Jet and Rocket Propulsion

Prof. Dr. A. Kushari
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 6

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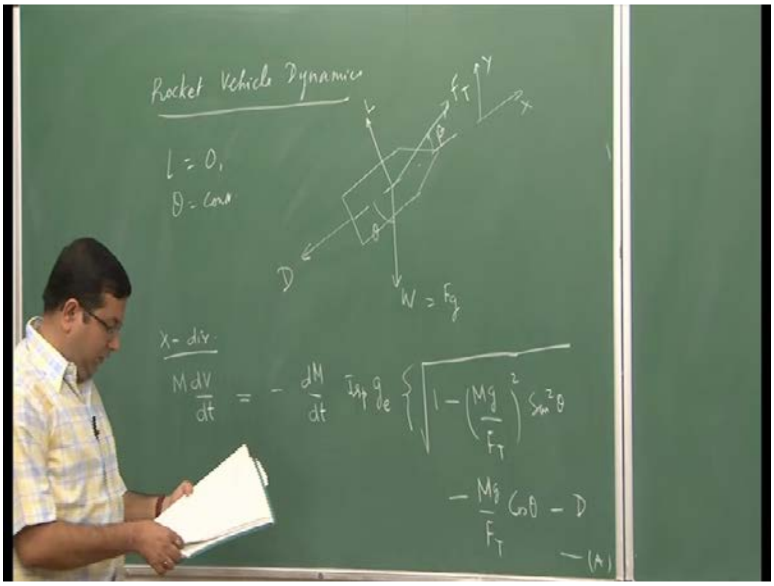

So, yesterday we were discussing rocket vehicle dynamics. And under this topic we derive an expression for the changing velocity of a rocket vehicle, while it is fired from certain condition and reach a certain condition. And we have shown that the expression when the vehicle has no lift, when L is equal to 0 and when theta is constant, where theta is the attitude of the vehicle. So, this is the vehicle we had talked about, where d is the drag, the rate of the vehicle acts down ward which is essentially the forces acting due to gravity lift is normal to this, and we have a thrust force acting in this direction, this angle is theta, and this angle is beta.

So, for this considering the reference frame like this, where positive x direction is considered to be positive. We had derive the expression for velocity change or the expression actually we derive from using Newton's second law of motion for M d V d T, this is along the x direction. Considering no lift and constant theta, this expression was equal to d M d t I s p g e then square root of 1 minus M g by F T square sin square theta minus M g by F T cos theta minus D and we had call this equation A. So, this is the equation we had derived yesterday, where M is the instantaneous mass of the vehicle, V is the instantaneous velocity of the vehicle. Therefore, d V d t is the rate of change of velocity, which is the acceleration of the vehicle, I s p is the specific impulse; g e is acceleration due to gravity at sea level. So, out of this, this term here represents the thrust, which this expression we have obtained by considering the y component of force balance and putting lift equal to 0. So, we had two component in the y direction; one coming from the gravitational force other from the thrust, and we equivated this two get, this expression essentially this is, this term comes from this angle beta, and then the second term here is the forces because of the gravitational pole. So, it is component of the weight in the flight direction or the flight path and D is the drag acting of the vehicle.

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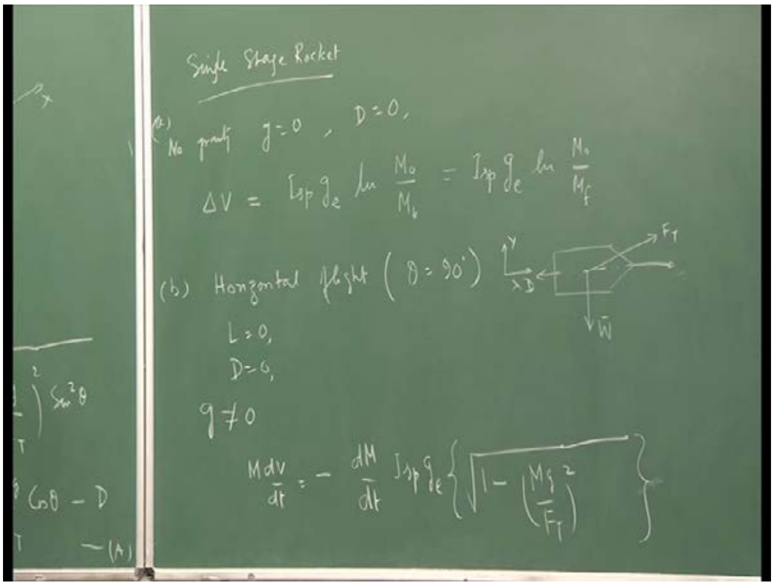

So, this is what we had derived yesterday. And then considering single stage rocket and assuming no gravity that is g is equal to 0 and we had derived the expression and also for no drag, drag equal to 0. We had derived this expression for the change in vehicular velocity del V equal to I s p times g e l n M naught by M b, where del V is the change in velocity and l n is the natural logarithm, M naught is the initial mass of the vehicle, M b is the burn out mass of the vehicle, that is the mass have to burn out, which is also equal to the final mass. So, this we can also write as M dot by M f.

So, this is the expression we have derived yesterday and next let us look at some other cases. The next case we are going to consider, so, this was case a, at this what the assumptions. Next case look at a horizontal flight, which implies that theta equal to 90 degree, which according to our convention of the system that we are talking about, this is our x and this is y and once again we consider there is no lift L equal to 0, lets us consider that the drag force is not present also so, D is also zero. However, we do not neglect the gravity like we did here so, in this case gravity is non 0, then our expression will be equal to M d V d t is equal to minus d M d t I s p g e then square root of 1 minus M g by F T square, and here we have sin square theta, here we are putting theta equal to 90 degree.

So, sin theta is then 1 so, sin square theta is 1. So, it comes like this and then the second term here was minus M g by F T cos theta, but when we put nice theta equal to 90, cos theta is 0 so, this term goes to 0 and we are neglecting drag. So, this equation will be only up to this. Now we can integrate this and we get an expression for the change in velocity, which I will write up here after integration. Final expression that we get for the change in velocity will be thus the final expression for the velocity increment can be given as del V by I s p g e.

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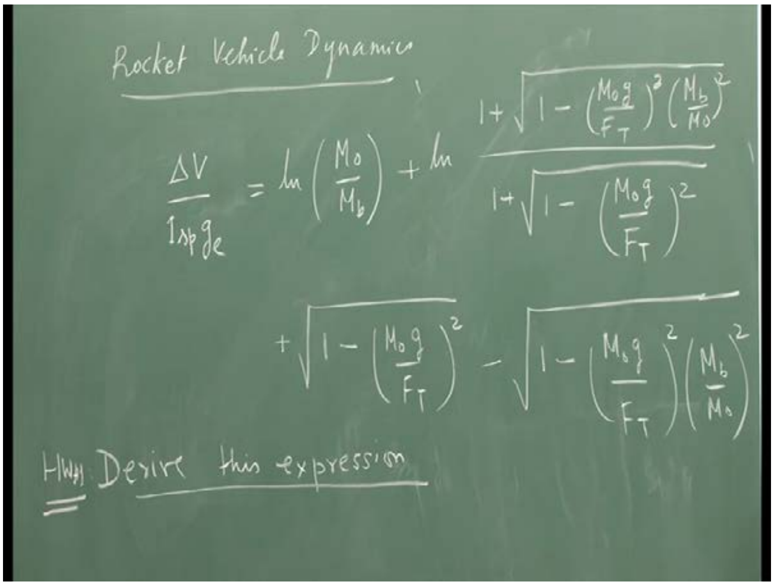

So, we take this I s p g e term to the left hand side, equal to l n M naught by M b once again M naught is the initial mass, M b is the burn out mass or the final mass plus now the integration of this term here will give l n 1 plus square root of 1 minus M naught g by F T square times M b by M naught square divided by 1 plus square root of 1 minus M naught g by F T square, we will have some more terms plus square root of 1 minus M naught g by F T square minus square root of 1 minus M naught g by F T square M b by M naught square .

So, this is the expression for velocity change, when we have power flight of a vehicle which does not produce any lift and drag is also 0. Let us say it is flying at age of the atmosphere and the vehicle is going for a horizontal flight. So, application of this will be, will say I c b m s for the substantial deviation of flight, the flight fly horizontally. So, that gives us the expression for velocity increment for that particular condition, when the gravity is also acting as we can see here the velocity increment is primarily the function of course, I s p then the initial mass, final mass and the thrust.

So, this is in the cruise condition ok, because if it is thrust is on; that means, it is in the cruise condition. So, this expression then will give us the power flight cruise condition for horizontal flight. Now I will give as a home work, home work number one: derive this expression, because I have given this expression, I have not solved it. So, I will ask you people to derive this expression by integrating this equation will get this expression. So, this is the second case that we talked about, where we are talking about a horizontal flight. As well as the space application is concerned one of the most important flight condition is the vertical flight, because typically the rockets are launched vertically upward.

So, the next case that we are going to talk about is vertical flight. So, the case three or the case c we will be discussing again we are still focusing on single stage rockets will come to multi stage rockets later.

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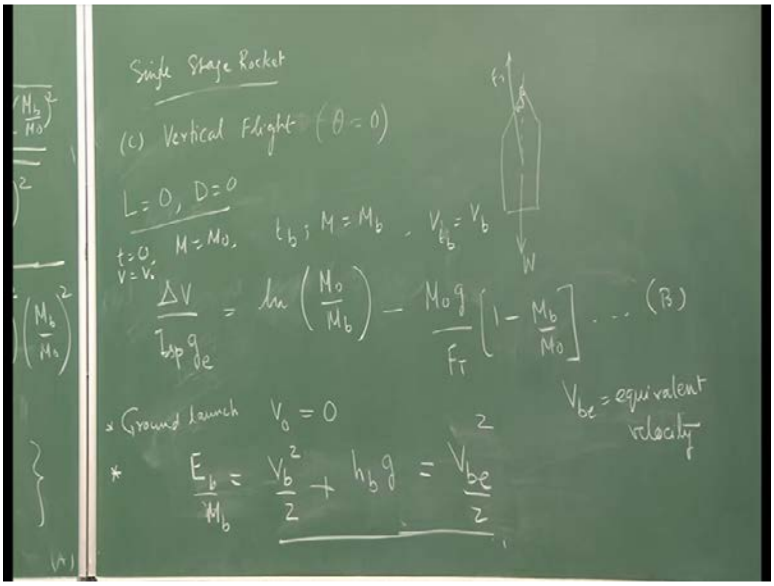

So, next case c we have vertical flight. So, for vertical flight we have theta equal to 0 degree right. So, if I look at my vehicle theta equal to 0 degree therefore, weight is acting downward and the thrust is acting at second angle F T given by beta. So, the vehicle is oriented like this. Now, let us consider the same case with no lift and no drag. Ideally the lift we do not want to have because, that will produce a side wise force, which we will read the vehicle out from the intended part drag also is. If we do a back of there will a comparison typically the drag forces are much smaller compare to the thrust or the weight.

So, therefore, drag can also be neglected, also drag will be there little bit of drag will be there because, if you are talking about within the atmosphere. Of course, outside the atmosphere drag will not be there, but with in that atmosphere drag is going to be there drag will try to slow it down, but the magnitude of this drag in comparison to the thrust force or the gravitational pool is much smaller therefore, for practical purpose is we can neglect the drag. So, therefore, these are conditions that we are considering; however, gravitational forces still are working. So, once again we had the same differential equation, which we need to integrate with these conditions already difference is that now, theta equal to 0 so, sin theta is 0 and cos theta is 1. So, when we put this conditions in our differential equation and integrate we get del V equal to I s p g e equal to l n M naught by M b minus M naught g by F T 1 minus M b by M naught, let we call this equation B.

So, here we are integrating from time T equal to 0, where mass is equal to M naught to let us say time equal to T b, where mass is equals to M b. So, we are integrating between these two types. So, the initial velocity let us say is something say V 1, final velocity is V 2, V 1 can be 0 as well. So, final velocity here is V b, V at t b is V b let us say now. So, this is simplified equation that we get for the velocity increment for a rocket vehicle, which is flying vertically upward. So, if we launch the vehicle from ground now, the special case ground launch. Then what is V 0 that is the initial at time T equal to 0, V 0 is 0.

The initial velocity is 0 at time T equal to 0 and then it goes as its going up the velocity is increasing. We often want to estimate the velocity of the vehicle at a particular altitude because finally, let us say, if we are talking about rocket launch then at sudden altitude it has to cross the escape velocity, first of all we have to see whether, we are able to cross the escape velocity or not and it has to be using certain altitude or certain time frame. So, therefore, that can be estimated from this equation. Now, we want to find out what is the velocity change in a sudden altitude, second point that like to point out here that by the time we reach that altitude, what is happening here as the velocity the vehicle is moving we are producing some thrust. So, some energy is being provided and that energy gets converted into the vehicle dynamics. Now, as the vehicle is moving the velocity is increasing therefore, its kinetic energy is increasing at the same time it is giving altitude. So, its potential energy is also increasing.

So, the total energy of the vehicle is the sum of this kinetic energy and potential energy. So, let us say the total energy of the vehicle at burn out let us say E at the burn out which, is point where all the propellant has been burnt. So, the total energy there is the kinetic energy I am talking about per unit mass. So, this will be per unit mass basis, let me look it as write it as by M plus h b g. So, M b V b square by two is the kinetic energy M b h b g is the potential energy right. Therefore, this is the total energy of the vehicle at the burn out height h b, where I am defining h b as the burn out height flight say.

So, this is the total energy that we are gaining. Instead of considering this energy separately we can define an equivalent velocity and that equivalent velocity then is the virtual kinetic energy with the vehicle attains and this altitude. So, we do not consider the potential energy separately, we club this two to together like, we have done for a definition of the equivalent velocity for the exhaust, where the potential forces a potential sorry the pressure forces and the momentum term where club together.

So, same thing we can do here also. So, we define component called equivalent velocity equal to V b e so, this V b e is our equivalent velocity. Then now, what do we see in this equation M b V b e square by 2 is the kinetic energy of the vehicle, if we do not consider the potential energy right. So, then this equation essentially with the V b e represents the equivalent velocity, which accounts for the kinetic, potential energy variation as well. So, then now, we can work with this equation little bit and see what are the consequences of writing is in this form and as I have said that, I have defined h b as the burn out height. Now, if we take that expression and put it back into this, we can get an expression for V b e equivalent velocity so, let me try and do that.

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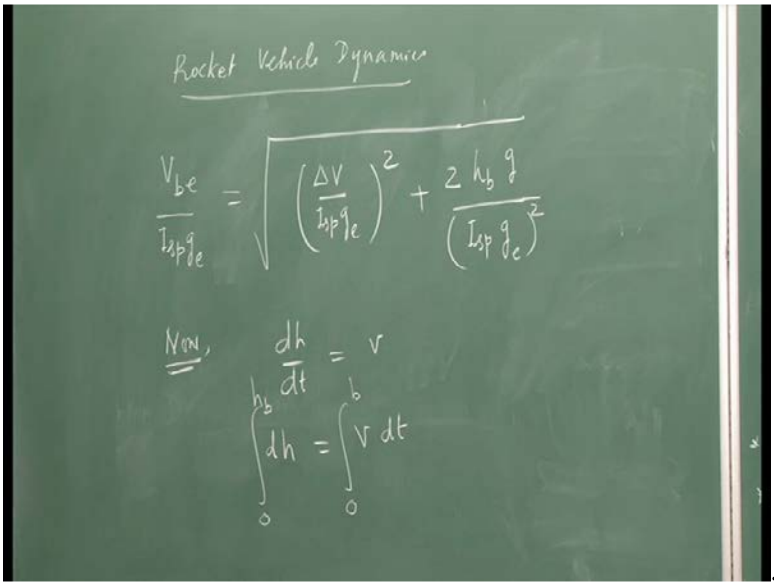

So, we can write V b e by I s p g e, which is equal to square root of, if I look at the expression for delta V e then delta V e is nothing but V b because, we started from V naught equal to 0 right. So, this term is delta V right. So, we replace this by delta V. So, then this will be equal to delta V by I s p square right. Because, from this what we are writing is first we can take this 2 on this side. So, two will cancel out, we will have a 2 here, this will cancel of and then V b e is equal to square root of g b square plus 2 h b g right. So, this is the expression for V b e coming from this equation.

So, now first term there in this equation are corresponds to V b square, which is essentially nothing but delta V square and the second term corresponds to the potential energy term 2 h b g. So, we can write it as 2 h b g by I s p g e square right. So, this equation is obtained from equation B and this equation, the second term here is obtained from this equation let me call this equation C right. So, this is obtained from B, this obtains from C. Now, the next step is to find this salvage B, how far will the vehicle go for a given velocity increment because, velocity increment as we have seen from equation B is a function of I s p the thrust, that is being produced or converted into the variation in mass M naught and M b right. So, that is the internal to the system design of the rocket. So, for a given rocket design and propellant being used how far the vehicle will be going that will be obtained from this expression.

So, let us say now, the rate of change of altitude d h d t is equal to V right, that is velocity of the vehicle. Therefore, h is equal to sorry d h is equal to V d t, if we integrate this from 0 to h b. 0 is on the ground, we are talking about the ground launch to h b the burn out height right. This is equal to 0 to b, the burn out state integral V d t. So, the first term here then is equal to h b. So, let us write this now, as h b so the first term the left hand side of that equation is h b.

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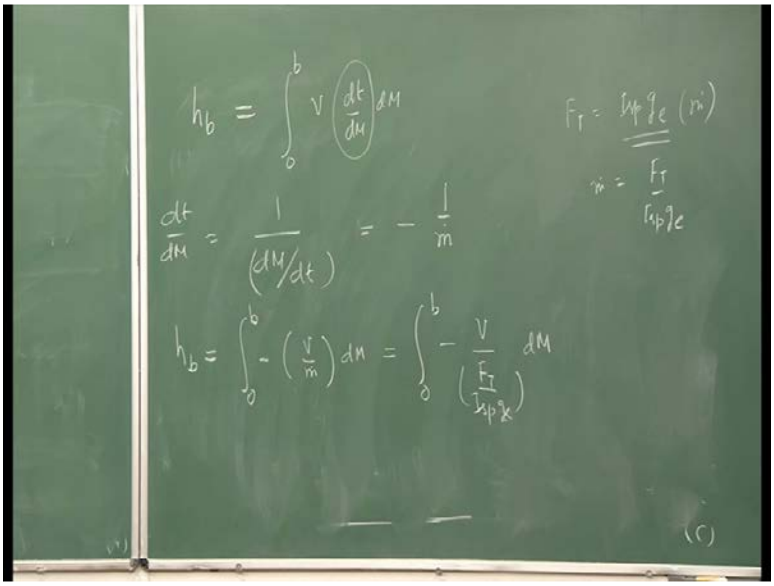

So, let me, first write this h b is equal to the integral V d t, but what I will do is I am write it differently d t, I will write as d t by d M times d M. Now, what is this term d t by d M? d t by d M is equal to 1 by d M by d t and d M by d t is rate of change of mass, which is equal to minus M dot.

So, therefore, this is equal to minus 1 upon m dot. So, going back to this expression then h b is equal to integral 0 to b minus V by m dot d M and d by m dot for this case can be obtained as function of the specific impulse right, because we have seen that the thrust is equal to I s p g e, which is the equivalent velocity times m dot right. Therefore, m dot equal to F T by I s p g e right. If I put it back into this equation I get 0 to b minus V by F T I s p g e d M. Now, what we can do is the expression for this velocity V now, can be obtain from equation B, the differential equation and we can replace m dot that was a peering in equation V by M and we can replace V b by V.

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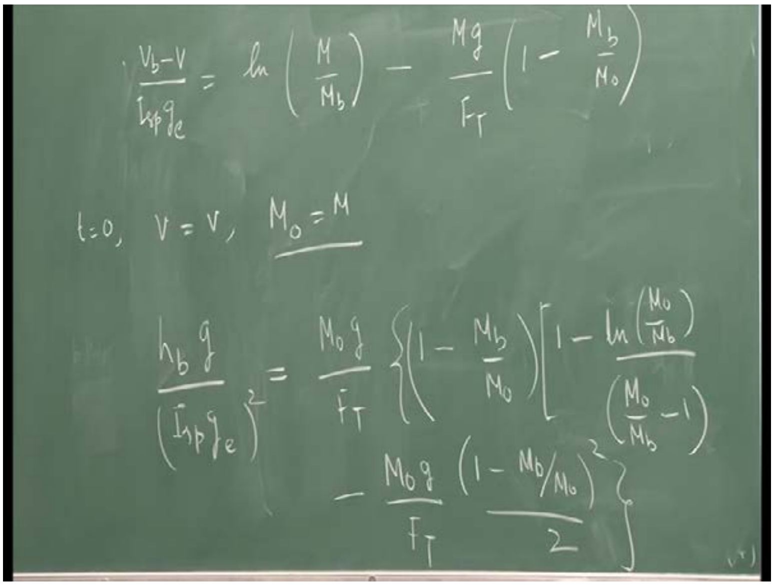

So, if I go back to the equation B, let me first write equation B here and then I will replace. Equation B was V b by I s p g e, which are this term was del V delta V, but V 0 was zero. So, this becomes equal to V b equal to l n M, just a second this will be V b by V we are starting from particular velocities of V b and then going towards V. So, this is becoming V b minus V by I s p g e.

So, the initial state now is our V. So, what we are saying is a time T equal to we are considering V equal to V. So, the initial mass for this case is M some instantaneous mass. So, that in this expression we replace m dot by M. So, that will be then equal to M by M b minus M g by F T 1 minus M b by M naught right. Now, what we do is we take this expression for V and put it into that and then we integrate. Finally, upon integration what we will get is let me write it here, h b g by I s p g e square is equal to M naught g by F T 1 minus M b by M naught 1 minus l n M not by M b by M naught by M b minus 1 minus M naught g by F T 1 minus M naught by sorry M b by M naught square by 2. This is the expression for the altitude gained by the vehicle starting from ground at 0 velocity till the burn out as you can see that is quite complex expression.

But the use of this expression is quite simple because, what we see here that the burn out height is the function of I s p, it is a function of initial mass; it is function of the burn out mass and the thrust. So, therefore, thrust is appearing as a very important parameter in the estimation of the burn out the height. Now, I would like to emphasize one point here that we worked with the parameter V b e right. What is V b e? What is the significance of this terms V b e? The equivalent burn out velocity V b e is the major of total energy of rocket at the burnout altitude right. Because, the rocket is flying with certain velocity V at the burn out altitude velocity is V b and it has a potential energy also.

So, V b e is the major of total energy of the rocket at the burn out or equivalent velocity of the vehicle at the ground to attain the given altitude and burn out velocity right. So, this is the energy that has to be provided on the ground. So, that it can attain the burnout height with the given burn out velocity. So, this therefore, is a significant or important parameter, this is energy that must be provided by the thrust. So, therefore, this becomes an important design parameter for completing a machine requirement. So, far we discussed everything during the burnout phase, I would like to point out one point here that this not the maximum height attained by the vehicle this is maximum height during power flight at the end of this machine we reach h b all the fuel is burnt now, the vehicle does not have any fuel. So, there is no power produced in that case F T goes to 0.

However, the vehicle has its own kinetic energy and potential energy now, what happens is that sharing of energy between these two. After this point there is no more increment in kinetic energy because, no more thrust is provided. So, now, the kinetic energy starts to decrease, but potential energy will be increasing because, kinetic energy is being transferred to potential energy. A point will come when the kinetic goes to 0 and that is then the potential energy is maximum. So, now, all the energy is converted to potential energy after that it cannot go anymore up. Now, it will start to come down so, therefore, the height at the end of this stage, where all the energy is converted to potential energy is the maximum height attained by this vehicle. So, now, let us look at the significance of this. This will then give us the maximum height that the vehicle will attained equivalent velocity will give us the maximum height the vehicle will attained, if it is launched with this.

So, this is the description of vehicle dynamics starting from a phenomenological approach, where we took a particular method followed a particular method and did some case study. Now, let us repeat the same thing again using a little different method and we will see that we derive all the same equations with a little different approach. So, the next approach that we are going to follow is essentially the primary objective, this approach also is took an expression for the velocity change V b right. Now, just to differentiate these two approaches now, I will be using u as velocity not V. So far I have been using V as the velocity.

Now, we are using u as the velocity. So, that we do not have the any confusion. Another point, I like to make here is that in this entire discussion, I considered that the thrust is slightly half axis slightly vector. Now, in this approach, I will not consider that I will consider that the thrust is acting in the direction of flight. So, going back to that let us, reconsidered the vehicle that, we have been discussing actually here, we will be just silent about the direction of thrust. So, we will talk about the same vehicle that, we have discuss so far, but follow a little different approach. So, so far we have discussed an approach, where we considered the thrust as the parameter and estimated the burnout velocity and burnout height considering thrust as an unknown parameter.

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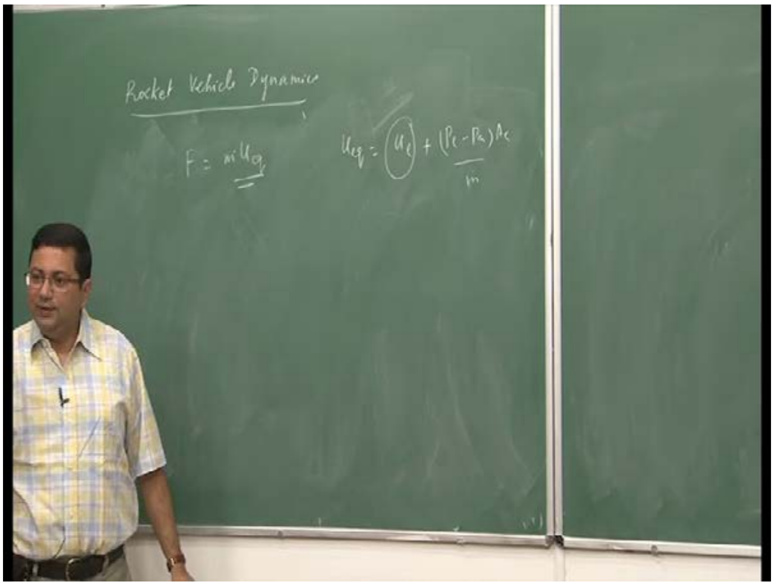

Now, let us look at a different approach of getting the same equations, but here if you recall we have derived in the last class. That the thrust is equals to M dot u equivalent, where u equivalent was equal to u e plus p minus p a by a e by M dot.

So, u e is the exit velocity. This is the parameter, which you are rocket engine is providing to you not the thrust that is the outcome of that what is the rocket engine providing is u e and M dot these are the two parameters that are coming from the rocket engine. So, now, what we would like to do is derive the same expressions that, we have derived for the velocity and height in terms of this parameters. So, we will reconsidered the same vehicle with the same flight conditions and consider we have an accelerating rocket.

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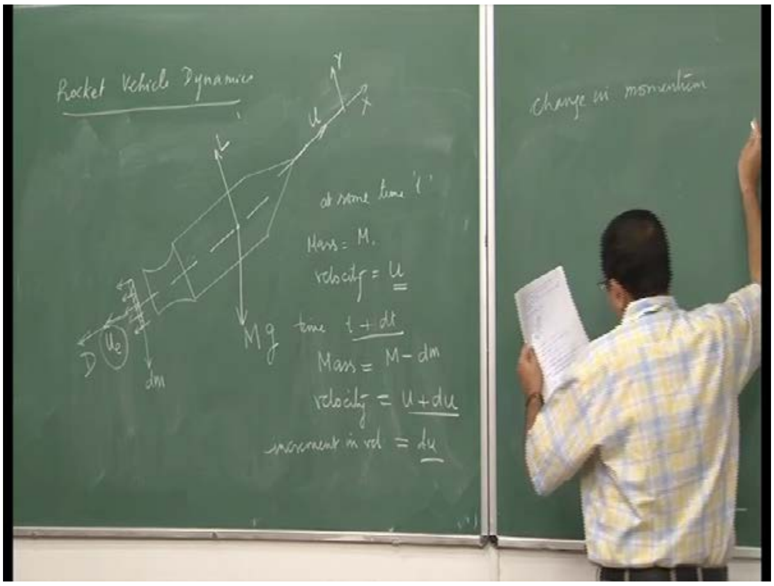

Let us consider an accelerating rocket given like this. This is the axis of this rocket along x direction, this is the y direction. Once again let us drop the forces acting on this rocket so, we have the weight of this rocket acting downward because, of the gravitational pole. If the instantaneous mass of this rocket is M then this weight is equal to M g, we have the lift acting normal to it like this, we have a drag force acting like this and let us say that the vehicle is moving with the speed u.

Now, we are not considering the thrust separately because, the thrust is produced internally in this case right. In the previous description we considered thrust as the reaction force now; we do not considered thrust as the reaction force we said that is being produced by this vehicle. So, now, let us look at how the thrust production mechanism, let us considered that the exhaust jet is moving in this direction with a velocity u e and let us now, look at different conditions that are prevailing in this. Consider at some time t, the instantaneous mass of the vehicle is equal to M and the velocity of the vehicle is given as u in the instantaneous mass and velocity. After a small increment of time d t so, at time t plus d t we have a change in mass because, let us consider a small amount of mass shown here, like this given by d M has lift this vehicle.

So, the total mass of the vehicle has reduced. So, initial mass was M after a small amount of time d t, during this small time d t, the mass d M has lift this vehicle. So, now, the mass of this vehicle after this time is equal to M minus d M. So, once again I would like to point out that d M is the mass of the exhaust in this small period of time d t. And then because, of this there is a change in momentum, it is the vehicle is becoming lighter. So, therefore, it will be moving faster. So, the velocity has also change and let us considered that the velocity at time t plus d t is equal to u plus d u. So, now, let us see here, u was the initial velocity of the vehicle; final velocity is u plus d u after this time. Therefore, the increment in velocity is d u so; increment in velocity is equals to d u. Now, the exhaust is moving out with a velocity u e this is the exhaust velocity with this formulation with this definitions of various properties. Let us now, look at the change of momentum of this rocket vehicle along the x direction.

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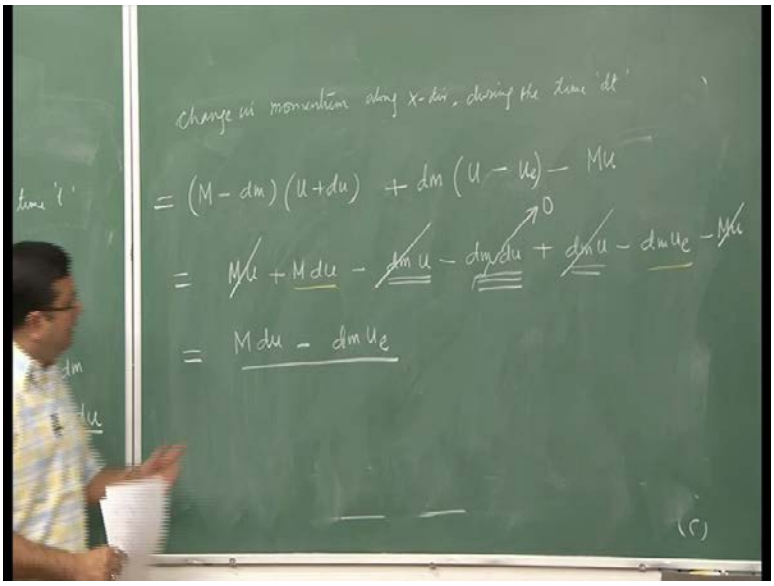

So, let us consider change in momentum along x direction during the time d t. So, our initial time was t, the final time is t plus d t. So, the time duration we are considering is d t during this time, how much change in the vehicle momentum has happened. So, the change will be the final momentum minus a initial momentum. The final momentum is given as the, as we know that, the momentum mass time velocity. So, the final momentum is final mass which is the mass at time t plus d t for M minus d m times the final velocity u plus d u. So, this is our final momentum. This is momentum of the vehicle right, but we have another thing the momentum of this exhaust gas that also needs to become considered.

So, this small piece of mass which, was there inside has now come out. So, that mass also has a change in momentum. So, what is the mass of that small piece is d m and now, there is a change in velocity of this small mass and this is at it is going in this direction right. So, this is the change is the negative x. So, let us say minus d m during this time what is the velocity of this small mass at time t plus d t? It is equal to the exits velocity because, this is velocity, which is come out right. So, this is the final momentum and at time t what was the velocity of the small mass u because, it was moving with the vehicle right. So, therefore, this is equal to minus d m u e minus u. So, this is the momentum of change in momentum of this small mass which, was inside. Now, notice one thing, we know that the mass is always conserved right, by writing it like this we are actually conserving the mass right.

As a total mass is M minus d m plus d m. So, mass is conserved. So, this can be re written not like this, but what we say is that, we take the negative sign out we make it plus d u they must becomes u minus u e there is the same representation. Typically there exist velocity is going to be higher than the vehicle velocity therefore, this term will be negative. So, we get the negative term anyway. So, we can write it like this, writing it like this essentially is the change in momentum in the positive x direction right. So, d m u minus u e is the change in momentum in the positive x direction, minus d m u e minus u was the change in momentum in negative x direction. So, therefore, these two are consistent with each other. So, this is then total momentum at time t plus delta t and the momentum at time t was M times u. So, the change in momentum is the final momentum minus the initial momentum.

So, this is the total change in momentum. So, I put the equal sign here to represent the total change in momentum. Now, let us expand this so, if I expand this, this becomes M u plus M d u minus d m u minus d m d u plus d m u minus d m u e minus M u. I have expanded this expression. Now, let us simplify this, this M u and M u will cancel off, then we have one term minus d M u and plus d M u, so this will also cancel off. Now, let us look at this term d M d u, d M is a small quantity, d u is also a small quantity. So, therefore this term is a product of two very small quantities in a second order term. So, what we can do is with respect to the other terms, this can be neglected. So, we can neglect this term with respect to the other terms. Therefore, now what we are left with is only this two terms, let me just highlight M d u and minus d m u e.

Therefore the total change in momentum during this small interval d t is equal to M d u minus d m u e. So, this is the change in momentum that we will be considering next for the estimation of velocity increment. Now, I would like to point out one thing here is now what we interested in estimating this term d u, this is the change in velocity we would like to estimate this term. So, let us stop here now, and we will continue in the next lecture.