Introduction to Flight
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Lecture No 48
Turning Flight Equations

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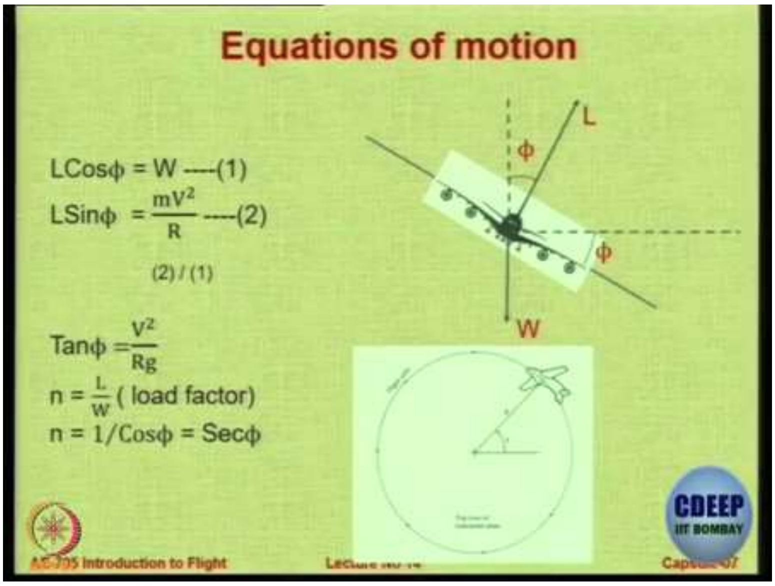

Let us look at some of the turning flight equations. So, here we have an aircraft. This is just a sketch, so do not assume that the aircraft fly like this which is at an angle $\emptyset$. So, it is banking at an angle $\emptyset$. So, the lift force is going to be perpendicular to its reference line, a component of that is going to overcome the weight and the remaining component is going to give you the centrifugal force.

So, $L\,cos\emptyset = W$, that is our first equation and $L\,sin\emptyset = \frac{mV^2}{R}$, which is the centripetal acceleration force acting on it, centrifugal force acting on it. So, we take a ratio of these 2 by 1, so you will get $tan\emptyset = \frac{V^2}{gR}$. So, if you want the angle $\emptyset$ to be small, then let your speed be small or let your R be large. This is what is there in transports. In transport aircraft, we go for very long turn because banking it by a large angle can cause discomfort to passengers.

So, there the value of phi is small but a military aircraft either want to turn very fast, they want to have high V in turn or they would like to have very small turn radius, so they have to bank it larger. So, for a given speed, angle of bank is directly connected to radius of turn or I would say inversely proportional to the radius of turn and the ratio of L/W is going to be the load factor. Now, this need not to be equal to the 1 in turning flight, okay? Actually, this will be equal to the secant of $\emptyset$, $^1/_{cos\emptyset}$. So, larger your turning speed or smaller turning radius, larger is the angle of bank and angle of bank directly gives you the value. So, if $\emptyset$ is 60 degrees, n is equal to 2, straight away!

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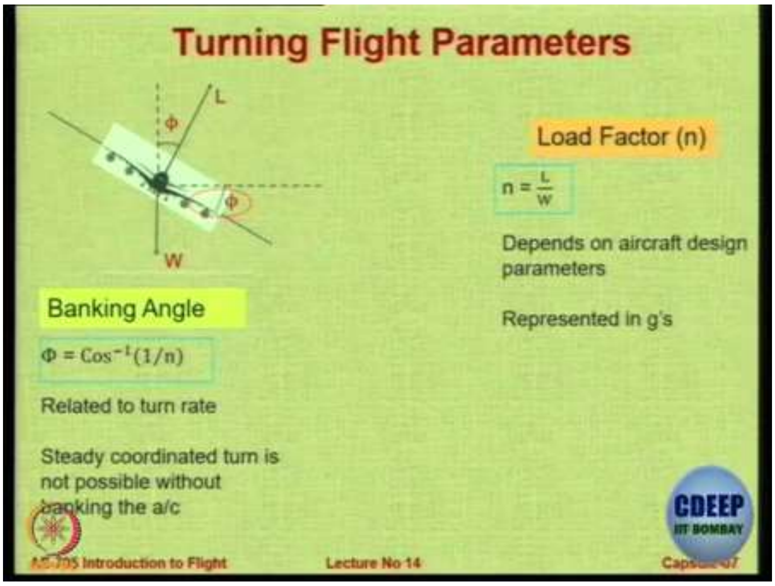

So, the banking angle is very important parameter, it is $cos^{-1}\left(\frac{1}{n}\right)$. So, you will not be able to, unless you have thrust vectoring, you will not be able to turn like this, you always have to bank to turn, okay? If you have direct thrust vectoring, then you can do, you are flying like this and now you have a thrust vectoring, so you can do this and then you can do this. That is not the topic of our discussion, we are looking at a conventional aircraft which does not have thrust vectoring, okay? And that too thrust vectoring normally is only in the vertical horizontal plane, side thrust vectoring is normally not provided in aircraft.

But there are some other aerospace vehicles for example in airship it is very common to provide a direct side force, in which case no banking is needed. You can go this way and then you can go this way, okay? So, load factor depends upon aircraft design parameters. Higher load factor means more load on the structure, the wing has to carry n times the weight. So, imagine, if W is 50 tons and if n is equal to five, that means you have to design the structure to carry 250 tons. That leads to you weight penalty.

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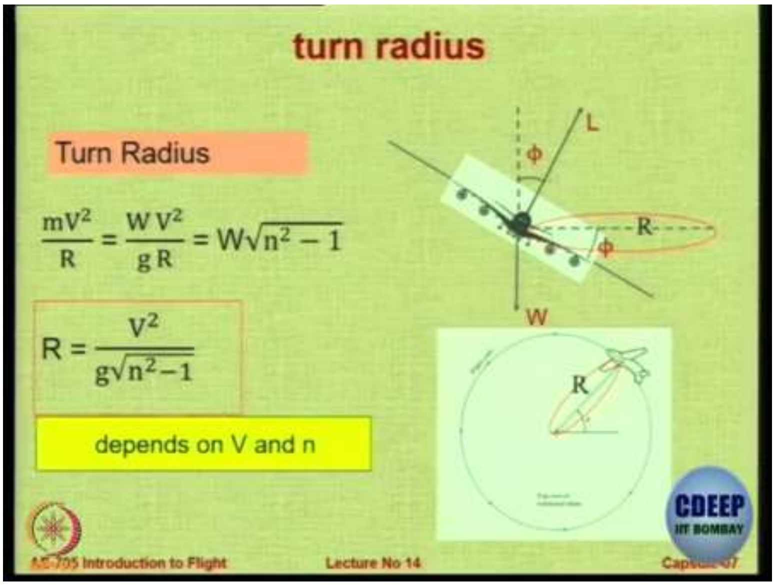

And normally the load factor is spoken in terms g(s), how many times the weight. So, 2g 3g 5g this is not telecommunication g. Yes?

Student: Actually I do not know what is thrust vectoring, that you said.

Professor: Thrust vectoring is word in English, thrust vectoring, thrust can be vectored.

Student: Means

Professor: Means thrust normally

Student: Means we are producing additional thrust in the opposite direction or

Professor: Thrust is basically a force produced by the engine, which is normally on the back of the engine.

Student: yes

Professor: Now suppose if I tilt the engine, then that force I am vectoring it, that is called thrust vectoring.

Student: But sir how can we provide this in a perpendicular direction to the flight?

Professor: By putting a special engine or any other force or any other device that produces force in that direction. That is why I said normally you do not see it in aircraft. That is why I dismissed it. I mention it. But if you have, theoretically speaking, if you have an engine mounted, let us say on the rear of the fuselage and now you can swivel the engine in any direction, suppose then you can provide thrust at any angle you want. But direct side force as I said is not in all very common in any aircraft.

So, that is why it is not relevant to our discussion right now. So, when I say that an aircraft cannot turn horizontally without banking, I want to be sure that I do not make it a statement true for every aircraft, so that is why I said unless you can vector the thrust. If you can vector the thrust then you can make it move this way, this way, this way. So, a helicopter for example, it can do a turn like this, because there you can tilt the blades so the helicopter is stationary, tilted it goes this way, okay? That is the point I was making. But good, it is good that you pointed out.

Such kind of interruptions are helpful for me, because I may assume as somebody mentioned on the moodle page, also in the feedback, that there are some terms which are not commonly known to the non-aerospace engineer. So, it is very good to interrupt me, then I can qualify my statements or make them more apparent and clear to other people, okay? So, thanks a lot for interrupting, right. So, let us look at term radius now.

So, this force $\frac{mV^2}{R}$, m can be replaced by W upon g, so it becomes $\frac{WV^2}{gR}$ and V, we have already seen, can be replaced. So, in other words, the radius of turn is directly proportional to the square of the velocity and inversely proportional to the $\sqrt{n^2 - 1}$, g being a constant. So, what does it tell us? It tells us that, if you want to have a tighter turn small R, either you fly at a higher speed, same aircraft flying at higher speed will give you a tighter turn or you can increase the load factor. So, when n is large then R is going to be small. So, just V and n are the two factors that define the radius of turn and that is the turn radius.

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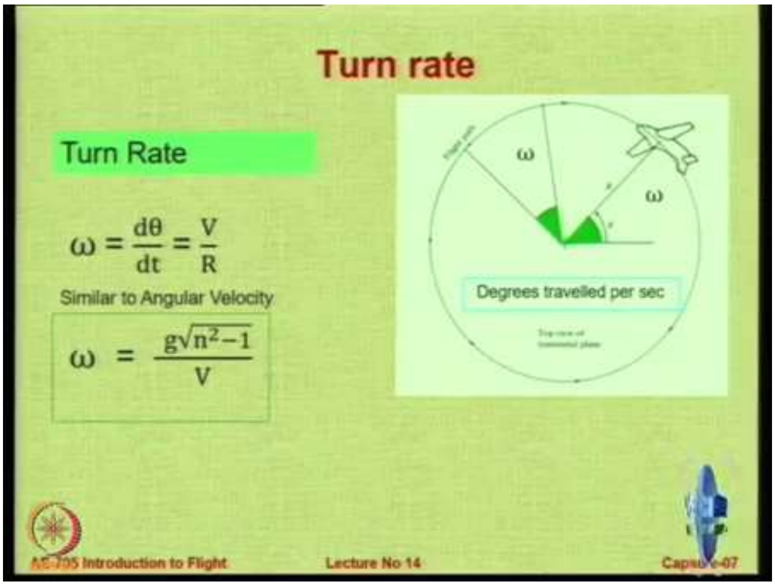

The radius at which you are turning, so that is, this particular definition is for the tightest turn or a shortest radius. The other aspect that we need, is called as a turn rate. That means when you are trying to close in to an enemy, you would like to have larger value of, change in this omega, d omega by dt. You want it to be larger. So, turning a smaller radius is meaningless, if you are not turning fast enough. So, degrees travelled per second, is the turn rate. Omega, which is d theta by dt, so that will be V by R. So, omega will be $\omega = \frac{g\sqrt{n^2-1}}{V}$, because as it divided V by R. So here, this is proportional to n, n square minus 1 under root and inversely proportional to V.

So, if you want to turn at a faster rate, then you have to be at a lower velocity. This is an interesting thing. Load factor is one aspect, but load factor you cannot change dramatically because that affects the structure. So, there may be a limit, what is the value of n max? It could be 6, it could be 8, it could be 9, that is it. Beyond that, we do not normally design the aircraft, okay? So, the load factor which they can take without any permanent deformation, is approximately 9 for the most agile fighter aircraft, you cannot go beyond that. But V is in your hands, however you cannot go below a V because then you will stall, okay? So, omega depends on V and n, R depends also on V and n both of them are interrelated, okay? Let us see the effect of bank angle. Now, the same aircraft, if you are banking at lower angle, you will have a larger radius, if you are banking at a smaller angle you will have a larger angle you will have a smaller radius. That is why, you saw the video that I showed you, where the aircraft was into a very tight turn it was almost 90-degree bank, almost, okay? Fine?

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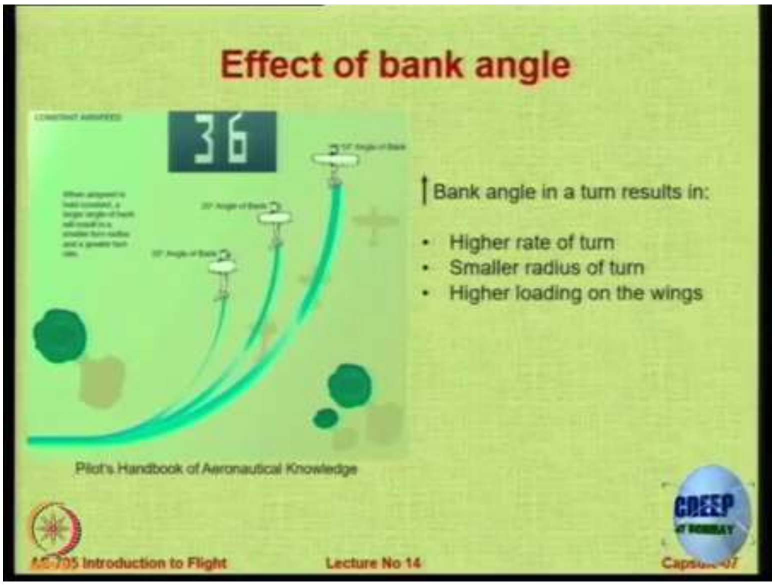

So, increasing the bank angle in the air turn it results in higher turn rate, it results in smaller radius of turn, but it also causes higher loading on the wings and stall speed also increases. But it is dangerous, because you will be stalling at a higher speed.

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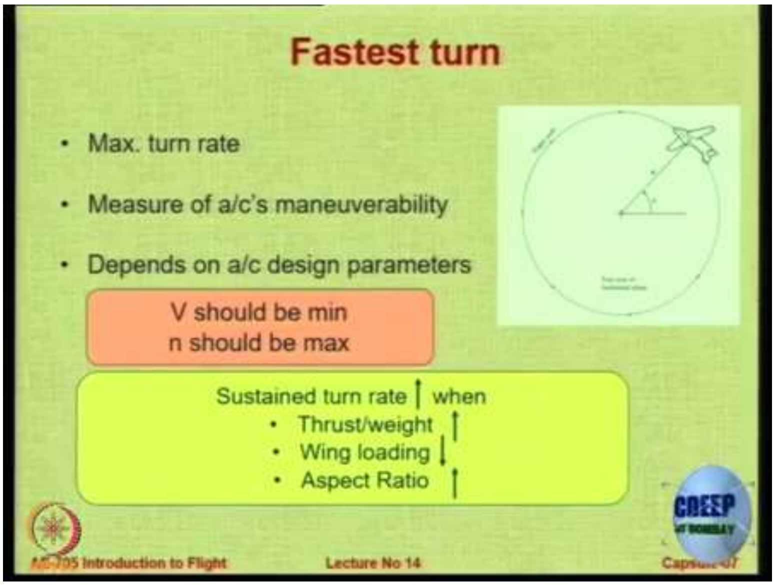

You are going to have higher stall speed, because lesser speed is now available for your overcoming the weight and the remaining is being consumed in taking care of the acceleration. So fastest turn will be the max turn rate. This is the measure of it is maneuverability of the aircraft and it depends upon V and n, okay? So, do you agree with this? V should be minimum and n should be maximum, if you want to have a fastest turn rate. Lower speed and higher n give you faster turn rate, higher speed and higher n give you tighter turn.

Now, there are 2 types of turns one is called as a sustain turn the other is called as an instantaneous turn. The sustained turn and instantaneous turn, the main difference is there's your operating condition remains same or you are allowed to change them. So, let us first take instantaneous turn. What this means is, you are flying at some speed, at some altitude and now you want to turn. And during this turn, you do not mind, if your altitude reduces or speed reduces or both reduce, if you do not mind this called as an instantaneous turn. And this is governed only by aerodynamic parameter, because you are allowing the aircraft to reduce in speed and reduce in the altitude. So, at any given instant.

So, can you tell me a scenario for a military pilot, where it will be helpful to have high instantaneous turn rate. Under what situation or for what application for a military aircraft, it will be useful to have a high instantaneous turn rate. Yes? To dodge missile, very good! Excellent answer! okay? Not in dog fights, that is a wrong answer. In dog fight, you need to have high sustain rate. Yes, to dodge the enemy missile! Because, at that time you do not mind losing speed or velocity you want save your skin basically. And you want to be away from missile.

So, at an instant you want to quickly turn. But if you are in the combat situation and if you are either pursuing an enemy or you are being pursued by somebody, at that time it is important that you do not loose velocity or altitude. Because in a dog fight situation, the one who is behind you and above is at an advantage, always! Always in dog fights, the pilots try to come behind the other guy and above the other guy, so that is the best condition to shoot. Now, if in this situation, you go for instantaneous turn, your velocity will reduce, you will become a sitting duck target. If your height drops, you will give a bigger advantage. So, in a dog fight, we are not interested to have, not interested to have instantaneous turn but sustain turn, okay.

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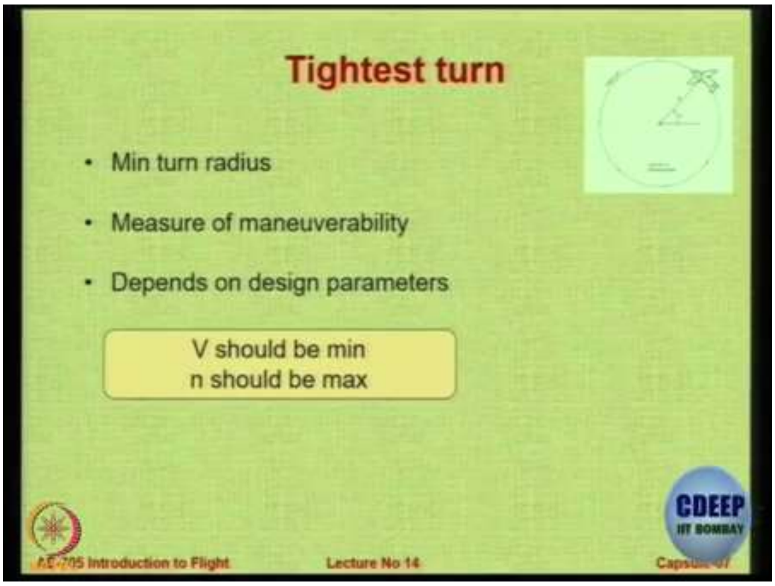

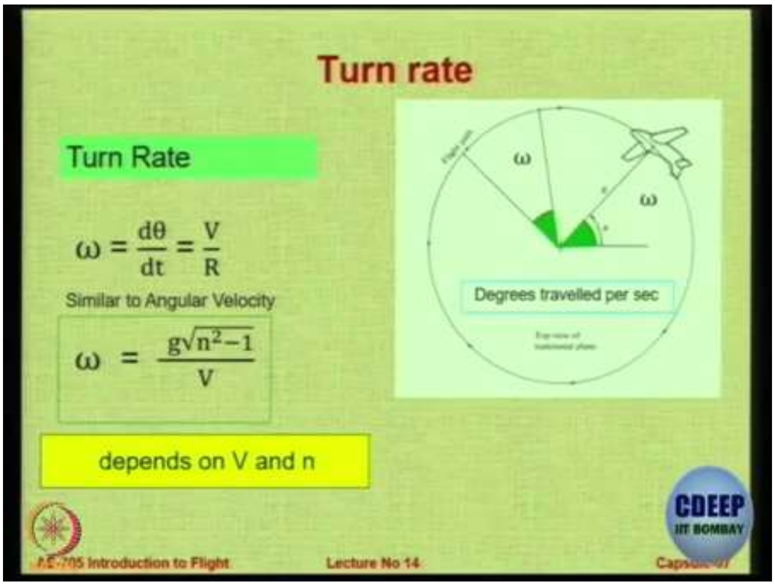

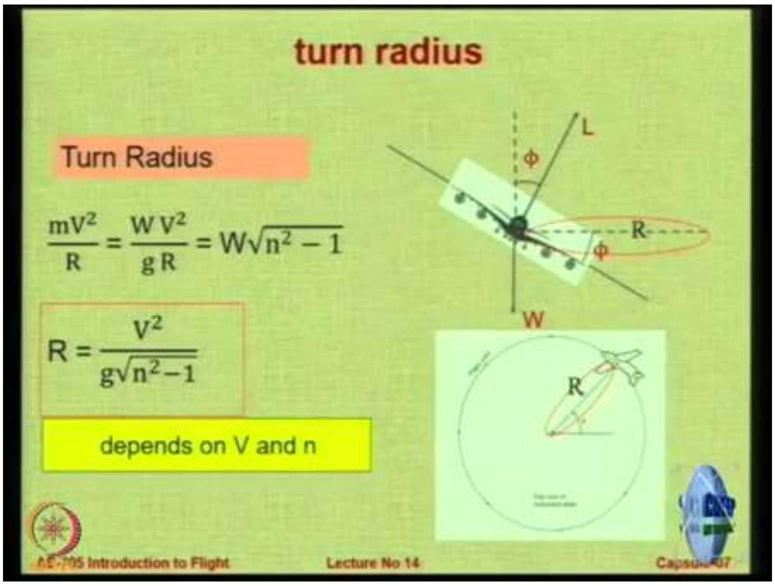

Let us also look at tightest turn. In this, you want to have minimum turn radius. Now, what could be the application for this in a military aircraft? When you would like to have a minimum turn radius? Rate is not important, radius is important. In which situation, yes?

Student-After completing the task.

Sir – Comeback? okay. So, after completing the task where you to comeback, that could be the one application. The other application will be to avoid any obstacle. Let us say, there is a mountain in front of you and you are now going at high speed, what you want to do? I do not want to turn fast, I want to turn at a shortest radius, so that I avoid the obstacle. So, this also a measure of maneuverability, okay? This also depends upon the design parameter, but here V should be maximum not minimum, correct?

V should be maximum and n should also be maximum. Is it correct or is it wrong? V should be minimum? In a tightest turn, with minimum turn radius, why is it so? Yeah. So, it is so the minimum the tightest turn. So, in both the cases you want the minimum speed, sure? Let us go back to the formulae. So, this is the turn rate. So, here you want V to be low, n to be high. This is turn radius, so in turn radius you want R to be less, so again you want V to be less and n to be high.

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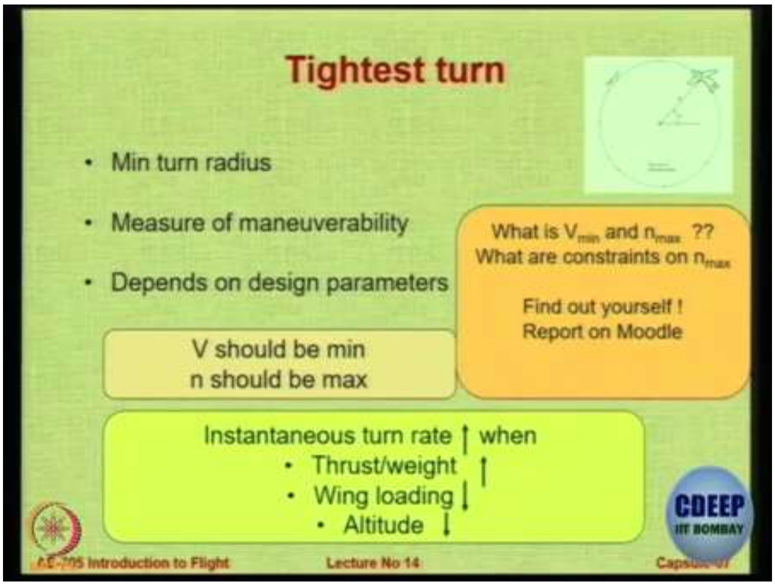

So the condition is same for both. Now, obviously when you have uh when you have uh high instantaneous turn rate, for that thrust to weight ratio is useful, wing loading, altitude and these two, what is the minimum speed and what is the maximum load factor? This is something that you have to tell me. For a given aircraft, you would like to have V to be minimum, how minimum can you go? You want n to be maximum, how maximum can you go? That is something, that you will tell me.

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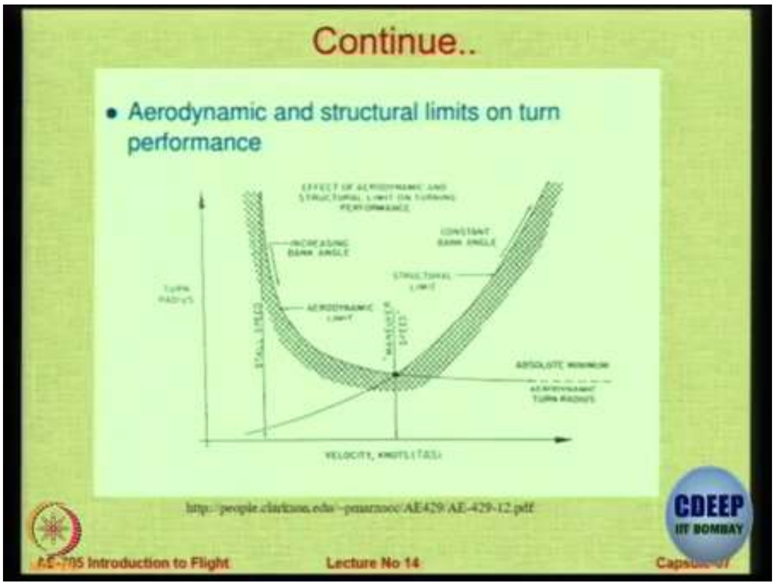

And, here is actually the answer. These are the limits which are imposed on the velocity. So, turn radius is going to reduce, you want it to be less, but you will hit two limits. On the right-hand side, you are going to limit, you are going to hit the structural limit. As the velocity increases, the dynamic pressure, half rho V square, on the structure will increase and the load that the structure can carry, has to be balanced. On the lower side, you have the aerodynamic limit. Because, the power required, power available, you know it also starts increasing. So, the bank angle is more, when you are going to turn at a lower velocity. So, this is a very nice curve, which shows that on the left-hand side it is the aerodynamic limit and on the right-side structural limits. And that sets you a boundary, for the V versus radius, okay?

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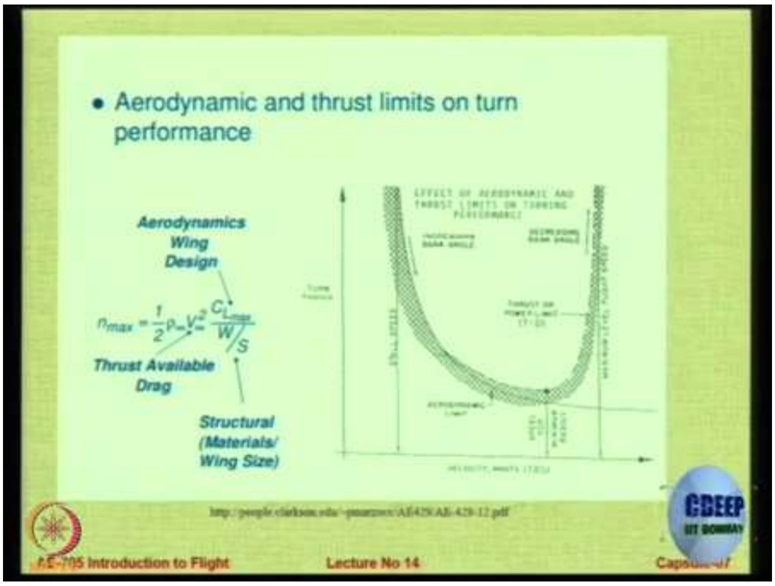

Similarly, you have also another curve which tells you uh thrust available and drag will limit V_∞, $C_{L_{max}}$ from aerodynamics, you can get a maximum value and $W/_S$ can be driven by the materials or by the size consideration.