0 is 9 point 807, L is 6 point 5 degree per kilometer and R is so just calculate, if you have a calculator you will get a number, that number you just remember, okay.

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{LR}} = \left(\frac{T}{T_1}\right)^{4.258}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{LR}+1\right)} = \left(\frac{T}{T_1}\right)^{5.258}$$

And you get this simply by two magical equations. One which we have already derived, the standard equation of hydrostatics and the other one is universal law which says that P is equal to rho RT, as long as the gas does not dissociate. So, using these two you can get these two expressions. Alright let us go ahead.

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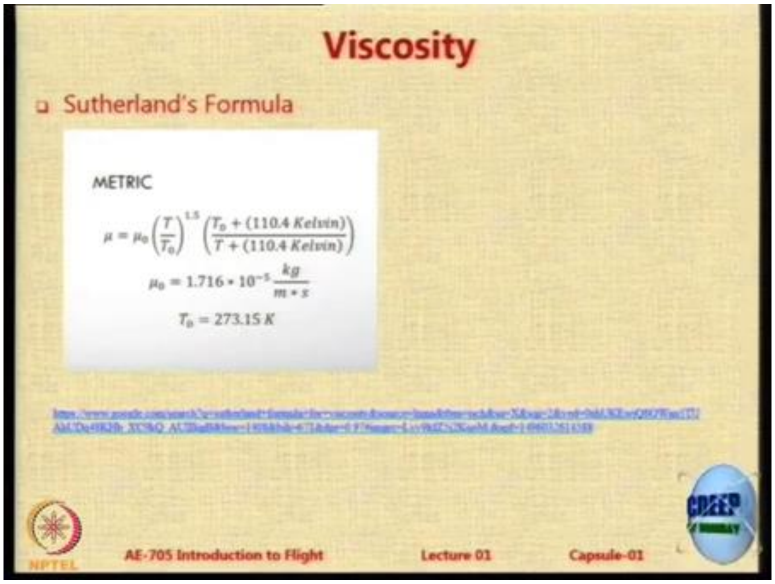

Viscosity, this is a very important parameter, okay. Sometimes in calculations when you do aircraft performance you need to know the viscosity of the air. So, why is viscosity important? Where does it play a role? Yes, but through which parameter it show up, through Reynold's number and that is where you need viscosity. So, viscosity calculation there are many formulae available, but there is one very elegant formula given by Sutherland.

I like it because it is very easy to code this formula, very easy to enter this data. So, it just says that μ at any altitude is equal to μ0 into T by T0 power 1 point 5 into T0 plus 110 and Tplus 110 point 4, okay. So, if you want to know how this has come there is link given below you can click and you can read it yourself.

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{1.5} \left(\frac{T_0 + 110.4}{T + 110.4}\right)$$

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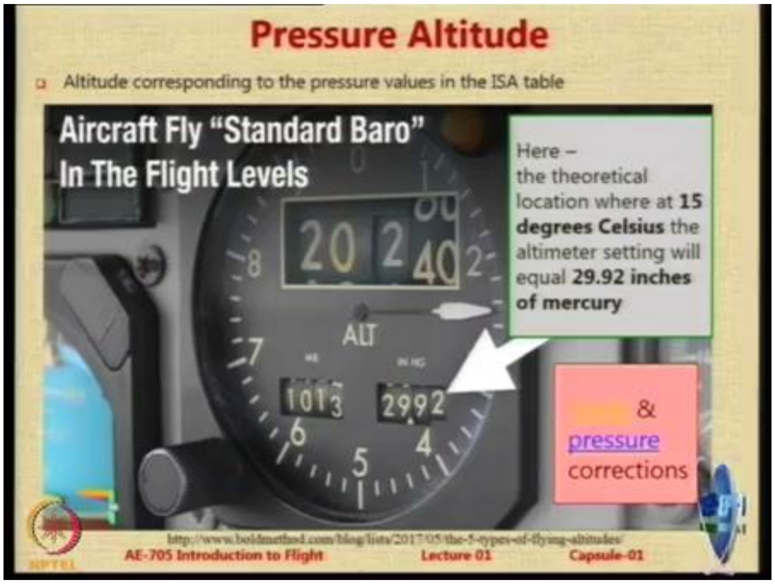

Now we look at the other three altitudes. Now the pressure altitude is basically an altitude depending on a standard pressure. So, you should look at the ISA, International Standard Atmosphere, the pressure at 0 is 101325 Newtons per meter square, that corresponds to 29 point 92 inches of mercury. So, with respect to that pressure at datum value, when you read the height that is called as a pressure altitude.

In other words, if an aircraft is flying at a height 'x' and in the ISA the or I should say the other way around, the aircraft is flying at a height 'h' and the ambient air pressure at that condition is 'y', you look up in the ISA at what height do you have 'y', that height is the pressure altitude, simple. So, I may be at sea level and the ambient pressure may be equal to pressure at 1 kilometer in ISA, so although I am at sea level, my pressure altitude is 1 kilometer because the ambient air pressure at the height which I am flying corresponds to a height of 1 kilometer under ISA, this is called as the pressure altitude. Do you understand this?

It is only a concept, but why is it given, why do we want to talk this kind of a complicated thing, the answer is the pilot does not know where the sea level is. The pilot only knows what the ambient air pressure is, through an instrument and assume there are errors removed etc. etc. Assume that the pilot knows the true pressure. So, when the pilot talks to the ATC, that is the question that you asked, how do you have reconciliation.

So, the pilot says I am operating at a pressure altitude of 3 kilometers. So, that is for uniformity, okay. So, the theoretical location, where at 15 degree Celsius the altimeter reading will be so and so of mercury, that is the altitude. In other words, look at ISA, find the pressure, can go to the ambient, and that is the value, okay.

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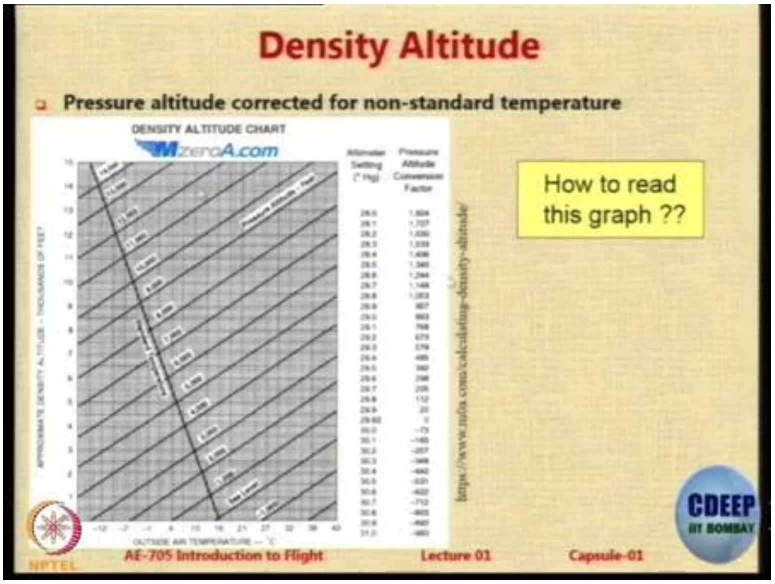


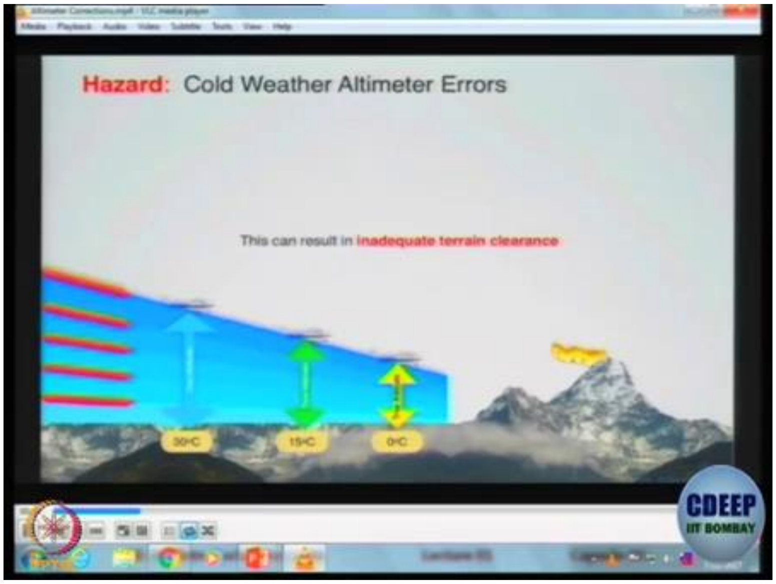
We also have density altitude; now can you guess what is density altitude? Can you guess?

Perfect. So, if I am at some altitude, at which density of air is 1 point 2256 kg per meter cube, I am flying at sea level density altitude. I could be on Everest, but my density altitude is 1 point 22, sorry is zero, correct. Here the reference is the density in the ISA. Now the first one, pressure altitude is useful for the pilot. Who uses this altitude? Density altitude, who, who is talking about it?

The airport people, because pilots cannot measure density of air, they do not have a small container and a mass spectrometer, you collect air, weigh it and then divide, woh nahi karte, they fly the aircraft. So they have a pressure instrument. But the people who are at the airport, they want to communicate with the pilot that the density altitude is 2 kilometers, that means this is a very hot, very you know, very less dense place. So your landing distances will change.

So, the density altitude is mostly used by operators, airport people, etc. So what is it, pressure altitude corrected for non-standard temperature. So for that there is a chart, so just to tell you how it is read, so what you do is you say okay, I am flying at some particular pressure altitude. Let us say I am flying at this pressure altitude, okay 9000 feet. And the temperature outside is not the temperature which is at 9000 feet under ISA. The pressure temperature outside is say 38 degrees.

So, what I will do is I will say okay the OAT, now the pilots can measure OAT right, because they can have a thermometer kind of a probe. So the pilot will say okay 38 degrees is where I am, I go to 9000 feet, okay. And then I can find out what is the approximate density altitude by going along this line. So, I will show you how this is read, there is a video which shows you. I will talk about that.



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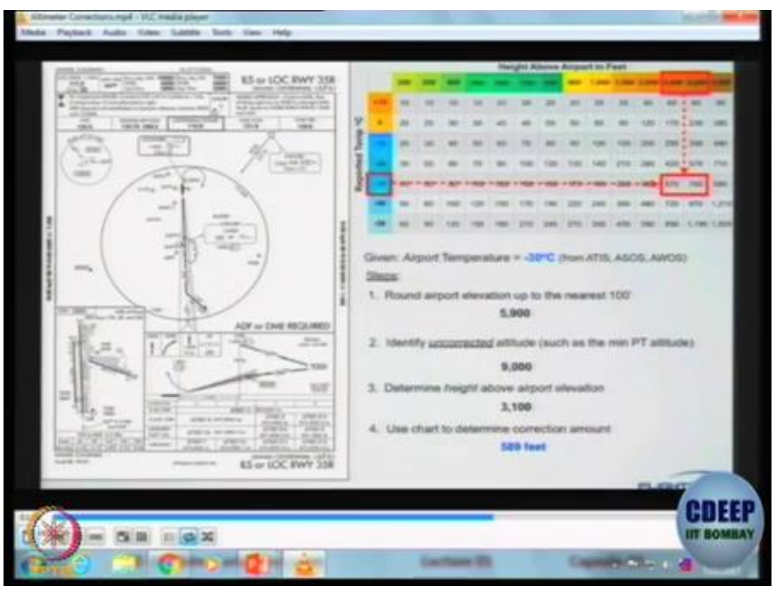
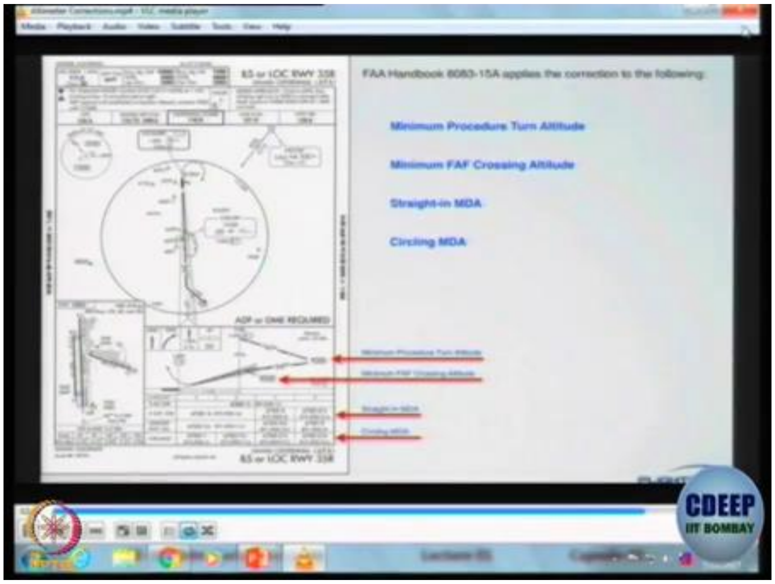
And then we also have temperature altitude, which is temperature based. If your temperature when you fly is 15 degrees centigrade, you are at sea level temperature altitude, okay. So, let us see now, let us see how this correction is carried out, okay. Now look at the cold weather, if you are flying in a cold weather, the true altitude may be lower than what is indicated, okay. You may get a false feeling that I am at high altitude.

If the temperature is colder than ISA, you will have this problem. So, what will happen, you will think you are at high altitude but actually you are at low, you will go and hit an obstacle. Very dangerous. So therefore, the pilots have to correct the altimeter readings for a non-standard temperature. So, this is how they do, okay. They use a chart or they use, nowadays they use a software. So, this is a typical airport chart, I will talk about it sometime later.





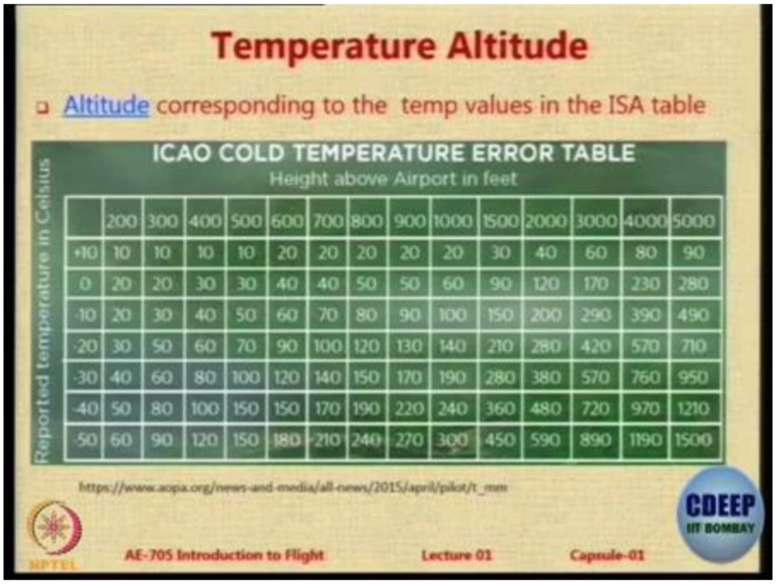
So, it shows here that the elevation of the airport is 5885 feet, that means this is in feet, right. So you are, you find the altitude of the runway. Then identify the correct uncorrected altitude which is 9000. This information comes also from the chart, see it is written here 9000, so this will tell you the height above the airport that is 3100 feet. I will just pause it here to explain to you. If some people are confused I will just tell you what it is.

This is basically a standard chart. This is for this is for Denver airport for example. Now I conduct a course on air traffic management or it is a part of the course on air transportation. There we go into more detail on how to read this chart, how to plan the routine, I will spend little bit time to tell you what it is. This number tells you the elevation 5885, that is the altitude at which the airport is located and this one tells you at what height you have to come in, when you come in to operate.

So, you should be flying at 9000 and the airport is at 5900 you just round it upto 100 feet. So you get 3100, that means the pilot knows I am atleast, I am roughly above the ground at 3100 feet. Now, we also look at at what temperature you are. So, you have temperature minus 30 degree centigrade. You have height 3100 between this two. So, then you are going to read out. You are going to just draw a line between 3100, 3000 and 4000 and minus 30 so it is between 570 and 760, this is just do the correction.

So, it is 589 feet which means, the error is 589 feet for this operating condition and with that, so this is the corrected altitude. In other words, because you are operating at a lower temperature minus 30 degree centigrade, you have to be, so this is the minimum procedure turn altitude, minimum PTA it is called and this is a final approach fixed crossing altitude. These are all the detail which we can probably skip right now, okay.

So, this is how it is done, you have a chart, where you have a reported temperature on the Y-axis



	200	300	400	500	600	700	800	900	1000	1500	2000	3000	4000	5000
+10	10	10	10	10	20	20	20	20	20	30	40	60	80	90
0	20	20	30	30	40	40	50	50	60	90	120	170	230	280
-10	20	30	40	50	60	70	80	90	100	150	200	290	390	490
-20	30	50	60	70	90	100	120	130	140	210	280	420	570	710
-30	40	60	80	100	120	140	150	170	190	280	380	570	760	950
-40	50	80	100	150	150	170	190	220	240	360	480	720	970	1210
-50	60	90	120	150	180	210	240	270	300	450	590	890	1190	1500

and on the X-axis you have the height above the airport, corrected between the height at which you are allowed to come in and the height of the airport rounded upto 100 feet. So, so much above ground level and so much temperature, it gives you what is the error and these charts are either digitally available or available in the form of hard copies which are used by the pilots to interpret it. Yeah?

Precisely, and that affects the pressure, that affects the measurement, precise that is the reason. So, what are the important points to be kept in mind? Whenever you refer to the tables, ISA tables, you should refer geometric altitude.

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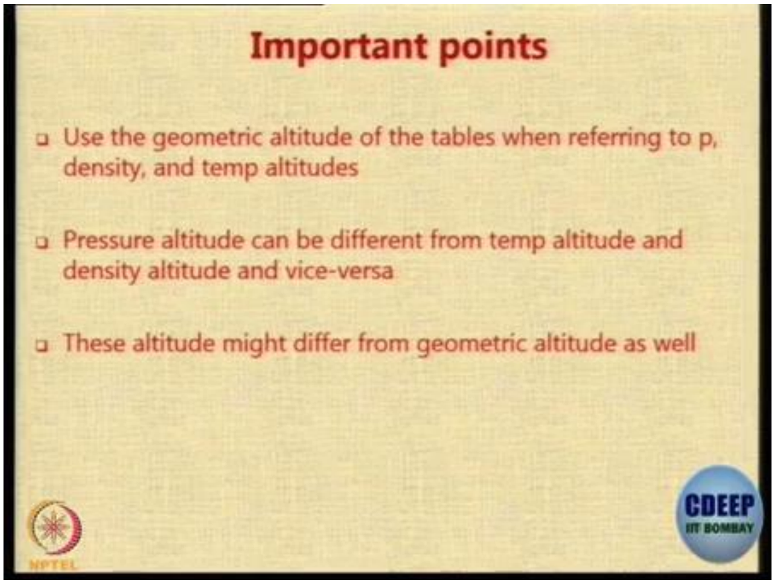

Whenever you refer pressure, density and temperature altitudes, you must use the geometrical altitudes. Secondly, there could be a situation when the aircraft is flying at a pressure altitude of 2 kilometers, density altitude of 3 kilometers and temperature altitude of 4 kilometers, all at the same time. Do you agree it is possible? So it is possible, because the references for them are the three parameters. So, this is very important, you can be flying at many different altitudes.

And of course, these altitudes may not be same as geometric altitude because the temperature, pressure and density do not follow ISA. They have their own wills and fancies, so you have to be careful.