NOC: Introduction to Airplane Performance
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Lecture - 10
Equations of Motion: Static Performance

Good morning, today we will be trying to write equations of motion for an aircraft which will be assumed to be moving in a vertical plane, that is it will be going like this. It is not doing any side motion.

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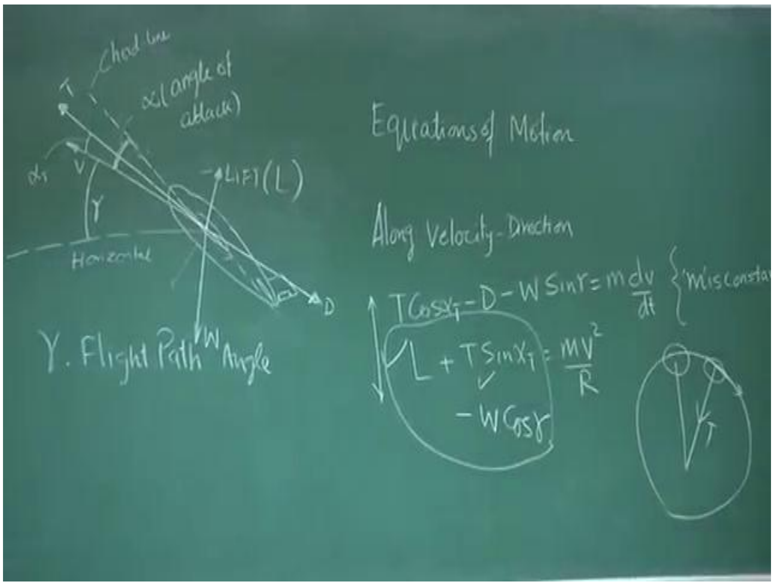

So, we are thinking, I am trying to model through equation of motion that as if the aircraft is on a climb like this, maintaining the vertical plain. And that is typically represented here before we try to write the equations of motion, let us understand few nomenclature, this dotted line what we are seeing this is basically the chord line that is I have joined the leading edge and trailing edge of the wing by a straight line and extended it, it has a significance, because wing is the primary lifting component.

So, what is the angle between the wing chord line and the free string velocity that gives you the angle of attack in a vertical plain and that is primarily responsible for the lift. Because, wing is primarily required to generate lift, now this is the chord line and every aircraft we will have an engine, because after all it has to have enough thrust to overcome the drag experience by the airplane.

So, let us say engine is installed in such a way the thrust vector is in this direction which is not exactly on the velocity direction. Because, this airplane will have not only thrust as a force it also will have gravity, we will have a drag. So, resultant velocity will be somewhere different than the thrust direction and let say the angle between thrust and velocity vector in the vertical plane is alpha T, T for Thrust and this line is the horizontal line and gamma by you know is called flight path angle, gamma is flight path angle.

What is the meaning of flight path angle? If the airplane is going up or climbing like this, then I can trace the centre of gravity of the airplane by following the velocity vector like this. So, this velocity vector, how much angle it makes with a horizontal is the flight path angle. And now if I see here, if this is the velocity direction by definition the drag will be exactly opposing the velocity and that is why drawn like this, the drag lift to be perpendicular to velocity. So, lift is perpendicular to this velocity vector and weight will be acting vertically down, so weight is here.

Now, I will write equation of motion along velocity direction, along the v direction and one perpendicular to v direction. So, what happens if I write along velocity direction, I could see the thrust cos alpha t that is component here which will try to accelerate this machine whereas, the drag will try to decelerate, so T cos alpha T minus D. Same time, the weight also will have a component along velocity direction which will also try to oppose it.

So, there is a W sin gamma and this net force will give an acceleration which is m d v by d t. But, please note what a caution here, we are assuming m is constant; that means, we are actually assuming that the weight change, because the fuel consumption is negligible. Now, we will be writing equations perpendicular to v and you could see forces acting on this vehicle perpendicular to v one of them is lift, another is component of thrust, another is component of weight.

So, if I want to write this equation now, this lift perpendicular to v, T is the component of thrust and then I have to add W cos gamma and this net force will generate an acceleration, centripetal type acceleration and which I am equating as m v square by R. Once you have writing this second equation, it will help you in reminding this thrust which is responsible for changing the velocity vector and that is why we have centripetal acceleration.

Similarly, here also this net force L plus T sin alpha T minus W cos gamma will try to change the velocity vector, direction of velocity and in turn it will generate centripetal acceleration. So, these are the two equations will be using to derive something which are of very, very importance.

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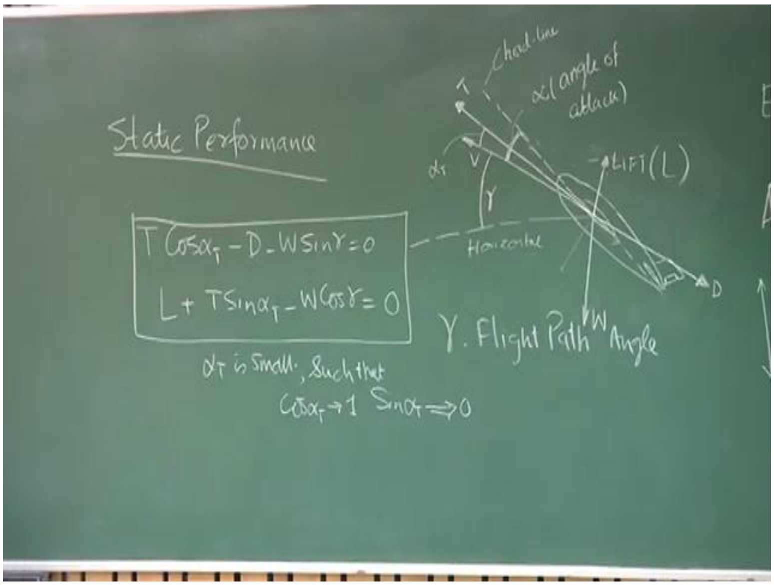

We will be primarily dealing with static performance. What do you mean by static performance? We are actually concentrating on performances which are non-accelerated, that is unaccelerated flight that is if I am cruising and cruising at constant velocity, if I am climbing and climbing at constant velocity, so we are not doing any flight where there is a acceleration. So, if I now come back to these two equation, the first equation for static performance become T cos alpha T minus D minus W sin gamma equal to 0 and second one becomes L plus T sin alpha T minus W cos gamma equal to 0.

Now, see if we impose some approximation, which are fairly good approximation for example alpha T, alpha T will be very, very small. So, for all simplification I can assume that alpha T is small, such that cos alpha T is 1 and sin alpha T goes to 0, goes to alpha T and alpha T small will goes to 0.

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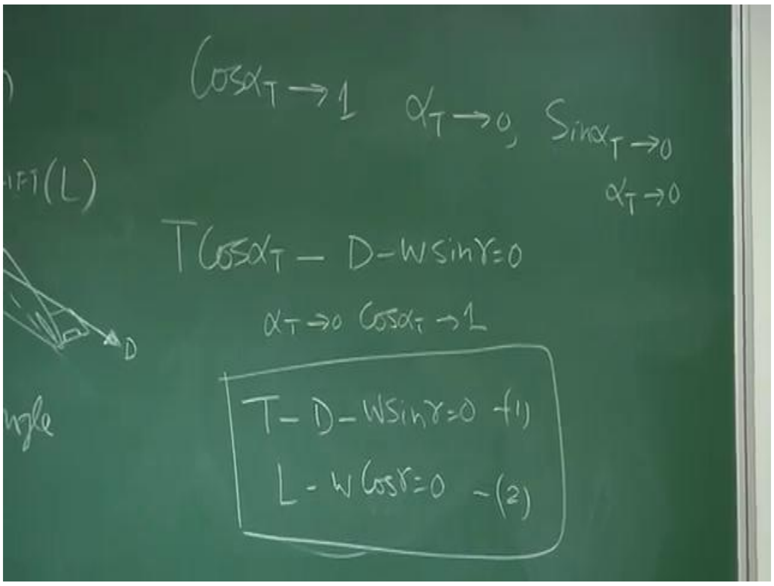

If I now put these conditions here, what do I get then I get, if I put cos alpha T tends to 1 and alpha T for tends to 0, sin alpha T tends to 0. Tends to 0, because it tends to alpha T and alpha T tends to 0, this you have to be very careful, because when you were making assumption alpha T small sin alpha T going to 0 or very small. But, same time we should be careful that this term is 3 sin alpha T, if the T is very large sometime it may not be very, very small to be neglected. But, we are assuming that good enough small, so that I can neglect this term.

One I write T cos alpha T minus D minus W sin gamma equal to 0, if alpha T 0 cos alpha T tends to 1, this gives me T minus D minus W sin gamma equal to 0. And the second equation, I get L minus W cos gamma equal to 0, this is second equation. Now, see here if I am talking about additional cruise that is I am cruising like this with gamma 0. Because, if gamma is non zero, if gamma is positive I am climbing, if gamma is negative I am descending. So, we are talking about a cruise where I cannot reduce the altitude, so I am going for a gamma equal to 0.

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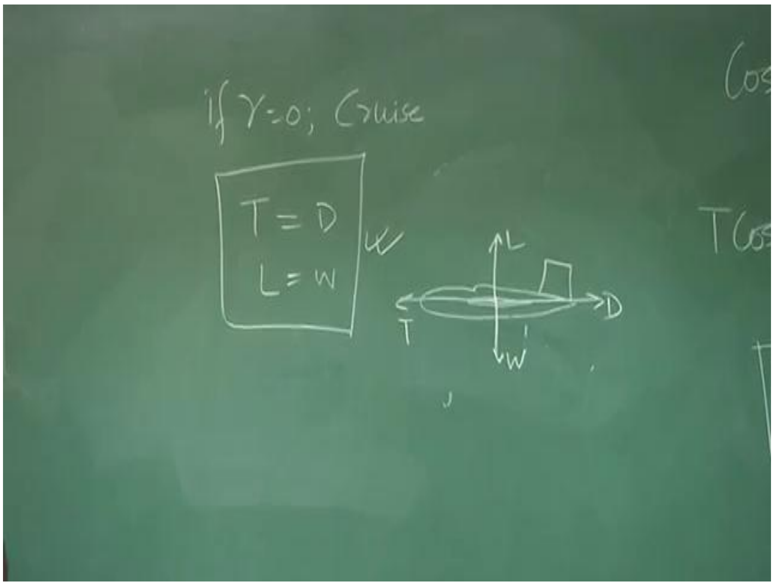

So, if gamma equal to 0 which is true for cruise, then I have T equal to D and lift equal to weight. So, now you understand when I draw a diagram like this, what are the approximation has gone into it. One is, if I repeat alpha T is small that is angle between thrust and the velocity vector is very small and also gamma equal to 0, then only I can directly draw this. This diagram should not confuse you in thinking that alpha is 0.

Because, if alpha is 0 where from do I get lift whole game, whole episode of airplane performance is towards generating requisite alpha or requisite see and you will understand soon. So, this is a very important relation and we will be using it rigorously.

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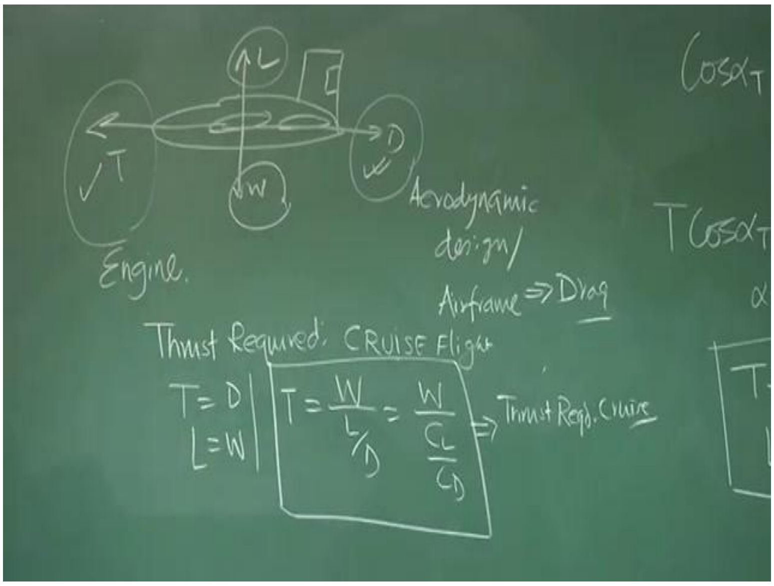

Now, it is very simple to write on the blackboard that thrust equal to drag and lift equal to weight, but first question comes who will give this thrust, I need an engine, what type of engine. So, this part is taken care by engine and the drag what should be our aim, mostly specially for cruise we should try to fly such a condition that the drag is minimum. So, this is more governed by the aerodynamic design, airframe design they actually attribute to drag in a sense that if I make an efficient airframe, I can have drag low, if I am making a airframe which is blunt I have more drag for the given speed.

Now, but the question will come it is fine for thrust I need engine, for drag I need aerodynamic design. So, that the total resistance is less, for lift also will be decided by aerodynamic design and more importantly lift means wing, whose mode is primarily responsible. So, we will design a wing area, so that they get sufficient lift for the given velocity and given altitude and with, why it is important. See in cruise, if I am cruising my aim is I must fly at a speed such that the lift is good enough to balance the weight.

So, if it is, if you are going on increasing the weight, then to generate that lift to balance the weight also will be demanding and we may not be able to fly at that speed to really balance the weight. So, we have to be very careful when we are deciding on the weight even at cruise conditions. Now, let us first try to understand what is the thrust required, we are talking about cruise flight.

As I told you, these are very important equation thrust equal to drag and lift equal to weight. So, I can club them, I can write thrust equal to W by L by D, I can write that W by C L by C D, this is an expression to get thrust required for cruise. But, how to calculate this question is this, how do I calculate. Let us go one step more to understand how do I calculate.

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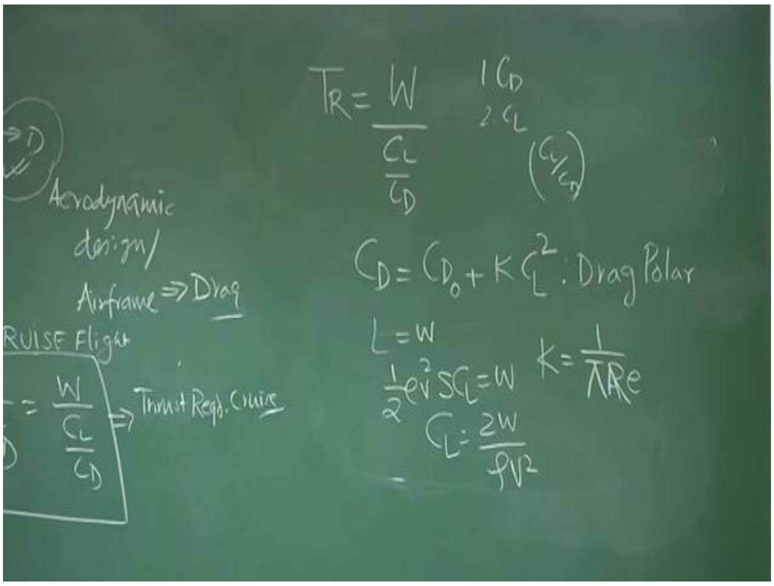

We say T R, we now represent thrust required that is for a given W which is given by C L by C D and by now, you know C D I can write as C D naught plus K C L square which is the drag polar of the airplane. How do I get C L? C L is who will give us the C L? Why do we need a C L? I need a C L to balance the weight. So, where from I get the value of C L? How much C L is required that is from lift equal to W, I know half rho v square S C L equal to W. So, I know C L equal to 2 W by rho v square.

So, now again see this, what is our aim, our aim is to calculate thrust required. So, I know, what is the weight of the airplane, I know what is the C L I should be flying. Because, I know what is the speed, I will be flying, I know what is the altitude I will be flying, I know the value of C D naught. Because, this is constant for a given airplane and also K you know K is 1 by pi aspect ratio e, where e is the off load efficiency, it could be between 0.7 to 0.9.

So, all these values are known, once I know this for a given flight condition, I know C D, I know C L. So, I can find out C L by C D as simple as that. So, what is that? I know C D, I know C L. So, I can find out C L by C D it is as straight forward and I can use this expression W by C L by C D is thrust required, but the story does not end here. We have to give a more insight into this expression, so that a designer can handle this situation.

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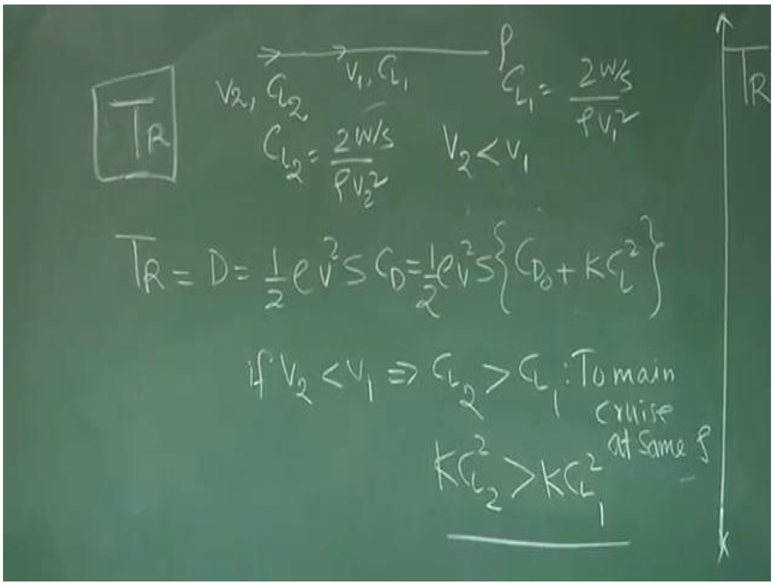

Let us ask our self, why do I need thrust, what is our aim. Our aim is, we are requiring thrust required just to balance the drag and who is giving us drag, drag is because of geo lift drag C D naught. And because we are requiring lift that is also giving a drag which I call induced drag. So, let us see if I write now thrust equal to drag and equal to half rho v square S C D and then, further I write as half rho v square S C D naught plus K C L square.

One thing I know, if I want to find out what is the thrust required for different, different speed, meaning thereby at different speed drag also will increase. So, to balance that drag I need to increase the thrust. So, I want to know the variation of thrust required versus speed through drag, this is important through drag I am trying to make an assessment. One thing physically I know that this gentleman C D naught is going to be constant for lower speeds.

But, as I am increasing the speed my C L requirement will go on reducing for same height, maintaining the same height that is if I am flying here at this altitude say rho and I am flying at a v 1. So, I need a C L 1 which is given by C L 1 equal to 2 W by S by rho v 1 square. Now, suppose I want to fly at same altitude again, but I want to fly at v 2 then I required the C L 2. So, C L 2 will be what, C L 2 will be 2 W by S rho v 2 square. Which one will be more, C L 1 will be more or C L 2 will be more?

If v 2 is less than v 1 that is initially let us say I am going at 80 meter per second at the height 10 kilometre I need the C L. Now, same height I am flying at 70 meter per second, then C L requirement will be more, because I want to balance the weight. So, if v 2 is less than v 1 that implies C L 2 is greater than C L 1 to maintain cruise at same altitude. C L 2 greater than C L 1 means, the induced drag at C L 2 at v 2 will be more, so K C L 2 square is greater than K C L 1 square.

So, what is the message? Message is, if I am maintaining the cruise at same altitude, if I am flying at a higher speed then the induced drag will be less. As the speed increases, the induced drag component will reduce, because C L requirement is less. If I am flying at a lower speed, then C L requirement will increase, so the induced drag part will increase.

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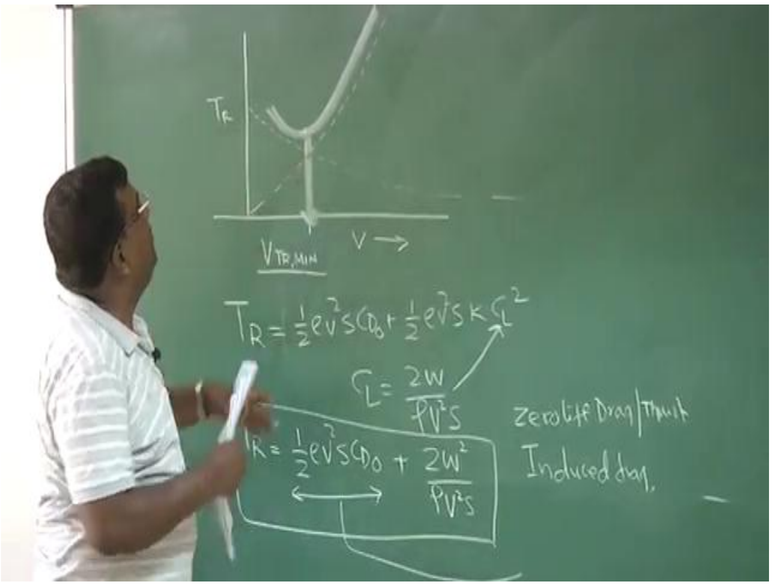

So, now, if I understand this and try to figure out how thrust required will change with v, one is by this understanding I can write I can get a feel of the variation. But, if I want to write explicit expression then I will do one more step thrust is half rho v square S C D naught plus half rho v square S K into C L square, but we know C L is 2 W by rho v square S, so I plug this here. So, what do I get? I get thrust required equal to half rho v square S C D naught plus 2 W square by rho v square S.

This is clear, how will you get this? I am substituting C L here by this expression. So, C L square will be 4 W square rho square v to the power 4 S square and this, they will get cancel from here, so you will get 2 W square rho v square S. Now, you could say explicitly as I am increasing the speed, this thrust required which is basically the drag goes on increasing with v. However, the second term as v is increased this term goes down which is by physics you understand, as I am increasing the speed, this term which is basically induced drag will go on reducing.

So, it is consisted with whatever physical understanding we have and we always see we will refer this first time as zero lift drag or thrust and second one is induced drag. Induced means who has induced this drag, this is lift induced drag, because from it is coming from C L and by physics also you know this comes, because if there is a lift, there is a pressure difference between the lower portion and the upper portion of the wing and there will be vertices forming at the wing tips and that we will take out energy, because this will have rotational energy. So, naturally there will be a loss of speed and we attribute that in terms of induced drag.

So, now if I see this I could see that the zero lift drag if I want to plot, it will be going like this it will go on increasing with v. However, the second term will go on reducing as v increases and if I find the net, net will be something like this and this is the portion which will be giving it name v for thrust required minimum. See by drawing these I have almost given the message that the v for thrust required minimum will be somewhere at the point where the parasite and the induced drag they intersect. Whether they will intersect here or not let us analytically see this.

How do I interpret this point? I am trying to interpret this point which is almost tangent here, this is the thrust required minimum. So, this is the velocity for thrust required minimum and I will do some steps algebraic steps using this equation only to find out what is the condition at which the v will be this v and the thrust required will be thrust required minimum.

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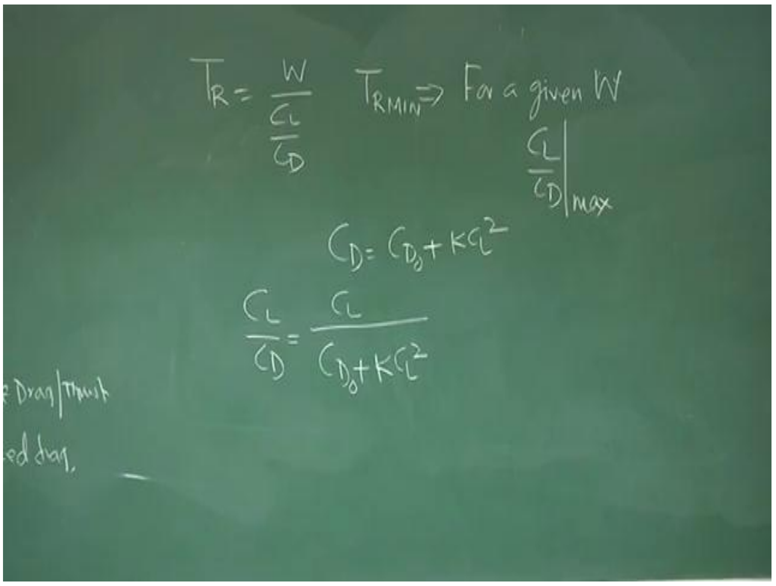

Let us first understand thrust required we have seen is W by C L by C D, one thing is very clear to us, if I want to have thrust required minimum this implies for the given weight C L by C D should be maximum for a given W C L by C D should be maximum. What is the meaning of C L by C D should be maximum question is this? We know that C D is basically C D naught plus K C L square. So, what is C L by C D will be C L by C D naught plus K C L square.

So, what is the question you are going to ask, we are going to ask question what is that C L I should fly. So, that C L by C D is maximum this clear, the question is what is that C L I should fly. So, that C L by C D is maximum, when I say what is that C L I should fly and for a cruise it is equivalent saying what is the speed I should fly for which C L by C D is maximum, because speed and C L are related.

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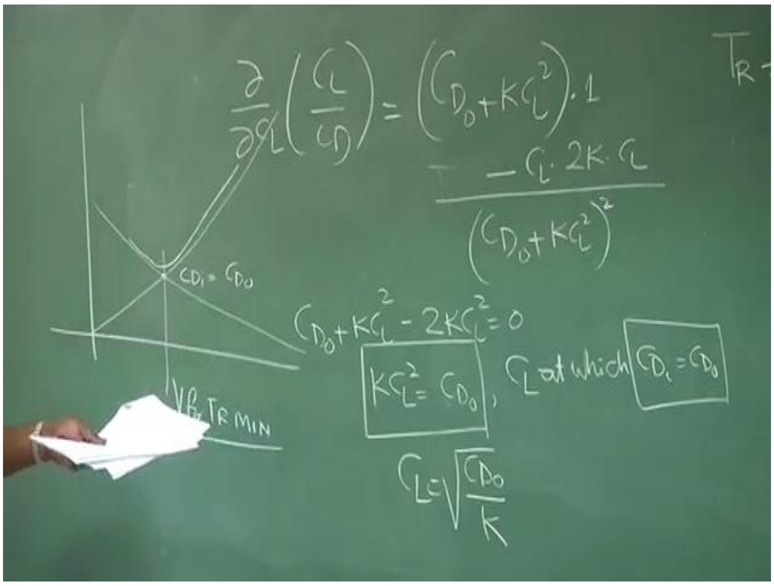

Now, how do I check for maximizing this C L by C D the rule is very simple, you differentiate then what do you get, you get C D naught plus K C L square into 1 minus C L into 2 K into C L divided by C D naught plus K C L square whole square and to see it is turning point or not I equate this equal to 0. So, I find C D naught plus K C L square minus 2 K C L square equal to 0. So, I get a condition K C L square equal to C D naught, you could see if I take the second derivative just check whether this turning point is a maxima or not only you have to see that second derivative should be negative which is straight forward from here.

If I take a second derivative of it I will again get a negative value less than 0, so it is indeed a maxima. But, let us come back to the physics of the situation, what is K C L square it was the induced drag, what was C D naught is the parasite drag. So, what is a message, it says it is that C L at which induced drag is equal to parasite drag or 0 lift drag that is the C L at which the thrust required is minimum. Now, you come back to this diagram, we recall we had induced drag going like this, parasite drag going like this, this is the point where C D I is equal to C D naught.

So, the net when you are drawing I know that this is indeed the v for thrust required minimum that is v at which K C L square equal to C D naught or I answer my question that to get thrust required minimum I must fly at C L which is given by C D naught by K under root is it clear, it is fine for us and it is clear that C L is C D naught by K. However, for a pilot if you tell dear sir please fly at C L equal to C D naught by K he may not understand, he is not a aerodynamic. So, how do I communicate to the pilot that is also an important question.

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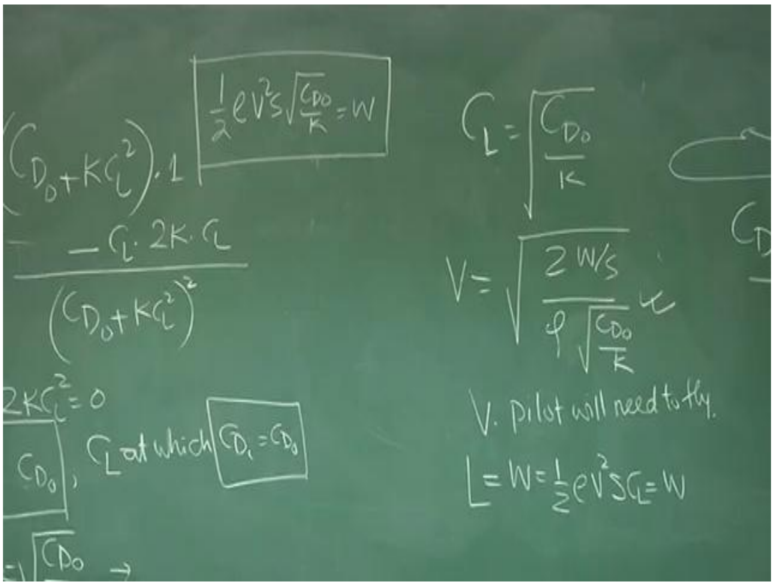

Now, let us see when I say C L equal to C D naught by K, what is the speed I should fly. So, that thrust required is minimum v is v will be under root 2 W by s rho C L that is C D naught by K. For a given airplane, any airplane which will be flying you will be knowing C D equal to C D naught plus K C L square this drag polar will be known to you, you know the value of K, you know the value of C D naught, you know the weight, you know the wing area and what altitude you are flying density also you know.

So, you actually know to fly at that altitude for thrust required minimum I must fly at this speed v. So, this will tell you what is the v pilot will need to fly do not forget it cannot be any arbitrary altitude, because we have to satisfy that it is following a cruise condition. So, you have to also ensure that lift equal to weight that is half rho v square S C L equal to W or this if I further write I will find that half rho v square S C L is nothing but, C D naught by K should be equal to W.

So, it is very clear from here that C L is fixed, because of C D naught by K you cannot have any other C L, C L is fixed value. As far as v is concerned it goes on changing with altitude that is if we are at some altitude rho 1 will have different velocity as compared to an altitude rho 2. But, you have to ensure that the row and speed combination should ensure that lift is equal to weight that is why there will be a particular v given by this equation for a particular altitude, where you can maintain cruise and same time thrust required minimum, this is clear this is very, very important.

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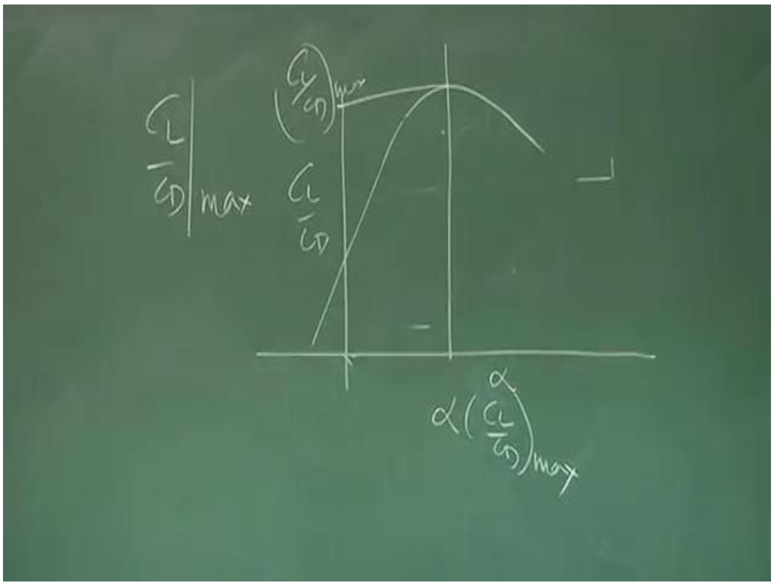

The C L by C D max if we plot C L by C D versus C L or let say for alpha then figure will go something like this. So, there is a typical alpha for which C L by C D is max, this is C L by C D max. What is a message from here? Let us understand one thing from pilot’s point of view.

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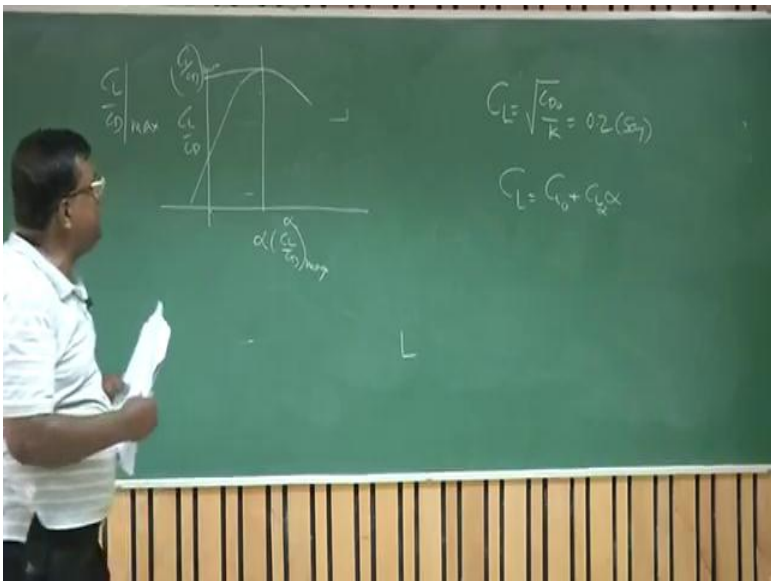

Suppose we are telling pilot to fly at C D naught by K and it happens to be 0.2 let us say. How the pilot will generate that C L? C L how do I generate C L for in aircraft, when your C L is C L naught plus C L alpha into alpha; that means, the pilot has to rotate the airplane to a requisite alpha, so that the C L is 0.2. Now, if you are telling that you have to fly at C L equal to 0.2 which is basically for thrust required minimum case then you must sure that this alpha has to be 0.2 for that airplane then only you can get C L by C D max whatever you are requiring for.

This value of alpha and C L by C D max combination will go on changing for different aerofoil, different aileron number. So, one has to really generate the huge internal database and create this database to understand what is that alpha, at which way I get C L by C D max and then, I translate this alpha to C L and from C L, I translate to a speed. Because finally, the pilot will be following the speed here the airspeed indicator he does not have any angle of attack indicator.

So, he does not fly with an angle of attack, he will see go to the altitude, he knows that at this altitude if I fly around this speed which was computed by the C D naught by K condition. So, he will maintain that and trim the airplane such a way that there is no change in the altitude. So, actually he will be flying at thrust required minimum.