Introduction to Flight
Professor Raj Kumar S. Pant
Department of Aerospace Engineering
Indian Institute of Technology, Bombay
Lecture 06.1
Critical Mach Number

Welcome to capsule number five, in this capsule we are going to look at some terms related to wings. So now we move from aerofoils to wings and today we still look at aerofoils but talk about critical Mach number wave drag on swept wings. In the next lecture we are going to look at finite wings and I think drag effects ok. So, let us have a quick overview of what we have in store today.

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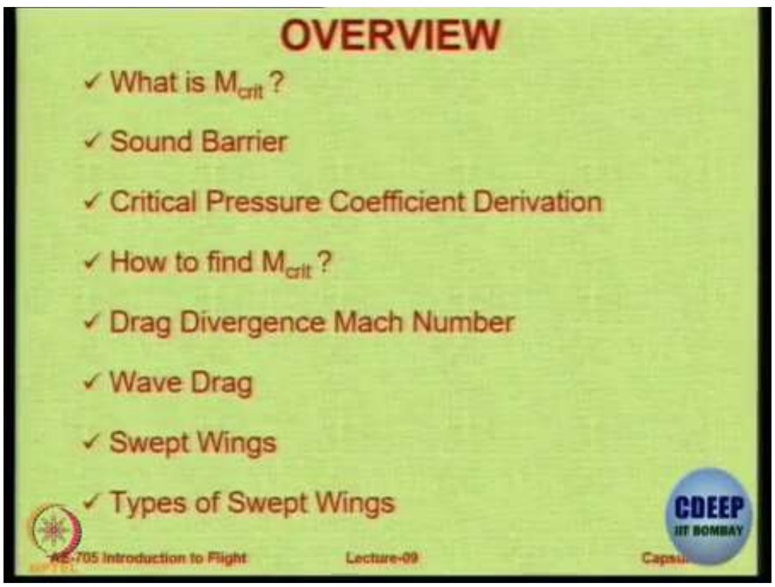

So, there are a series of concepts to be covered mostly to do with critical Mach number and the drag divergence Mach number, and then we look at some types of wings which are available to try and give us better control over wave drag. So what is M critical? Ok, so M critical there must be something critical because of the name itself. So to see what is M critical we will watch a short video.

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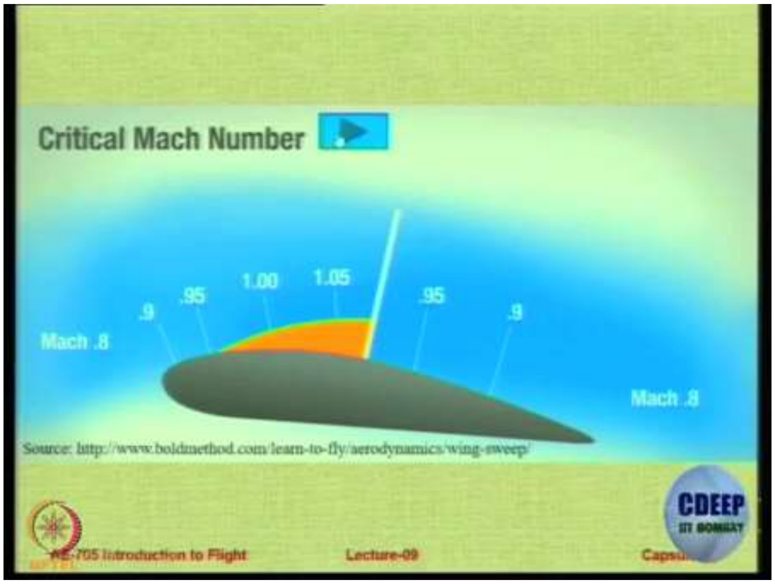

Video: What actually happens to the air and the aircraft when approaching the speed of sound?

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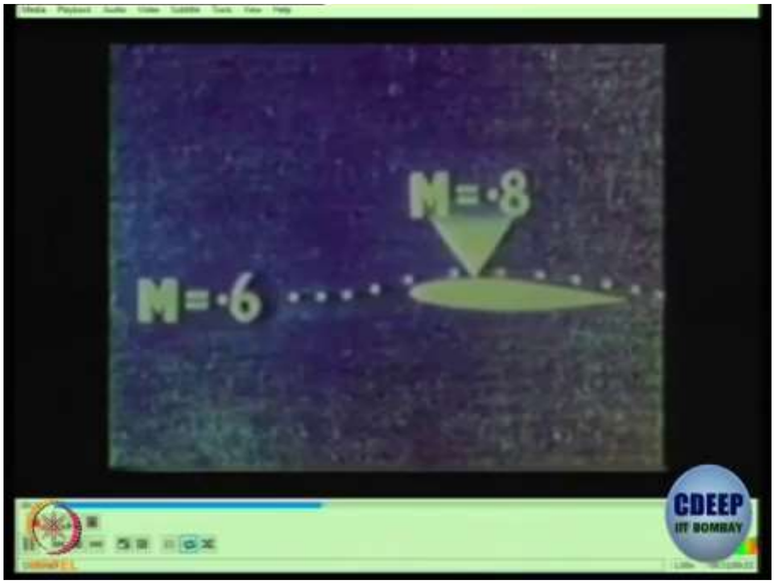

The air flow speeds up as the air passes over the wing and reaches the maximum speed at certain point on the wing. So, the Mach number of the air at this point is greater than that of the aircraft as a whole.

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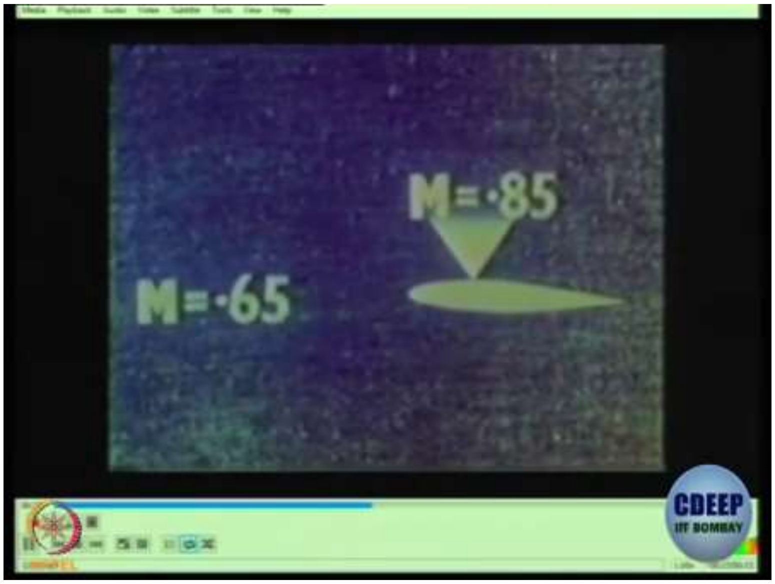

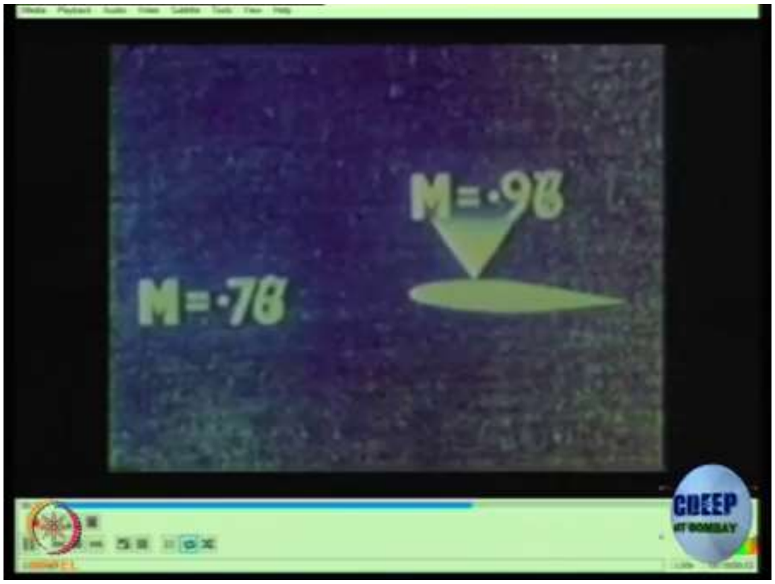

As the aircraft speed increases so does the local Mach number at this point on the wing. Eventually, just at this point on the wing the air reaches the speed of sound.

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Although the aircraft as a whole is at the lower Mach number. The aircrafts Mach number when this happens is called its critical Mach number, usually written M CRIT.

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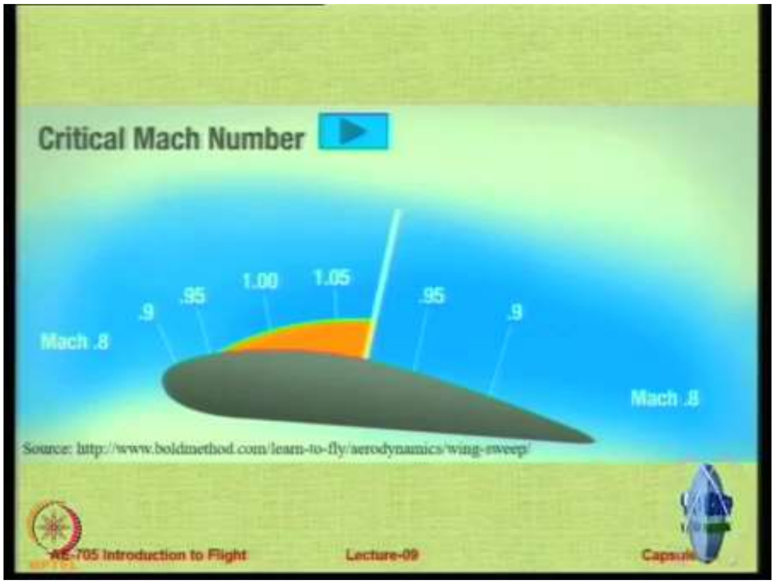

Ok, so this is very clear because of the angle of attack or curvature or camber, there is going to be acceleration of fluid flow both above and below the aerofoil. So, the local Mach number is going to increase from the free stream Mach number. There will be some free stream Mach number at which sonic conditions are first reached anywhere on the aerofoil , remember it could be upper surface, it could be lower surface, it could be front, it could be behind. The lowest free stream

Mach number at which sonic conditions are first reached anywhere on the aerofoil is called as the critical Mach number.

But the question is why is it critical? It is critical because, it tells you that from now onwards there is going to be some portion on the aerofoil which will have sonic flow, Mach number more than 1 locally. Sonic flow is no problem except the fact that it results in a shock wave and across shock wave there are serious problems okay. So that Mach number is the critical Mach number and the pressure coefficient value. So the pressure coefficient where you reach the sonic conditions first is called as the critical pressure coefficient or CP critical, the local V is 1, Mach number 1 and V infinity is critical Mach number and the place where it hits or the place where the sonic conditions start and the shock wave is presented that particular place is where we have the most negative pressure coefficient ok.

So, this free stream Mach number 0.8 is critical Mach number and this Mach number we would like it to be as high as possible so that we can fly faster and faster without encountering the problems of sonic flow ok. So, if there are two aerofoils, aerofoil A, aerofoil B, and aerofoil A has a higher critical Mach number, it is a better one because, it allows you to travel faster without encountering. Now, when you have M equal to one you get a weak shock wave, ok. So, the actual value of M critical it varies from wing to wing depending on the wing profile, wing geometry. The two main parameters that affects the critical Mach number of the aerofoil are its camber and thickness.

Basically, camber and thickness are the ones that create acceleration, at a given angle of attack if you have a thicker aerofoil there will be larger acceleration, if you have more camber there will be higher acceleration. So, the thick aerofoil because it deflects more it is going to have a lower critical Mach number that means at a lower value of M you will have sonic conditions first reached ok. So, with this we come to the concept of something called as a sound barrier ok.

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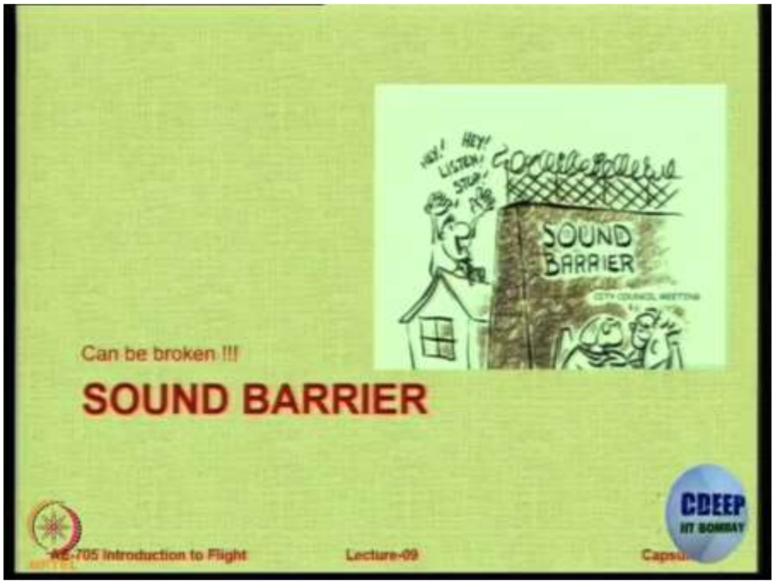

So, sound barrier is a barrier but it can be easily broken. Now, this was one of the very interesting misconceptions in early days of aviation. So, what used to happen is as the aircraft began flying faster and faster, earlier the maximum speed was limited by the power available of the power plant. They were mostly using turbo props, piston props so they were not able to fly faster than say Mach number 0.6, 0.5 etc. But when we started getting more powerful power plants and eventually the jet engine the thrust available became much larger, so aircraft could fly faster. So, they were able to fly Mach 0.8, 0.9, 0.92 and then pilots reported that as soon as you cross a particular value of Mach number below 1, the whole plane starts shaking, there are lots of vibrations and you just cannot fly faster than Mach number 1.

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So many people thought there is some kind of a physical barrier, speed of sound is a barrier which cannot be surmounted, but then some people said “Hey, we have bullets which fly faster than speed of sound and bullets are also flying objects.” If bullets can fly that means even sufficient force you should be able to fly. So, we knew that we can fly from anecdotal evidence of bullets, but we were just not able to fly. So for many-many years there was this so-called sound barrier and then it was finally broken in 1947 when Chuck Yegar flew an aircraft called BELLX1 for the first time at Mach number more than 1 so yes it can be broken and let us see what it is and why we call it as a barrier?

So, let us revisit the pressure coefficient to get an understanding. So, at very low speeds and when I say low speed, I talk about Mach number upto approximately 0.3. At these low speeds you can assume Cp to be constant ok. So Cp will be equal to some Cp called Cp knot, does not change with Mach number. Now, when you go beyond Mach 0.3 or so and up to around Mach 0.7, remember I am not using absolute values I am saying approximately.

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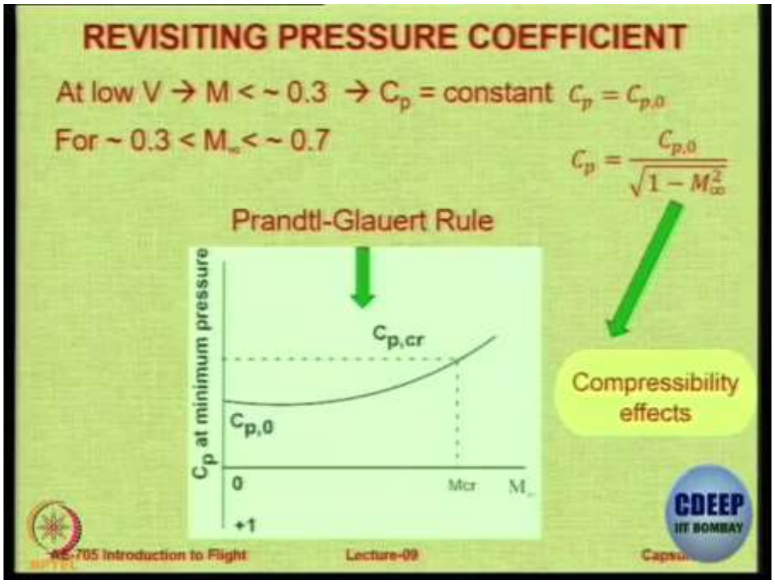

During these conditions, the Cp starts changing it starts actually reducing and the formula which is called as the Prandlt Glauert rule is a very simple formula that correlates the Cp at any Mach number with Cp at very low Mach numbers and this is because of the compressibility effect. This change in Cp occurs because of compressibility effects and actually if you plot the value of Cp versus Mach number you will see that it is almost constant Cp zero and then it starts increasing and there is 1 Mach number called M critical at which Cp is Cp critical and it keeps on increasing further. But actually the rate of change or the rate of increase in Cp will not be like this, it will actually be little bit more non linear after little bit after Cp critical. Remember, you cannot apply this formula in supersonic flow where M infinity going to be more than 1. In fact, interestingly you just reverse and call it as $\sqrt{M^2-1}$, ok.

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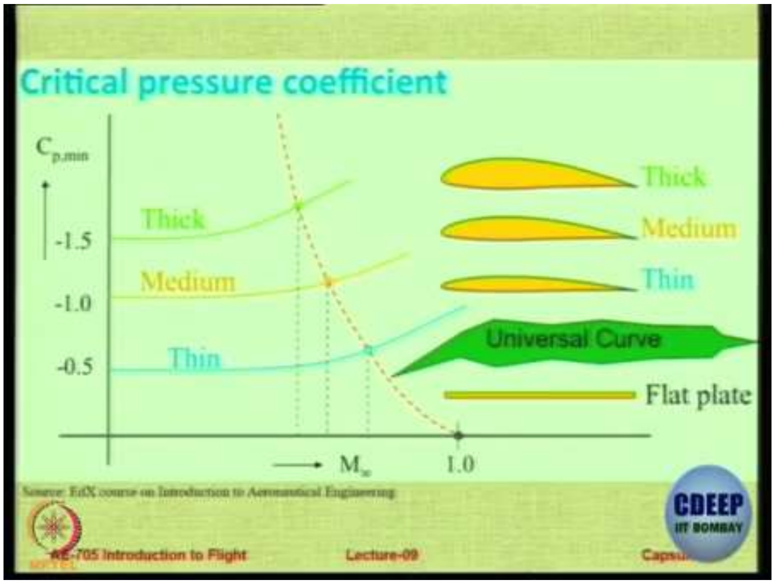

So, let us see the critical pressure coefficient so if it is a thin aerofoil you can fly a longer a higher Mach number in the free stream to achieve the critical Mach number. If it is medium it is lesser, if it is thick then it will start increasing. So therefore, that is a line which will give you locus of the location of the critical Mach number. And this line is a universal curve, it does not depend upon it does not change with whether it is a thick aerofoil or a thin aerofoil in the sense that there is a curve which can be applied to almost any geometry. So, this line is available, if you have the geometrical data, ok you can get the value the value of Cp will change depending on thin, medium and thick and it will increase as per the Prandtl Glauert rule and it will reach the critical value at some particular point, so you can apply this uniformly ok.

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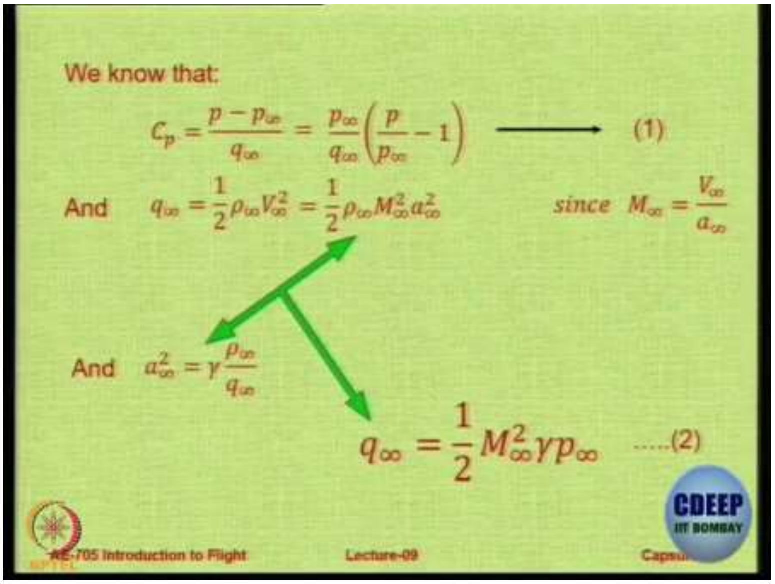

So, let us see this we will just you little bit of maths to get the value of CP critical. So, we know that the pressure coefficient is defined as the difference of pressure between the local pressure and the free stream pressure, non dimensionalized by free stream dynamic pressure q infinity ok this is just revisiting last times one. So now what I do is, I do some mathematical jugglery so that I can get $\frac{P_\infty}{q_\infty}$ outside so its ratio of $\frac{P}{P_\infty} - 1$, this is our first equation.

$C_p = \frac{P_\infty}{q_\infty}\left(\frac{P}{P_\infty} - 1\right)$, okay. We also know that dynamic pressure is defined as $q = \frac{1}{2}\rho V^2$, therefore $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$.

Now since Mach number, $M = \frac{V_\infty}{a_\infty}$,local Mach number therefore, you can reproduce or replace and say that $q_\infty = \frac{1}{2}\rho_\infty M_\infty^2 a_\infty^2$. Now you just plug it in that equation, so if you define Gamma is the ratio of specific heats so the sonic speed is defined as $a_\infty^2 = \gamma \frac{\rho}{q}$. So, from these three equations you can get $q_\infty = \frac{1}{2} M_\infty^2 \gamma P_\infty$, okay.

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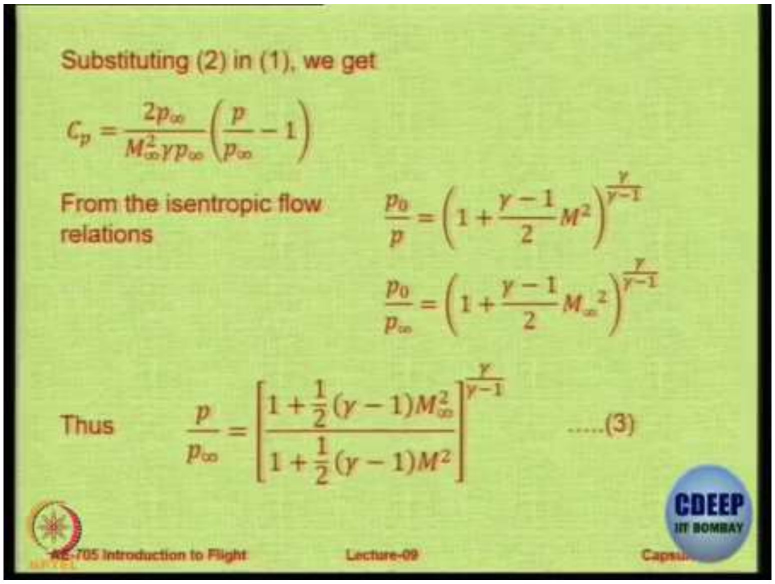

So we substitute that in the previous expression. Now you will explain this better to yourself when you derive it yourself. I am going to upload the slide so you will be able to derive it and then you will be able to get a feel much better. It is just a simpler replacement. But now

$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$ or $\frac{P_0}{P_\infty} = \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)^{\frac{\gamma}{\gamma-1}}$ for isentropic flow relation. So now we have to assume if the flow is isentropic then replace inside now, so P by P infinity will be what? It will be P by P infinity will be just replace M by M infinity okay. So, therefore you put it inside the expression take the ratio of P and P infinity and put it inside.

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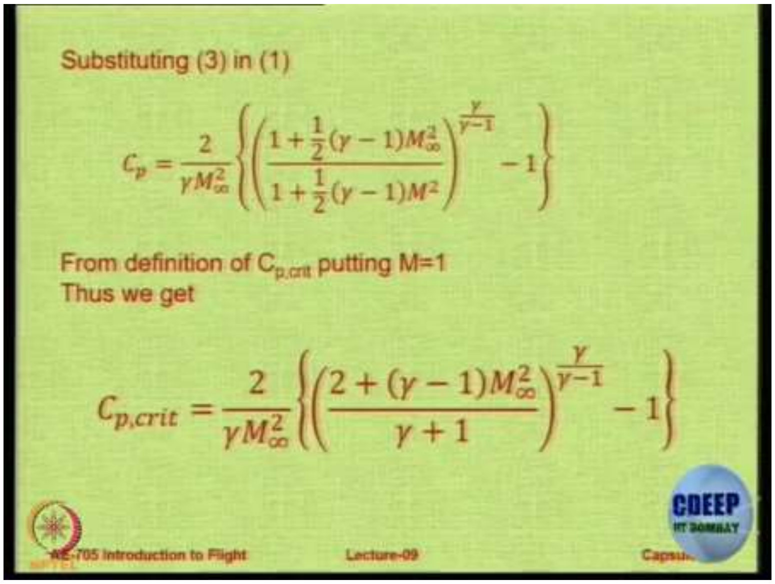

Finally if you define Cp equal to Cp critical by M infinity equal to 1 okay. So, you can get

$$C_{P,crit} = \frac{2}{\gamma M_\infty^2}\left\{\left(\frac{2+(\gamma-1)M_\infty^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right\}$$

where the two conditions M is at the point where you reach critical Mach number and that point M is equal to 1 by definitions so therefore, you can get an expression. So this is a neat expression it only contains Gamma which is a gas constant unless the gas dissociates and Gamma changes from 1.3 to some other number, it is valid and that is true in isentropic flow so all you need is M infinity. So now where is the aerofoil in this where is the thickness, where is the camber? Nothing, it is just a function of M infinity, so you can call it has a universal expression for Cp critical ok. Just put the value of M infinity and you will get cp critical.

Remember it is only Cp critical not M critical, just the value of Cp. So, the value of pressure coefficient at critical Mach number correct, or at the place where sonic conditions are first reached is always the same, it is just a function of M infinity. If your critical Mach number is higher M infinity is higher, so Cp critical is going to be got by this expression. So, as M infinity becomes M critical as M equals to 1 so it is a straight forward expression.

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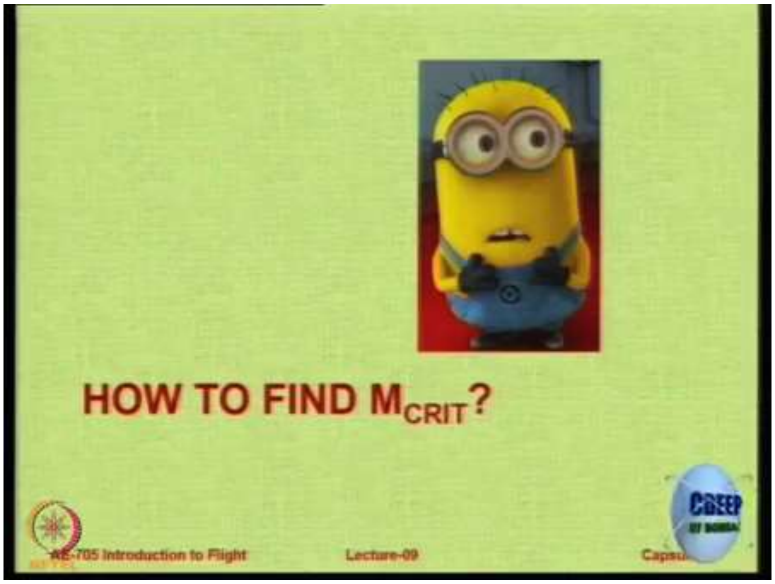

Now, problem is how do you get M critical? How do you know at what free stream Mach number, the flow will accelerate so that sonic conditions are first reached because at a velocity higher than M critical there will be larger areas of flow exposed to sonic flow. So what is the point at which it happens for the first time for that we need a method? So there are two methods which are available, we will discuss both the methods, one is a simple graphical method the other is using the equation which we just now derived.

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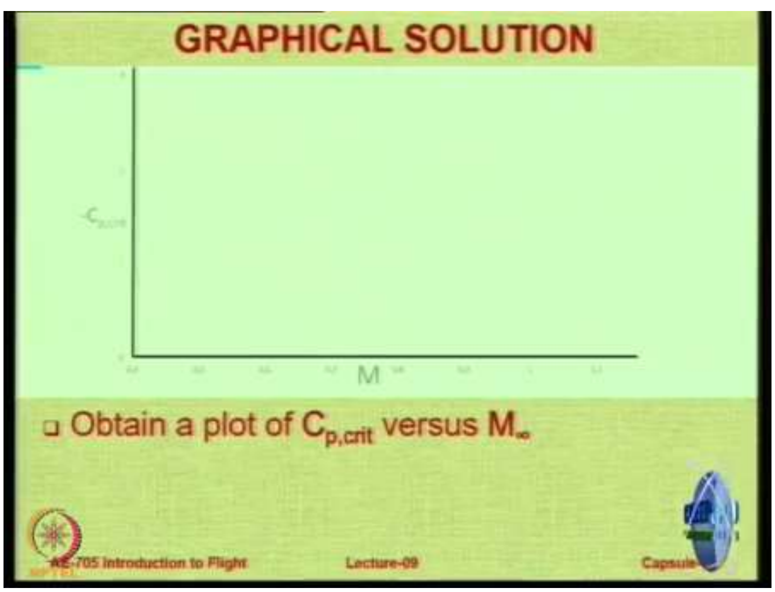

So, graphical method is actually very elegant and simple. On the Y-axis you plot Cp critical, on the X-axis you plot Mach number okay. So, obtain a plot of Cp critical versus Mach number. How do you get this plot? Yes.

Student: From the derived equation.

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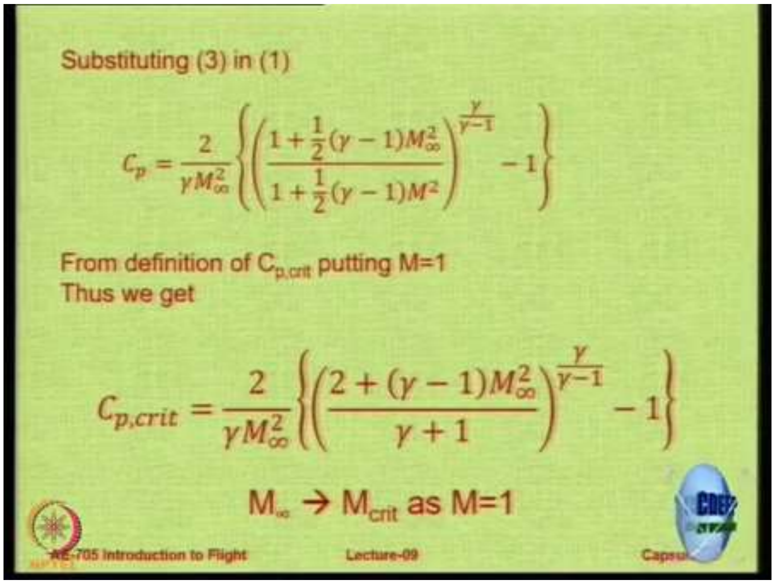

Professor: Exactly, are you guys sleeping? From the equation that we just derived. What is the equation, I will show you again. From this equation I just now told you that Cp critical is a function purely of M infinity for any aerofoil, so you have this expression just put the value. So using this expression you can get the plot it is a simple plot okay, it is a quadratic plot because there is M infinity square.

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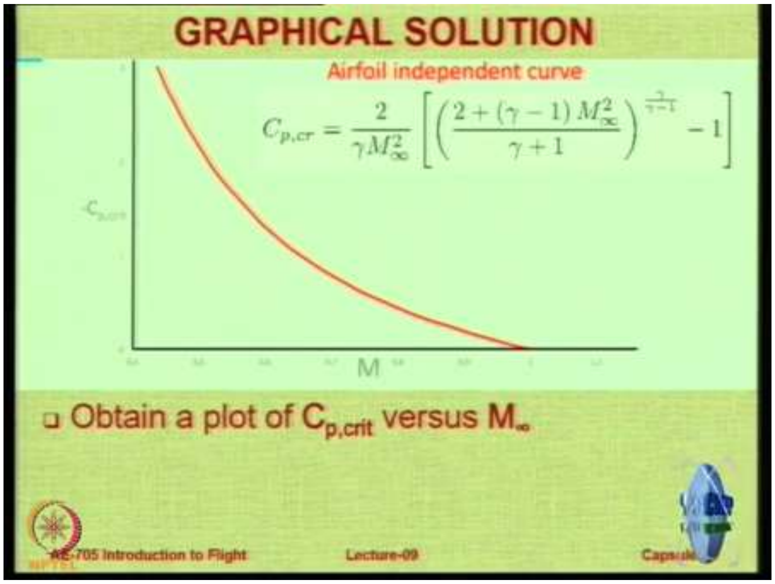

Okay no questions? Now the next is obtain the value of Cp knot that means Cp at lower Mach number. Now usually this is given, usually it is available from the aerofoil data, but suppose you do not know okay. Suppose you do not know, then you will have to do a guess, then you plot C p versus M infinity from the Prandtl Glauert rule.

So, what you do? You get the Cp knot which means you get the value at which you cut the y-axis, because the Cp knot is going to be a function, it will not be the same for all aerofoils. So, Cp knot has to be got from either experimental data or from some online calculator or some way you have to get Cp knot. But how does Cp vary with M infinity, from Cp knot is by the Prandlt Glauert rule so you will get one more line like this. Now at the intersection of this you can get the value of Cp critical, because the red line is a locus of Cp critical for all wings or aerofoils which fly at a particular Mach number and the blue line is the locus or the value of variation of Cp with Mach number for a particular aerofoil starting from a value which is the intersection on the y-axis, wherever they intersect that is the critical Mach number, this is one way of doing it. There is also a method to do the whole thing mathematically, which means just solve these two equations simultaneously okay.

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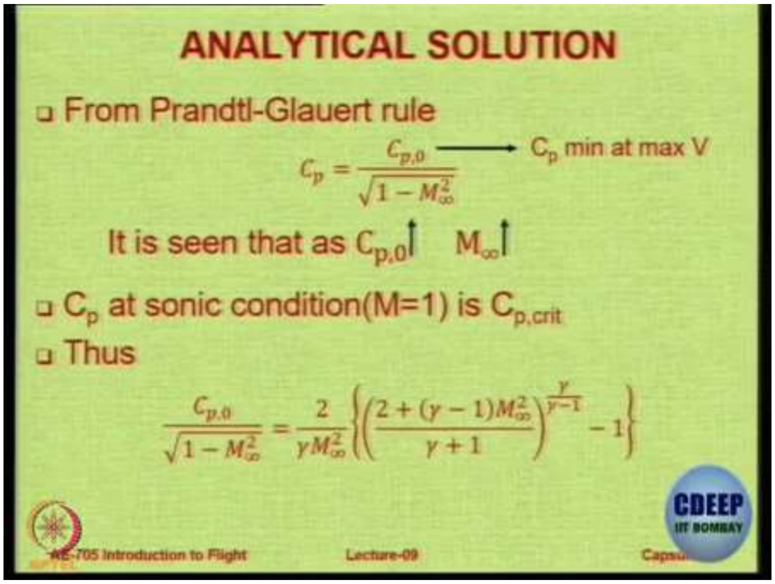

So, what you can do, you can say okay this is a Prandtl Glauert rule and you can notice here that Cp knot is going to increase as M infinity increases or as Cp knot increases the M infinity will increases or vice versa actually, so at sonic conditions so you just equate them at M equal to 1 and by equating them you can actually solve and get the expression, so it is the same thing either you do it graphically or you do it by simultaneous equation.

Now some people have this misconception that it is very easy to get a critical Mach number location. Just look at the place where the thickness of the aerofoil is maximum, that will be the place where you will have the maximum acceleration so that is the place where the Mach number will be equal to 1. So as the free stream Mach number increases, sonic conditions will first be reached at the maximum thickness point, correct. So many people say it is very simple, it is not true, I will show you an example.

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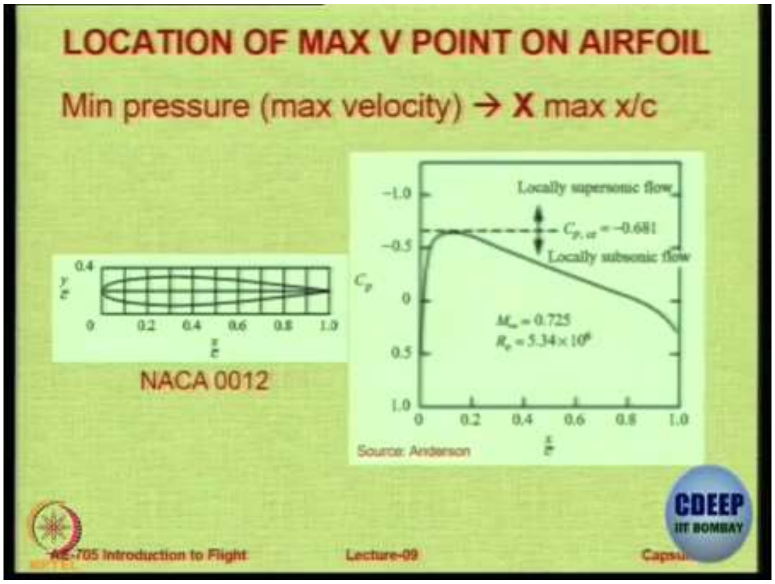

For example, look at this aerofoil okay. This is a simple NACA 0012 symmetric aerofoil 12 percent thick, so you notice that the Cp is maximum at X by c equal to 0.11 that means at 11 percent location of the chord from the leading edge the Cp is maximum, but thickness is not maximum at 11 percent, the thickness is maximum at 30 percent. So even though the maximum thickness occurs at thirty percent of the chord, the highest Cp has already been achieved at an upstream location. So do not have this misconception that the location of the maximum thickness is the location where the sonic conditions are first reached or where the location where the cp is maximum so it depends on local acceleration. You have already drawn these plots last time in the tutorial okay so you have to be careful

So now if you want to know more about why this happens, I would encourage you to go to the main source Anderson which is the basic textbook we are following and it is nicely explained why this happens so there so that is a self-study for you, I do not want to talk about it more here so they are not corresponding and this is a very interesting observation. So, to get the value to get the location where the velocity is maximum you actually have to look at the complete geometry of the aerofoil and not just the maximum thickness location. Okay so critical Mach number is a Mach number at which sonic conditions are first reached.

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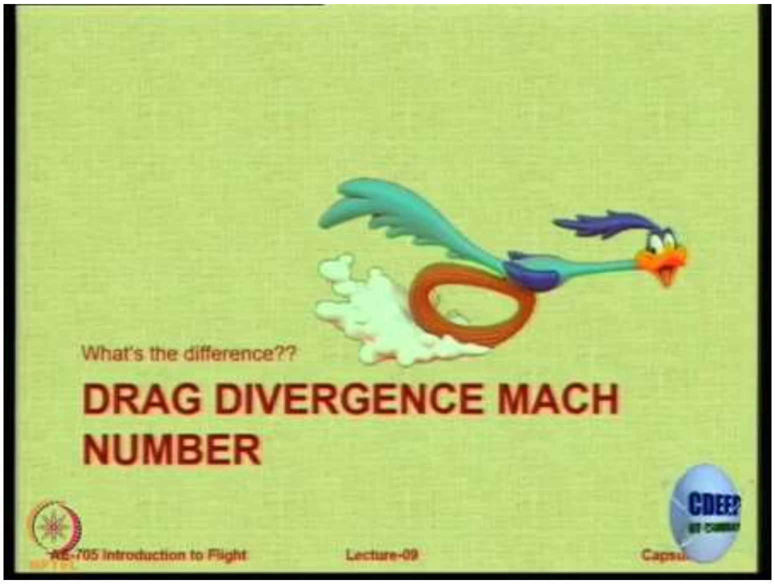

Why do we care? That is not what we really care. The designers or the pilots are mostly interested in this particular Mach number which is called as the MDD. The drag divergence Mach number and the name itself tells the whole story. It is a Mach number at which there is a shooting up of drag and that is the reason why people thought there is a sonic barrier. Now my question to you is, do you think that the drag of an aerofoil will start shooting up at a Mach number where at a critical Mach number or will it be before it or will it be after it. What do you think? At what free stream Mach number will you have excessively higher drag built up, before M critical, at M critical, after M critical or none of the above we have a fourth choice also. What do you think?

Student: After.

Professor: Why after why not at M critical? Why not at M critical? Because at M critical you have M equal to 1 which results in a weak wave, a weak shock wave and a weak shock wave will not give you the maximum drag. As the Mach number goes slightly beyond this M critical you start getting higher drag, okay.

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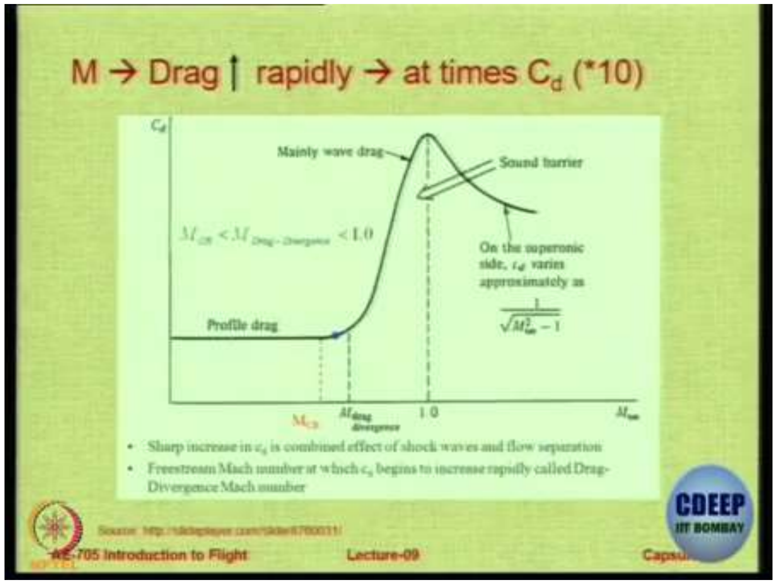

So, the Mach number at which drag shoots up, and shoots up means what rapidly, it may become 10 times the drag at M critical that Mach number is the critical Mach number but that is not the definition, okay. The definition is the point at which the graph suddenly undergoes a change in the slope and this particular sharp increase occurs not only because of shock waves but also because of the flow separation that is induced by the shock wave. So typically this drag divergence Mach number will be M 0.02 beyond M critical typically. So, if the critical Mach number is 0.83 it will be approximately 0.85 approximately it can change slightly also depending on the geometry of the aerofoil.

But generally, it is 0.02 Mach number, so slightly beyond. So sonic conditions appear and then very soon you start getting a fantastic increase in the drag. that is the problem. As I said, shock wave is one reason but that is not the only reason. Shock wave is only a generator of problems, it is not the problem in itself because of shock wave very soon you will experience flow separation and that comes because of the adverse pressure gradient, okay. But there could be a weak shock wave which may not lead to flow separation, so therefore it will create additional drag but not make it so bad that the flow separates okay. So, across a shock wave we know that the pressure is going to increase and the velocity is going to decrease.