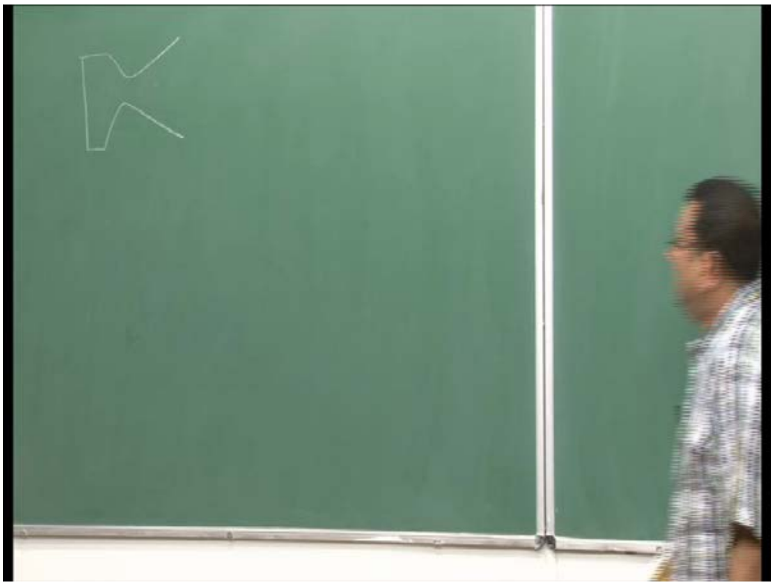
Jet and Rocket Propulsion
Prof. Dr. A. Kushari
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 29

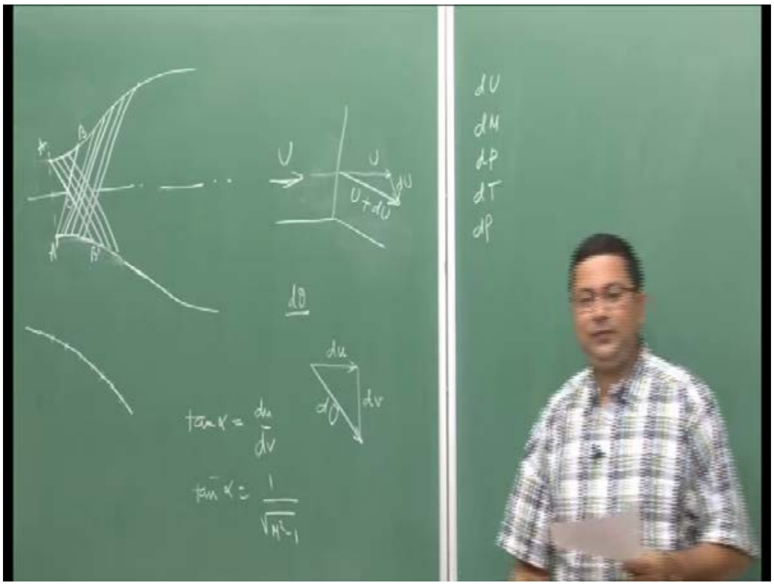
(Refer Slide Time: 00:28)



Good morning. So, in the last couple of lectures we have been discussing shaped nozzles. We had first discussed a conical nozzle and but the nozzle is a cone and we have shown that because of the three dimensional effect. There is a loss in the performance, which was given by a factor lambda, which depends on the half cone angle. Then we have said that this loss is primarily, because of the flow deviating from the nozzle axis, because of the three dimensional effect and what we said is that? If you go for a shaped nozzle, if we contour the nozzle wall in such a way that the flow turns and towards the exit it leaves parallel to the axis of the nozzle as well as we get a uniform flow, then these losses can be minimized.

And based on that we started our discussion on design of such contour nozzle or shaped nozzle. We started discussing the use of method of characteristics for nozzle design. We have shown, we have said that the method of characteristics is applicable only for supersonic flows it cannot be used for subsonic flows. And the inherent assumption in method of characteristic is that the flow is isentropic. Based on that what we have done in the last class is we have started discussing the design of a curved nozzle using method of characteristics.

(Refer Slide Time: 01:53)



We said that let us say, this is our nozzle shape and apply like this. And we have also said that the converging portion is irrelevant, because the flow is accelerating there is a favorable flow pressure gradient and the flow is subsonic. So, that is not much of problem in designing that. So, we can just choose a contour which is rather not a contour that is the converging portion and get away with that. The diverging part; however, is more critical. So, we have to put in more effort in designing the diverging part. Then what we have said is that, when we start to design, let us say we choose a particular point here. So, let me once again draw the contour.

Let us say, this is the inward inlet part, this is the nozzle axis. Let us say that the characteristic lines are formed from here, because of the slight change in the angle. If I expand this, if a supersonic flow is coming like this and then there is a slight change in angle about this turning there is an expansion fan that is created, because of that expansion fan. The flow turns and becomes parallel to this side at the same time it accelerates. So, mach number increases. So, towards the beginning section then we have, what we said is that we take say this section and break it into many small sections. So, that there is a gradual change in the angle.

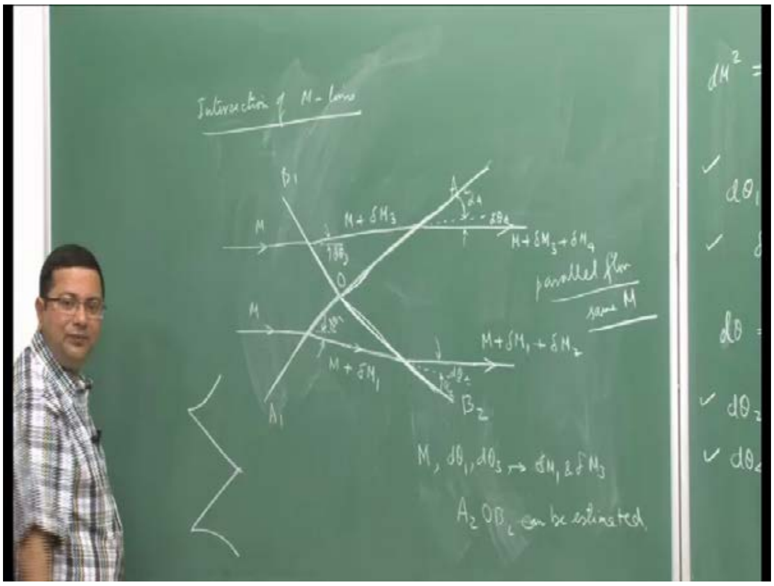
And then we said that starting from both this points the characteristic lines are formed which are going to, go like this and form a kind of grid. It is not exactly cutting the way I wanted to, but anyway it forms a grid like this. We also said that in this location let me call this a, this is c, this is b. Similarly this is a prime, this is b prime, this is c, in this location, in this area rather portion of the segment, if we see that, we have only one sided characteristic lines. On this side it is left running and this side it is right running there is only one sided characteristics that are present. So, then we had estimate, since the valuation in angle theta or other d theta here in this case is specified we can estimate the changes in properties. So, how we will do that? Let us say, if I look at this flow again we have said that this flow will be turning, if the flow velocity coming here is U and then turns becomes parallel here at the same time it gets accelerated.

So, this velocity becomes U plus d U. Now, initial velocity was U so, there is an increase in velocity d U and this d U can be split into two components, one is the horizontal component d u, other is the normal component d v. So, therefore, this is our d U. Based on that, then we use the definition for mach angle, which was tan alpha is equal to d u by d v and at the same time tan alpha equal to 1 upon m square minus 1. This is the definition of mach lines and this is the mach angle. From there we can, we have. So, using this definitions as well as the definition of speed of sound or the mach number and isentropic relations. We have derived relationships for the change in velocity, change in mach number, change in pressure, change in temperature and change in density.

So this is what we have done in the last class. So, by doing so, what we done is till point b this entire zone here all the conditions now are known. Till starting from point a to point b all the conditions here are known. And here the mach lines are not crossing each other. Now, the next thing that we have to discuss is, if I look at this zone the mach lines are crossing each other. If they are crossing and if I look at this after every crossing there is a change in property.

So now, everywhere there is a change in the properties like this. This of course, this will not be the same equations, same expressions, but the properties are going to change. So now, what we want to do is? Let us discuss what happens when the mach lines intersect each other. So, this was the recapitulation of what we have done in the previous class, let me just remove this. And focus on the intersection of mach lines and how the properties change. So, the region downstream of this region, which we have been discussing so far the mach lines cross each other. So, therefore, conditions downstream of these mach lines are no longer uniform, which were uniform previously, because mach lines were not crossing.

(Refer Slide Time: 08:20)



So, now let us look at intersection of mach lines. Let be consider two mach lines, one left running one right running. And they are crossing at this point o and after crossing the mach lines change they deviate from their original path and go like this. So, this is the incoming mach line A 1 and B 1 after crossing it become A 2 and B 2.

So now, let us see what happens to the flow? Here the flow is uniform in this domain coming with a mach number M. Now, first let us look at the bottom one, this flow comes to this mach line, when it crosses this mach line it gets deviated and it goes say like this. Let me consider this deviation for this mach line is d theta 1. And then after crossing the mach line not only it gets deviated it gets accelerated. So, the mach number is going to change. So, in this branch let us say the mach number is let me call this M 1, M only. So, M plus delta M 1 so, initially this was M, now it is incremented by the amount delta M 1.

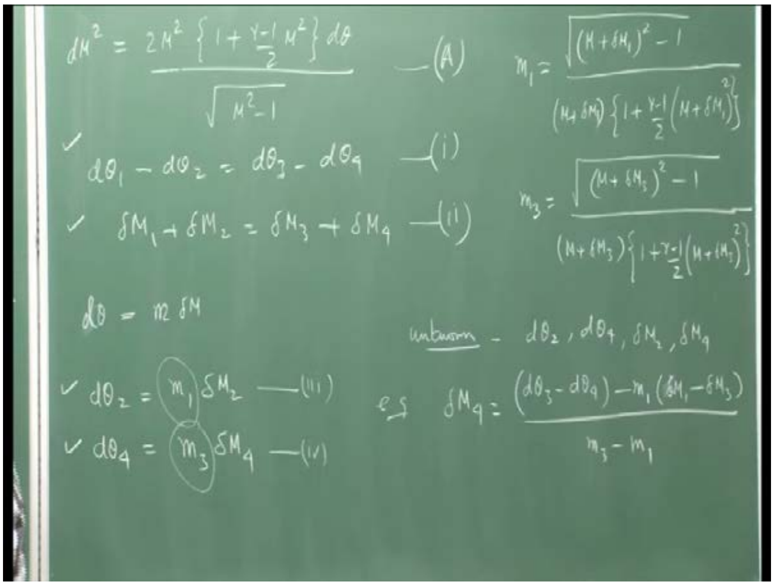
After that, this mach line, this flow crosses another mach line, which is o 0 o B 2 or 0 B 2. So, when it crosses this mach line o B 2 then it is going to be another change again it will get deflected at the same time the flow will get accelerated. So, let me, this was the initial path and if I extend it, this is the initial path of this flow coming into this mach line, now it gets deflected say like this, let me say this angle is d theta 2. And now, the flow here after it has crossed this o B 2 line has further accelerated. So, initially now, when this was M plus delta M 1 now it becomes M plus delta M 1 plus another acceleration delta M 2. And now this angle that is the, see this is the flow direction for this mach number and this is the mach line. So, therefore, this angle alpha 2 is the mach angle for this mach line with respect to the incoming flow. Same thing happens for this sum.

So, let me do it here. First it gets deflected by an amount, let us say d theta 3 and gets accelerated d M 3 then coming here it is further changing. So, this was the original direction now, it changes like this and this deviation is d theta 4 and this angle is alpha 4 and the mach number after this deflection is M 4. So, essentially what happens is that in this region there is no mach line. So, therefore these two flows should be same at we do not want to have a difference in mach number.

So, initially it comes with a given mach number then this mach line and this mach line may have difference at different mach angles. So, because of that deflections may be different, but finally, what we get is this. So, the flow condition upstream of this line o A 1 which is this side, as we can see here the flow condition upstream of this is uniform, but upstream of this if I look at o A 2 there is a change between this and this. So, right. So, therefore, this mach line and this mach line have different and the flow conditions are different between this and this region. So, because of this the mach lines are changing the directions right the mach line is changing the direction it is not going straight in this direction, because the flow conditions are different right. Because this mach line is determined by this mach number this mach line is determined by this mach number and since this and this are two different mach number mach line they deflect or they deviate from the original path. Now, if we know the flow in this region this as well as in this region A 1 o B 2.

So, we know the flow between B 1 o A 2 and A 1 o B 2 then we can determine the flow in this region also that is the point that is if we know this change that is occurring to the flow we can get what is going to be conditions here. So, that is what the bottom line is of all these analysis that how do we get these changes. So, the deflection of the stream line occurring in this o A 2 and o B 2 these deflections are not arbitrary.

(Refer Slide Time: 14:44)



They are actually determined by this at let me write this expression 1. This we had derived in the last class, what does this say? That the change in mach number depending on a change in theta and if the theta changes the mach number changes. Now, if I look at this region itself, what this region is seen essentially is the same region done there, that a supersonic flow crossing a mach line right nothing else. So, therefore, this is applicable to the flow here, similarly it is applicable to the flow here.

So, therefore, what will happen to the flow here is dictated by that, that expression. That we have a change in mach number and that change in mach number for this region is delta M 2, for this region is delta M 4 and this is not arbitrary, it will be governed by this equation. And there is a requirement that all the stream line in this region are parallel. So, after turning the flow should become parallel here. So, flow should be going in one direction that is what we want to get right we want to get a parallel flow. So, let us say that we design in such a way that the flow becomes parallel. So, we have a parallel flow in this region in A 2 o B 2. At the same time since we are having a parallel flow, we should have same mach number also once again that is the requirement we want to have uniform parallel flow right. So, therefore, in this region where there are no mach lines.

So, just to rephrase what we have been saying when the region where there are no mach lines if the flow is not crossing any mach lines or the region between different crossing and different mach lines the mach number should be same uniform flow at the same time the flow should be parallel. We do not want to have because if the mach number is different we have different mach lines right. Similarly if the flow is not going to be parallel we are going to have different mach lines. So, therefore, we want to have parallel and uniform flow between the mach lines. So, similarly in this region there will be parallel and uniform flow. here flow is parallel and uniform here of course, we are assuming the flow is parallel and uniform right. So, between every crossing the flow should be parallel and uniform.

So, now let us look at this domain, if I look at A 2 o B 2 we want to have parallel flow what does this mean? Parallel flow means, the flow was parallel here it was deflected twice and then it takes a final direction. Similarly the flow was parallel here it was deflected twice and then it takes a final direction. What we are saying is that? This deflection for this flow is equal to the deflection for that flow. Now, how much this flow was deflected d theta 1 minus d theta 2, how much this flow was deflected d theta 3 minus d theta 4. Now, the flow was parallel here so, you ordered for this to be parallel this deflection should be equal. So, therefore, we write d theta 1 minus d theta 2 is equal to d theta 3 minus d theta 4 let me call this equation 1. Similarly we are saying that the mach number is same here in uniform flow; that means, whatever increase in mach number has happened from here to here is same as the increase in mach number from here to here. What is the increase in mach number for this branch, it is delta M 1 plus delta M 2 for this it is delta M 3 plus delta M 4 so, this two should also be equal.

So, therefore, delta M 1 plus delta M 2 equal to delta M 3 plus delta M 4, let me call this equation 2. Now, if I look back at this equation let me give me the name equation A. If I look back at this equation this gives an expression for change in mach number with respect to theta right. So, I can write this as d theta is equal to some constant m times d m right, d theta equal to some parameter m times d m, but constant it will be function of mach number also right. So, therefore, if I use it for different parts now, let us say first let us use it for this one d theta 2. So, our d theta 2 will be equal to some parameter M 1 times delta M 2 right. Similarly for this one d theta 4 is equal to some parameter M 3 times delta M 4 right, let me call this equation 3 and this is equation 4.

Now, this values of M 1 and M 3 comes from this equation right. These are not arbitrary and those values can be obtained as putting these equation, I just write down the expressions M plus delta M 1 square minus 1 divided by M plus delta M 1 minus, I am sorry, 1 plus gamma minus 1 by 2 M plus delta M 1 square, where is it coming from, from this, from this equation. Where if I look at d theta the incoming mach number is M plus delta M 1. So, that is what we have put there. So, we get this expression.

Similarly, we get M 3 which is appearing here is equal to square root of M plus delta M 3 square minus 1 divided by M plus delta M 3 1 plus gamma minus 1 by 2 M plus delta M 3 square. So, as we can see here M 1 and M 2 are functions of the incoming mach number M and then the deflections or that the change in mach number that has occurred, because of the first crossing right so, I get this expressions.

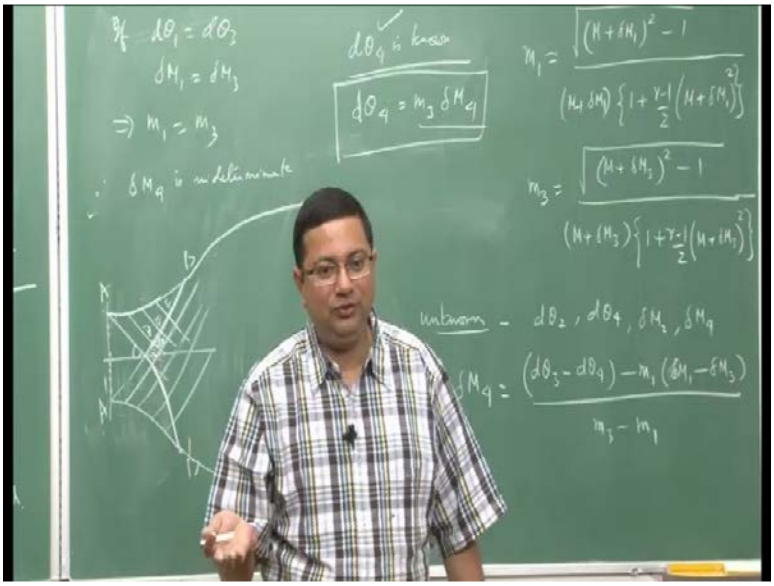
Now, let me look at this we have 1, 2, 3, 4, four equations right. And what are the unknowns that we have? So, we have four equations, equation 1, 2, 3 and 4 and my unknowns are d theta 2, d theta 4, that is this change and this change, because this is known, this change is known, we know exactly because this is a single type of mach line it is not crossing. So, we have already derived all the expressions for that we know how much theta is going to change and everything. So, this is known, we are interested in this zone. So, we need to know this angle and this angle plus this two mach numbers del M 4 and del M 2.

So, these are the four unknowns that we have. Now, we can solve for these four equations with four unknowns and we can get all the values. That is how much change in angle will be there, that is how much flow is turning change in angle is how much flow is turning and how much change in mach number is there. Once we know these two we can use the other equations, equations last lecture, I had listed five equations. We can use now those other equations and get the change in pressure, density, temperature, velocity, mach number everything right. So, as soon as we get this to solve other things can be obtained. So, for example, if I am interested in getting delta M 4 then from these four equation, delta M 4 will be equal to divided by M 3 minus M 4 right. So, from this four equations this is the value of delta M 4, once again M 1 and M 3 as you can see is function of incoming mach number and the first change which is known. So, delta M 1 and delta M 3 are known in this equation.

So, therefore, M 3 and M 1 are known. So, I can get the change in mach number after it crosses the second one. So, essentially if I can summarize it that, if I know the incoming flow mach number, uniform flow mach number, if I know the initial angles d theta 1 and d theta 3 then of course, from here I know delta M 1 and delta M 3 right using that equation I get these parameters. Once in know this then the entire flow in this region that is A 2 o B 2 is can be estimated. So, if I know the flow in this region and what is happening across these mach lines.

So, if I know the characteristics of these mach lines for that I should know the characteristics of these mach lines then the entire flow can be estimated. Now, there is one glitch here, if this and this mach lines like we have done here, if I look at this mach line and this mach line from the first point. Then this and this mach line are originating from exactly same conditions. So, therefore, this mach line and this mach line will bring in exactly same change right. So, what is happening there in that case that is there originating from the same condition and bringing in the same change.

(Refer Slide Time: 26:30)



Then what happens is that d theta 1 is equal to d theta 3 right as well as del M 1 is equal to del M 3. Now, if del M 1 is equal to del M 3 then from here what we see is that M 1 is equal to M 3. So, therefore, M 1 is equal to M 3. Now, on that such scenario this is zero right. So, this is indeterminate right.

So, under such case del M 4 is indeterminate. Then how do we get this, how do we solve for this? If they are coming from the same and similar thing will happen here not only in this zone. When we draw this here in this zone right, right the mach lines are crossing and then we are getting in to this condition, how do we get this thing. What we will do is under such scenario, we know that the flow is parallel here flow is parallel here. So, if the flow is parallel here it is parallel to this here also is a parallel flow. So, therefore, the flow in this region is parallel to the incoming flow. So, because of that since the flow is parallel to the incoming flow this value d theta 4 is known right, because the final turning is known, how much is it turning at the final condition. So, d theta 4 is known in that such scenario then we do not have to use this equation to calculate instead what we can do is we use we have the fourth equation in our list.

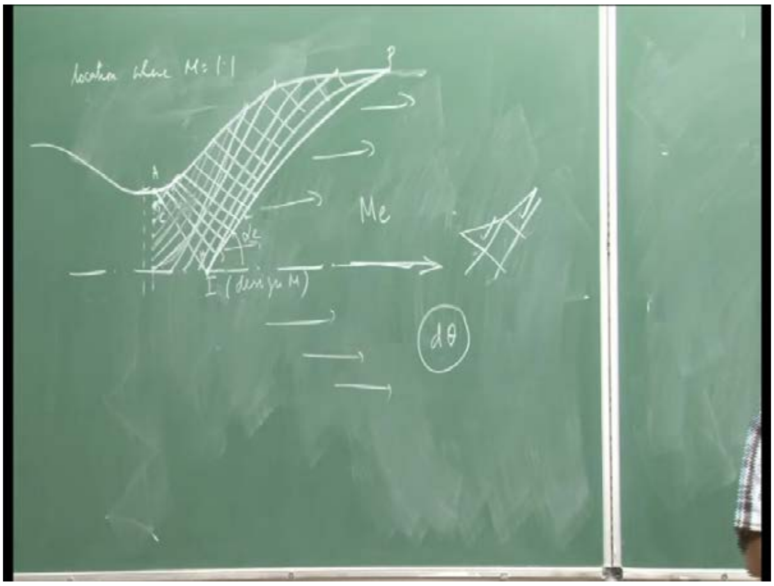
So, if d theta 4 is known we can just use this equation to calculate the change in mach number we do not have to solve all four together. So, essentially that simplifies that if we know for example, for this case, that the flow is originating this two mach lines are same then straight away we use this, but it is not applicable to flows here, because here this mach line is different, this mach line is different. So, for this zone, I have to use solve the four equations, but only here is use this to get this condition. So, therefore, doing like this we can solve for the flow downstream of this condition also now. So, this was our initial domain where all the mach lines as we have seen were same type. So, this was a, b, c, a prime, b prime up to this we have seen that the deflection is known. Now, beyond this, beyond this domain b, c, b prime here the mach lines are crossing right.

So, now we can solve for theta also. Earlier our d theta was specified now, we are solving for theta. So, when we solve here now this also emerges as a solution. So, we now can calculate the flow downstream of this crossing point downstream of c. After that we can calculate downstream of each of this point d, e, f, etcetera.

So, now we can get the full flow domain solution, we know now the pressure everywhere. Pressure, mach number, density, velocity, temperature everything can be obtained. So, what we have seen now is that initially this has to be specified after that we can design the flow in such a way that we get a uniform flow at the exit. Let me now, just list the procedure that is to be used to do that. So, first of all we need to know this conditions, not only that, this I have taken is this is essentially making a computational grid, we can also do is we can take conditions in between also and make a final grid the resolution will be better right. So, we can have conditions here from here the mach lines will immunate etcetera, etcetera.

So, now let us say for the time being we are just focusing on this. So, we need to know the mach number here and here also that is my inlet condition. Now, there is a problem here, from where we should start, should we start from the throat, should we start before the throat, should we start after the throat. If we start after the throat, what is the mach number, there it is less than 1. So, measure of that is not applicable. If you start at throat mach number is 1. So, therefore, what is alpha, 90 degree, again that cannot be used. So, we have to start slightly ahead of it, slightly ahead of the throat, instead of starting from slightly ahead with a length what we say is that we start from a mach number slightly higher than 1. So, the standard practice is to start at a location where mach number is 1 point 1. So, we start at a location where mach number let say is 1.1.

(Refer Slide Time: 32:51)



So, this is now, our assumed initial point. The number of starting points now we can select at that plane. So, let me just draw a schematic here, this is our initial rather the access the converging portion throat and the diverging portion. So, this is my throat, I am starting somewhere here, this is my point A, where mach number is say 1 point 1. Now, what we do is, we can choose many many points here depending on our computational facilities available, how complex computations can be done. So, we choose multiple points in this line, everywhere in this line mach number is 1 sorry, 1.1 is uniform flow coming here, that is what we are assuming.

Now, let us look at what is happening? From here there will be a mach line going like this, from each point here there will be mach line going like this, straight right, these are going to be straight lines only here it will curve. So, let me call this point I. Now, similarly from all these points here there are going to be mach lines coming like this. Similarly from different points coming from the bottom, from the bottom half also there will be mach lines which coming in. So, now, what we can see here is we have created a grid of mach lines starting from this point A, let me say this point is B, this point is C etcetera, let us say this point is H, this is I, similarly I have J, K etcetera up to P. So, now, we have created the entire mach line grid, where the mach lines are starting from different locations. So, now, we start our calculations from here and proceed downstream. Note that the flow is always along the stream lines right.

So, essentially we are getting the solution for the stream lines. And since the flow is along the stream line we have one stream line at the center, but the flow is like this and this direction we know. Another thing we know is the initial curvature, this wall is also a stream line right. So, we know the flow is going to be along the wall. And so, initial this information is known. Now, after making this grid following this method of characteristics, we can consider the flow to be isentropic we can get the two d flow field right. This will as, this is the two d method that I have already said the two dimensional flow field can be estimated completely by following this method. And that is how we designed a perfect nozzle.

So, what we are doing is, we start from this line initial point and start to solve this problem. At some point here I when we reach the flow field I, let us say at this. So, now, when the flow is going here it gets accelerated right, at some point say I we say that we get the designed mach number. So, as we marching our solution, we get the desired mach number at point I. Now, we know that we have reached a condition where we have our design conditions. Then this is the exit mach number, designed exit mach number, I like to point out here that this designed mach number may not exactly coincide with the point of intersection it may be in between two or in between the two intersections.

So, then in that case we have to interpolate we have to interpolate and get the designed mach number. Now, downstream of this I once we have reached this after that this mach cone that is emitting from here, let me draw the mach cone here, this mach cone that is emitting from this now, we have to ensure that everywhere here now the there is no more mach lines coming to this zone right. So, we have identified this now we have drawn a mach cone if there is no more mach line this entire domain here the flow will be parallel and uniform right that is what we want, because we have shown that once it crossed the crossing of two mach lines or the domain after that the flow is going to be parallel and uniform. So, we are going to have mach number equal to M e and it is parallel flow.

So, now we have identified the exit mach number and now we know up to what point we have to extend our solution domain. So, the downstream of this point I the correspond and the in the corresponding mach cone the velocity is constant and the flow is parallel to the exit. We can now draw this mach line I P as shown here, at an angle alpha e right and that depends on the mach number that we have got here M e right. So, we can draw this mach cone alpha e. Now, once we have this then along this line every point here the properties are known, because they depend on the mach number right. So, the properties at this mach line is known mach line I P are all known.

So, that is how we get up to the exit point let us for the time being just take a step back here, how do we reach this contour or attain this contour for that now let us look at this wall. Initially this wall curvature must be provided like we have already discussed. Initial curvature for the wall must be provided, but after we are out of that influence zone where we have only single type of mach lines after that the solution will tell us what should be the wall contour. So, let us say downstream of this point the wall contour is no longer arbitrary, because as we can see here there are no multiple lines crossing right.

So, it is no longer arbitrary. Arbitrary contour will be chosen between this two points after that now we do not have arbitrary contour. Now, the stream lines in this region which we are having here the that will be determined by the known properties at all these points a b c d etcetera all these points here the properties are known. If I look at now this let us say let me remove this if I look at this let us say consider this is a mach line stream line we have two mach lines crossing here right this and this. So, we can estimate the properties here right and we can estimate how much curvature will be provided based on the analysis we have just done right.

So, we know how much turning is to be provided. Then we solve for this let us come to this position now. So, now, here again we have one mach line here one mach line here we can estimate what will be the condition. So, like that close to the wall we have like this. So, depending on this we calculate for this location then depending on this we calculate for this location like that. So, we progress calculating the wall contour d theta all along from point here to point here A to P. That is how for every point here we estimate how theta has to be varied then finally, we get this contour and at the end what we say is that now we stop here this is our nozzle exit. Everywhere here there is a contour the flow streamlines act parallel to this wall. And finally, here we get a uniform parallel flow.

So, from the known values now, which we are calculating from different location, we can fill in the solution between this entire domain from here to here, here to here and everything. Once this is obtained the solution is obtained now we can draw the stream lines also we can draw the stream lines and we can get the wall contour. The stream line between A and P is my wall contour actually right. So, if we can once we solve for this, we know how much the flow is turning what are the flow properties. So, this solution will give us all the properties, it will give us the wall contour, it will give us the mach number variation, it will give us the pressure variation, density variation, temperature variation, everything in one go as a package. That is something that is very useful information. Now, the accuracy of course, of how accurately it is predicting depends on how many points we choose here. So, how fine is our mesh finite the mesh better will be the accuracy, but then beyond the certain number of mesh the incremental advantage is going to diminish therefore, we do not want to go to very fine mesh, because as we increase the mesh the computational time increases.

So, we do not want to go to very fine mesh. One thing likes to point out here that if I look at the solution procedure, we are not solving any differential equation any where right. Essentially the equations we are solving are all algebraic equations. So, that is a very simple technique. So, therefore, there is no finite differencing or anything required it is just solving set of algebraic equations again and again therefore; the solution is also very very fast. And that is one of the major advantage of this method. We do not have to solve full navies stokes. Thing is that; however, there are some assumptions that we have met which are going to be problematic for example, we have assumed that the flow is sleeping, this is the stream line the flow is sleeping right.

We have assumed that the velocity at the wall here is the sleep velocity. This is something that is not going to happen in practical cases, we are going to have mostly boundary condition there. In practical cases there will be a boundary, where the velocity is 0 or then slightly increases in the boundary layer right. That we have not accounted for, not only that close to the wall in reality even if the flow here is supersonic we have subsonic flow close to the wall that is something that is not accounted for here and actually this method will not be even able to capture that, because the flow becomes subsonic the method is going to collapse right. So, therefore, this is applicable only in the supersonic domain. That we have to understand and not only the supersonic domain it is completely outside the boundary layer, where the flow is essentially not effected by the boundary layer at all; however, by using this method we get the contour we get the length also as you can see from here to here we get the length of the nozzle also. So, as the initial estimate this is a good technique right.

So, we can get the undisturbed or feasting flow without the boundary frictional effects we can get the contour we can get the length. Later on we can correct it by solving full navies stokes, but then the design is there. Initially I said in the last class, that for solving full navies stokes we need to have this geometry. Now we have the initial geometry which has been obtained from a simple method. now we can correct our predictions for this geometry by incorporating the real flow effects. So, what I will do is I will stop here now. In the next lecture, I will talk about real flow effects and how do they effect the performance of the nozzle.

Thank you.