Jet and Rocket propulsion

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Lecture – 20

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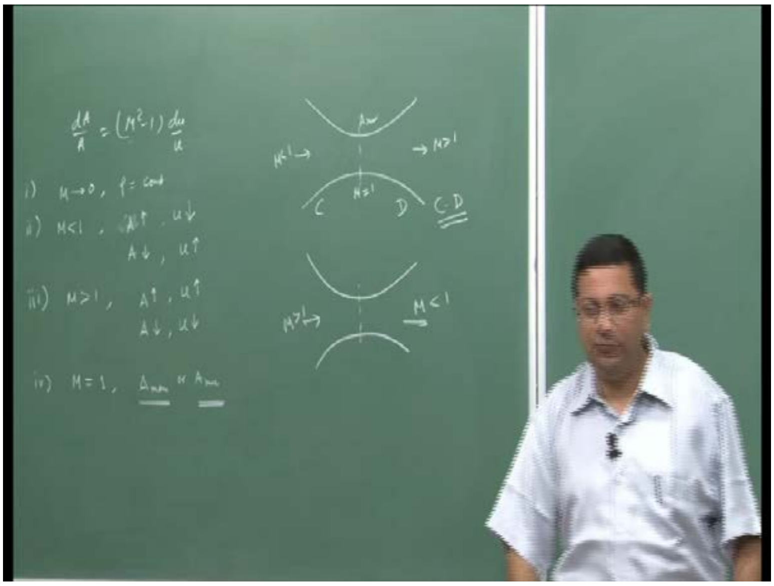

Welcome back in the last few lectures we have been discussing the quasi one d flow through variable area in ducts, we have derived the area velocity relationship in the last class, which is d A by A equal to M square minus 1 d u by u. After that in the last class we have shown that there are 4 possibilities of the mach number variation, if mach number is very small ending to 0, we have shown that density is constant. So, we have an incompressible flow.

Then we have shown that if the flow is subsonic, then as area increases velocity, sorry area increases velocity will decrease as area decreases, velocity will increase. So, for a subsonic flow a diffuser need to have an increasing area or a diverging passage, and nozzle needs to have a converging passage. Then we have shown that is other way round for a supersonic flow, for a supersonic flow as area increases - velocity increases, as area decreases - velocity decreases. And then we have also shown that the limiting case when mach number is equal to 1 area is either a maximum or a minimum. So, this we have shown from area velocity relationship.

Then what we did in the last class is that we looked at various combinations of area variation. So, that we have either a maximum or a minimum area in between. So, we looked at various variable area duct, and then we have shown that this maximum area is not physically possible either for a nozzle flow or a diffuser flow, because it does not give a monotonic variation in the velocity. On the other hand, if we take it through a minimum area then it gives a monotonic variation in the velocity, that is if we have a subsonic flow we take it through a minimum area and then expand minimum area. So, that if we go through converging diverging passage, then the flow will accelerate in the converging area come to sonic speed at the minimum area, and then it will further accelerate in the diverging area, because at the diverging portion it will be a supersonic flow. So, for taking a subsonic flow and converting to a supersonic flow, we need to have a converging diverging area with the minimum area at the throat, where mach number is equal to 1, we have shown this in the last class.

We have also shown that if you have supersonic flow, and if you want to slow it down to a subsonic flow, then also we need to go through a converging diverging area, we have shown all these cases in the last classes. So, now let us focus on this, because our discussion on a topic is nozzle flow. So, nozzle flow essentially is accelerating the flow. So, here we look at this case that we have a subsonic flow, we can one referred to a supersonic flow.

Then we need to go through a passage like this, which is a converging diverging passage this side is converging, this side is diverging and we have a minimum area in between. So, therefore we have proved that in order to get a supersonic nozzle, we need to have a converging diverging passage or a converging diverging nozzle. This concept was first proposed by a person called de laval, therefore this nozzles are called as de laval nozzles, converging diverging nozzles are called de laval nozzles. So, therefore this is something that is absolutely essential, particularly for rocket propulsion because we want to increase the exit velocity as much as possible, which means we want to take it to supersonic speed exhaust, and in order to get that we need to have a converging diverging nozzle. So, this is kind of a recap of what we have discussed till the last class.

Let us now proceed from there, now I would like to get the actual relationship, so that we can get the velocity. So, in the derivation of all this where we got in the mach number, we have considered we have assumed that the flow is adiabatic and irreversible sorry reversible, that is frictionless therefore we have considered to be isentropic. So, now let us take it further and look at the isentropic flow through the variable area ducts that we are discussing.

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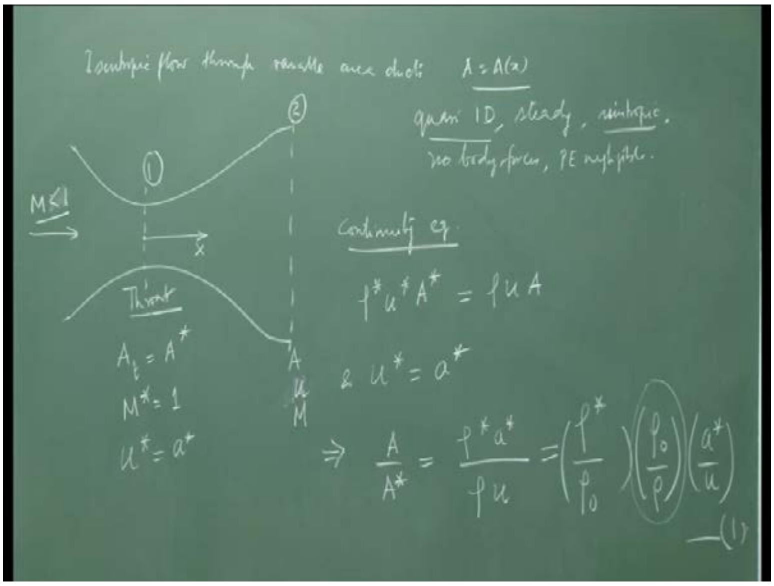

So, the next topic now is isentropic flow through variable area ducts. So, let us consider a duct a converging diverging duct like this, and we have some flow coming in here some mach number, we have a minimum area at this point 1, and we have certain area at the exit 2. So, let us consider that we have a subsonic subsonic flow here, inlet as subsonic flow M is less than 1, we are not specifying the area let say the inlet flow is subsonic. We have a minimum area at 1, we call it the throat; throat of the nozzle.

And let us say that since it is a throat the throat area is the minimum area given by A star, then what is the mach number at this point is 1 that we have shown already therefore M star is equal to 1. So, we are representing the throat properties by this designated star. And if the mach number is equal to 1, therefore the speed of sound is also equal to the velocity or velocity is equal to speed of sound at that location. So, these are the conditions at the throat at 2 at the exit of the nozzle, let us say that the area is equal to A, the velocity is M, the velocity is u, mach number is M or it may not be at the exit at any location inside the nozzle; these are the conditions.

Let us now consider the flow to be quasi one d steady and isentropic with no body forces and potential energy negligible. Since it is isentropic, that means it is adiabatic and reversible; reversible means it is friction less. So, it is inviscid. So, this is inviscid adiabatic flow. Now, we want to analyze the flow for this case.

So, first from the continuity equation, we have discussed this flows in before. So, we have shown that the continuity equation can written as rho 1 u 1 A 1 is equal to rho 2 u 2 A 2 in this case my 1 is at the throat. So, A star u star rho star is equal to rho u A, that we have shown. And we have this condition here that u star is equal to a star, therefore we can write A by a star is equal to rho star a star upon rho by u. Then we can write this as rho star by rho naught rho naught by rho a star by u, let me call this equation 1. So, let us see what we have done here. Here a star is the throat area A is the area anywhere in the diverging path of the nozzle, we use the continuity equation to relate this two, then we have written A by A star, so area at any location divided by throat area.

We want to find out now that if area is given, what should be the flow properties there. So, area as we know for a quasi steady quasi one d flow, area is a function of A x a function of x, right. So, if I start from here at x equal to 0, if I go in the x direction at every location at every x, if area is specified we know the area. So, let us say this area is known, therefore this quantity is known, because we know the minimum area. We are going to find out what will be the density here? What will the velocity here at a particular x location, for that what I am doing is that I am writing it first as a star rho star a star by rho u, a star is a fixed quantity, because throat properties are known. Rho star is also a fixed property, because throat quantities are known, rho and u are the variables.

Now, after that what I have done is I have divided and multiplied by rho naught, where rho naught is the stagnation density here in the diverging portion. Now, if I look at this relationship rho naught by rho, the flow is isentropic right everywhere is isentropic. So, for any point I can define the stagnation density rho naught, and then we know a relationship between rho naught and u in terms of the local mach number right and that is what we trying to get.

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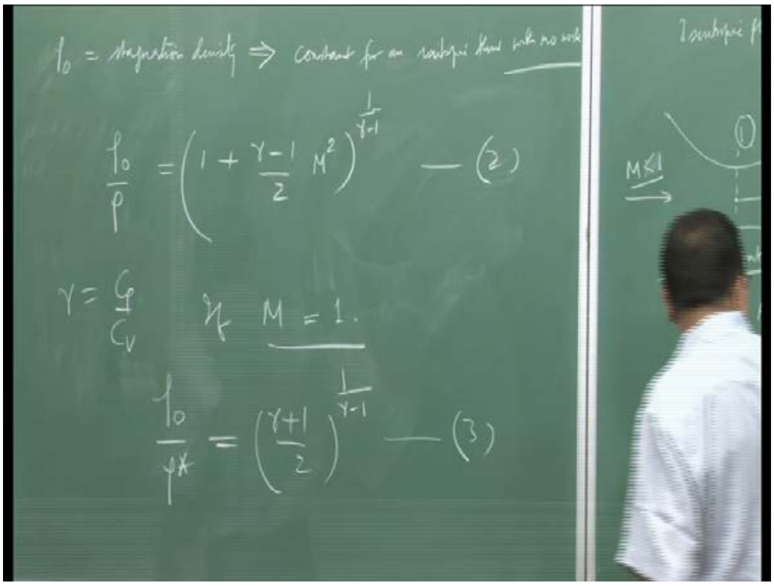

So, what we can do is now this rho naught by rho is a rho naught by rho is constant for an isentropic flow right, rho naught is the stagnation density. And therefore, it is constant for an isentropic flow with no work, and I like to point out here one thing that if we are considering an isentropic flow. Then the pressure can change if and only if there is a work right, otherwise the stagnation pressure will remain constant. Similarly stagnation temperature will remain constant, therefore stagnation density will remain constant, but if there is work done then the flow can remain isentropic, but pressure temperature and density will change. So, therefore, in this case there is no work done. So, therefore, the stagnation pressure temperature and density are constants.

So, therefore from isentropic relationship we can get rho naught by rho equal to 1 plus gamma minus 1 by 2 M square to the power 1 upon gamma minus 1 let me call this equation 2, this is coming from isentropic relationship which you must have seen in gas dynamics courses, in aerodynamics courses, etcetera. So, I am not going to definition of that this is the isentropic relationship getting the stagnation density, and local density to the local mach number, and gamma is the ratio of specific heats. So, gamma is equal to c P by c v, this is something that should be known to you.

Now, if mach number is equal to 1, then what happens to this relationship? If mach number is equal to 1, this rho is equal to rho star our throat condition. So, we have rho naught by rho star is equal to in this equation I put gamma equal to 1. Now, what we see is that rho star rho naught is constant rho star is independent of gamma, sorry independent of mach number and this is the relationship which is just a function of gamma. So, you can directly solve for this. So, the first term in this equation right hand side of this equation can be obtained from this. So, I can write it equation three. So, first term of this equation is obtained, second term of this equation is obtained in terms of the local mach number, now we have to get the third term A star by u either typically in terms of the local mach number. If we can do that then what we have is the area relationship in terms of local mach number. So, let us try to do that.

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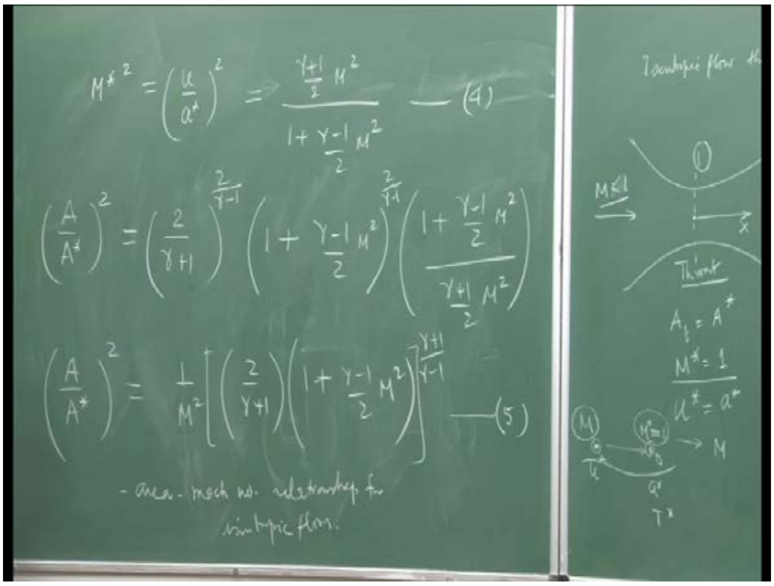

So, now here I write some equations, which I would say that you read off from the text books, because I am not going to derive that it will take one day to derive that M star square is equal to the mach number at the throat square. So, we have here relationship for rho star by rho naught and rho naught by rho, next we have to get a relationship for a star by u, now in isentropic flows M star which is the mach number where mach number goes to 1 is a defined property. All this stars are defined properties how it is defined is that if we have a flow field, if we have a fluid particle moving with certain mach number M, it may be subsonic it may be supersonic. We catch hold off this fluid particle and then either accelerate or decelerate it by traversing of course a certain distance.

So, that at this point the mach number is equal to 1, then this variation where M star is equal to 1 is defined as the is essentially a property of the flow at this point A is has been either accelerated or decelerated to mach 1 isentropically. So, if it is moving with a speed u then corresponding to that there is an M star where corresponding to that M star there is a corresponding to this u there is an M star value. Similarly there is an a star value also which is the local speed of sound here, because we can define a T star for this u. Now this things you can study again in any book in isentropic flow and gas dynamics, I am not going to a details of that what can I show is tell you that that is M star can be expressed in terms of the local mach number M here.

So, now M star is equal to u by a star whole square M star square, which you can get this in the text book on gastronomic and isentropic flows will be equal to let me call this equation 4. So, as you can see that this M star is defined in terms of the local mach number M. So, now this definition here is my M star right u by a star. So, now, looking at that equation one I have every term in the right hand side as a function of the local mach number M. So, now I can put them back and get an expression for the area ratio A by A star is equal to I will just right down 1 plus gamma minus 1 by 2 M square 2 upon gamma minus 1 1 plus gamma minus 1 by 2 M square upon gamma plus 1 by 2 M square.

So, here this term here is coming from rho star by rho naught, which is independent of local mach number, this term is coming from rho naught by rho which is a function of local mach number. This term is coming from a star by u, which once again is a function of local mach number as is given here. So, then what we get is A by A star, where A is the area anywhere which as I said will come from that area relationship and A star is the throat area.

So, if I know the area a star A and A star, I can get the local mach number by solving this equation. So, what will be the mach number here can be obtained by solving this. So, therefore I can simplify it little more A by A star square is equal to 1 upon M square 2 upon gamma plus 1 1 plus gamma minus 1 by 2 M square to the power gamma plus 1 upon gamma minus 1, let me call this equation 5. This relationship is called area mach number relationship for isentropic flows.

So, once again what it is giving is that the local mach number as a function of local area or local area as a function of local mach number and gamma. So, this relationship is also very important relationship, that is why typically in the isentropic flow tables, where P naught, P T naught, rho naught, etcetera for different P T values mach number are given; this values are also give the area relationship. So, these are given in isentropic flow tables which you can look up from any table available in any gas dynamic books gas dynamics book. So, this equation actually tells us how the local mach number is going to vary with the variation in area right.

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So, at every point here in the nozzle what should be the local mach number we can estimate from this. So, it also shows that the local mach number is a function of the local duct area a and the sonic throat area a star. So this relationship shows that the local mach number at any point is a function of this area, and this sonic throat area. We have now what we have shown earlier that the area for mach number equal to 1 can be either a maximum or a minimum first point. Second point we have shown that the maximum area is physically not possible, it is the minimum area is only possible. Therefore, now if you are saying that the mach number is the function of the local area, and the throat area, and we have already shown that this relationship that we have derived there shows the mach number as a function of A upon A star first of all.

We have shown that area is always greater than A star, because A star is the minimum area right ((Refer Time: 22:22)). This we have already discussed in the last class that the A star is the minimum area. So, everywhere here the area is greater than this. So, this is one point, therefore this term here is greater than one agreed.

Now, if I look at this relationship for a given value of A star there will be 2 solutions for M M, it is a quadratic equation or quadratic relationship. So, there are two values of mach number corresponding to a given value of A by A star; one of them will correspond to the subsonic, other will correspond to the supersonic solution. So, one of them will correspond to this side of the flow, other will correspond to this side of the flow. So, if I plot now this variation now I can remove this one and go into little bit of physical understanding.

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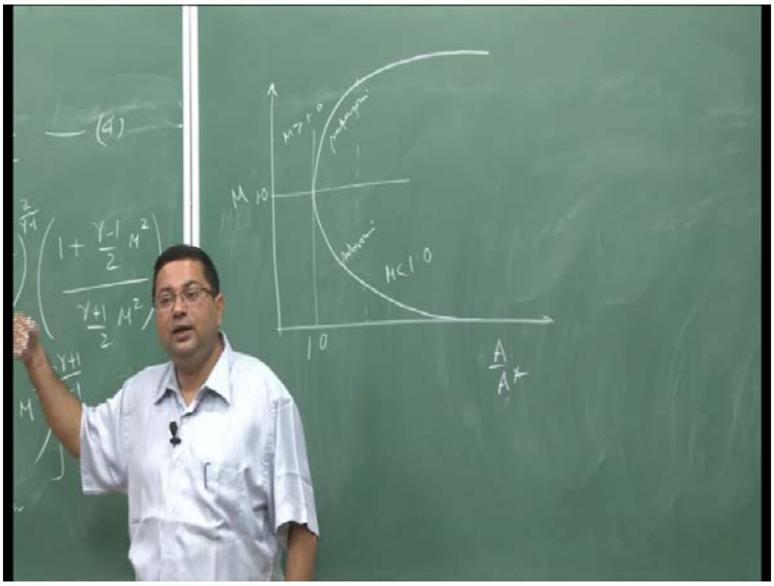

So, if I plot this area ratio variation versus the mach number, then what we will get is at mach one what should be this area ratio at mach 1 1 right, this is here. Then from the subsonic side this should go like this, right. So, this is mach number less than 1 subsonic side area must increase, here in the supersonic side also area must increase. So, this is the subsonic, this is supersonic, as you can see here this mach number is greater than 1, here mach number is less than 1.

So, the solution of this equation will take a form like this. So, for a given value of A by A star as we are seeing is there are two possible solution, if I take look at any one of them. Except from the sonic case, everywhere there are two possible solutions; one is subsonic, other is supersonic. Now the question arises that what value this flow will take how will the flow know that it has to be a subsonic or supersonic flow that depends on the boundary conditions. So, that is again something that is quite critical that what kind of flow will exists depends on what kind of boundary conditions that we have. So, therefore the next thing we are going to discuss are this boundary conditions. So, what we have established is that for a given value of A by A star; there are two possible solutions; one will be a subsonic, other will be a supersonic solution. Next we are going to discuss how this flow is established whether subsonic or supersonic. So, for that let us consider once again a converging diverging nozzle. So, let me know remove all these portion and start that discussion.

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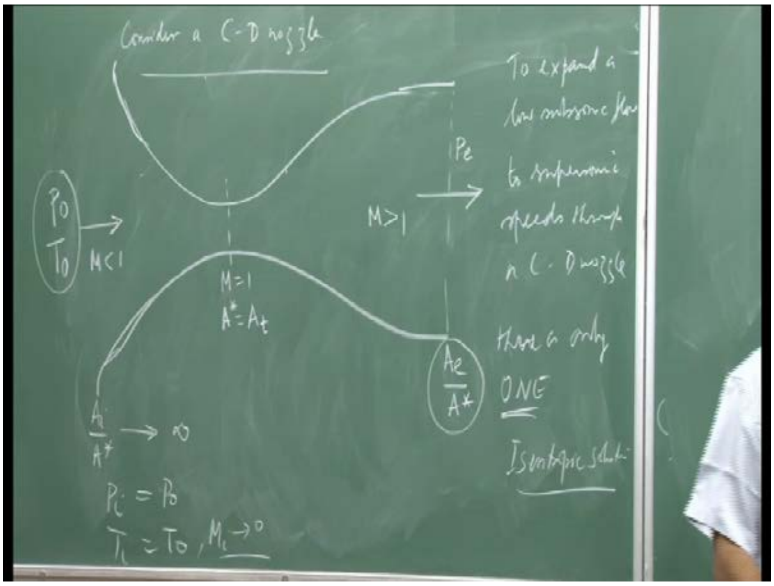

Let us consider a converging diverging nozzle. Let me say it is like this, something like this, we have certain stagnation properties at the inlet of this nozzle P naught P naught, and the flow is in this direction the mach number is less than 1 here. Then the flow is going through the throat where the area is equal to A star is equal to the throat area mach number is equal to 1, then it is further expanded in this section where the exit area ratio is A by A star, the mach number here is greater than 1, the flow is going in this direction and exit pressure equal to P e.

Let us consider that the inlet area we have something like a bell mouth shape, we have something like a bell mouth shape. So, a bell mouth shape what it does is that what is the inlet area ratio length, if it is a bell mouth shape it is pointing like this, right. So, for a bell mouth shape nozzle the inlet area ratio is infinity, because A i can be considered to be infinity. So, the inlet is fed from a reservoir, but the gas is maintained at a pressure P naught and T naught, the inlet area is infinity.

If the inlet area is infinity, what is the pressure and temperature there? Is the stagnation pressure and stagnation temperature, therefore my P i is equal to P naught, T i is equal to T naught right. So, and also my mach number at the inlet tends to 0. Then we have nozzle inlet like a bell mouth inlet then we can get this conditions established, P naught and T naught are equal to P i and T i, and the mach number tends to 0. Now for this case let us look at how the flow is changing. So, from here to here there is a converging area right.

So, therefore the flow gets accelerated there is an expansion of the gasses till it reaches the throat, and after that there is a diverging area. So, now here the flow is supersonic. So, there is further expansion. So, the given nozzle essentially expands the gases to a supersonic speed at the exit. So, at the exit will get a supersonic speed, and then there is only one possible isentropic solution, if it has gone to the exit condition at supersonic. Because for the given area ratio A by A star we are supposed to have two solutions; one is subsonic other is supersonic. In the case if it has gone to supersonic, then it has taken only one solution. So, that is the one solution, but that is the isentropic solution.

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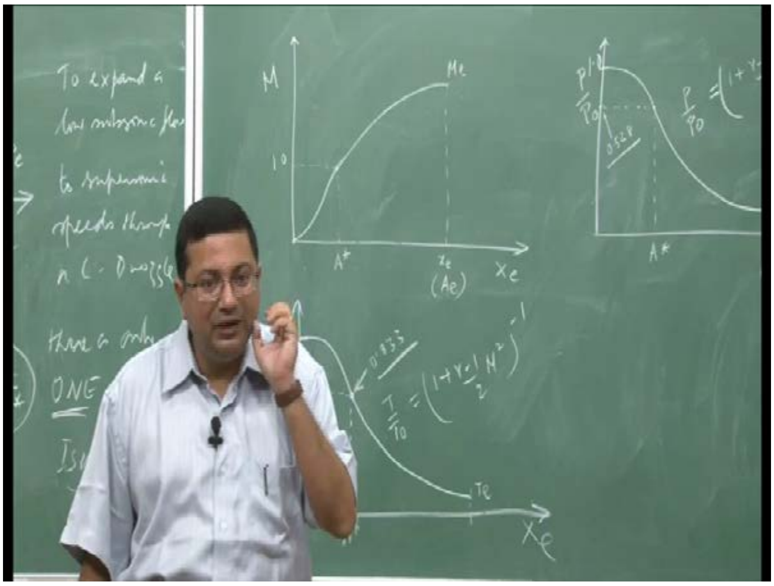

So, there is for this case to expand low subsonic flow to supersonic speeds through c d nozzle there is there is only one isentropic solution. This is important that the solution is isentropic. So, we can have only one way of expanding it to an isentropic flow, if we have to go through this process, there is one solution possible. In the converging section as we are seeing here the flow is accelerated to the sonic speed at the throat, in the diverging section on the other hand the sonic flow is further accelerated into supersonic flow.

Let us consider that my origin is somewhere here, and this is my x direction, this is my origin 0. Now let us plot the variation in properties along x direction starting from x equal to 0, we go up to this point up to the exit, let me say that this is equal to x c up to the exit of the nozzle. So, first let me draw this we will be drawing the variation plotting the variation of three properties x c, first let us look at the variation of mach number then the static pressure we will just normalize it with the stagnation pressure, and the static temperature once again normalized by the stagnation temperature. So, these are the three properties will be plotting. First let us look at the mach number variation at x equals to 0 my mach number is 0, then from here to A by A star equal to 1, the mach number is increasing, but it remains subsonic right. So, if I plot it here.

If say this point corresponds to my A star, then it goes from 0 to this point. So, at the throat where A equal to A star I get mark one, when I go beyond that there is a further acceleration. So, that mach number increases now and becomes supersonic. So, it goes like this till the exit where the exit I get mach number equal to M e, this is my x c and correspondent area is corresponding area is A. So, this is the isentropic mach number variation in the nozzle.

Next let us look at the pressure variation; the pressure at the inlet of the nozzle is equal to the stagnation pressure P naught therefore, this ratio is equal to 1, as the flow is expanding the pressure is going to drop, right. So, as the pressure drops the pressure is going to drop like this till the throat, this is once again my throat a star. And it will take sudden fixed value at this point I will come to that value little late, then it further continues to drop till the exit and reaches a given value P e at the exit at x e, where area is A e. Now this variation is isentropic right. So, if it isentropic variation we can use the isentropic relationship P by P naught is equal to 1 plus gamma minus 1 by 2 M square upon gamma minus 1 upon gamma, thus the isentropic relationship.

Now, let us first look at this point, here at this point mach number was equal to one. So, in this equation if I put mach number equal to 1, I get a value for P by P naught in terms of gamma. If you consider the working fluid to be air then gamma is equal to 1 by 4 – 1.4, then this is a fixed value and that value is equal to I will right it here0.528. So, the value of the pressure static pressure ratio static to throat pressure ratio at the throat is equal to 0.528, where mach number has reached 0.5281. the pressure drop continues to move along this, at the exit this what pressure it will take is a function of the exit mach number given by this relationship and the exit mach number comes from here which will be actually coming from our area rule, right. So, therefore, this variation is fixed.

So, if our pressure at the exit is equal to these pressure we get an isentropic flow nice and smooth isentropic flow supersonic flow. Next let us look at the temperature variation, temperature at the throat rather the at the inlet is equal to 1, because it is temperature ratio is 1, because the inlet temperature is equal to the stagnation temperature. Once the flow is expanding in the nozzle. So, the temperature is going to fall, it will fall up to the throat a star and beyond that it will continue to fall till the exit temperature T e, once again this is isentropic mach number is varying like this and the temperature ratio is function of mach number for an isentropic process given as power minus 1. So, depending on this mach number we get certain value of temperature. Once again here if I put mach number equal to 1, I get T star by T naught.

So, that is the temperature here, this will be a fixed value 0.833 for gamma equal to 1.4 and beyond that it is going to fall according to this relationship, and at the exit for the given mach number it will get certain temperature T e. So, what we are seeing here is now as long as this process is isentropic depending on the area it must have a given mach number. And then if the given mach number is there the exit pressure must be a fixed value the exit temperature must be a fixed value it cannot take arbitrary values, because it is bounded by the isentropic relationship which guides the flow or dictates the flow.

So, therefore at the throat we have the throat area a star mach number equal to 1, and then beyond that at every location either before the throat or after the throat, the mach number is function of the location x, because the area is function of location x right. Because here let us say x has certain value therefore, there is certain area and that area will dictate what will be the mach number similarly here. So, therefore, everywhere in this plots the mach number is dictated by area which is the function of x therefore, at every location we have certain mach number which corresponds to certain pressure, temperature, etcetera. The exit ratios on the other hand, that is the exit temperature, exit pressure, and exit mach number depends on the exit area and the throat area. Throat area is fixed depending on exit area, we get certain values of the exit ratios.

Now this shows us what kind of process is established, but 1.1 more point I would like to emphasize here that in order for this force to be established, there needs to be a force that will move the flow. So, that force which moves the flow is the pressure force. So, now what we have discussed so far that we considered there is a pressure difference or pressure force, which will moving it that is we have certain inlet pressure P naught certain exit pressure P e which will moving the flow.

Now, what we will do is in the next class we will discuss what is the value of exit pressure which will give us this effect. So, essentially I would like to find out what will be the pressure force that will establish a required flow which means. Now, if I look at a practical scenario, we have a reservoir, where temperature and pressure are maintained, we have attach a nozzle which kind of flow, we will get depends on what pressure we have at the exit right. So, therefore this exit pressure now we are going to vary all in other words we are going to vary the pressure difference between this and this, because that exerts the flow the force to move the flow.

So, in the next class what we will do is? We will look at the effect of back pressure in establishing a flow through a nozzle, and that will again bring us to the discussion of our over ventilated and under ventilated. Previously we had talked about over ventilated ideally ventilated and under ventilated case, we have shown that the thrust is maximum for the ideally ventilated. Now, we will see the physical meaning of this types of flows. So, in the next class we will start from there try to establish the flow. The flow that we have here. Now we will see that how this flow gets established. Under what conditions we get this case, and then we discuss various consequences of the back pressure. So, we will stop here do you have any questions, otherwise I will stop here.

Thank you.