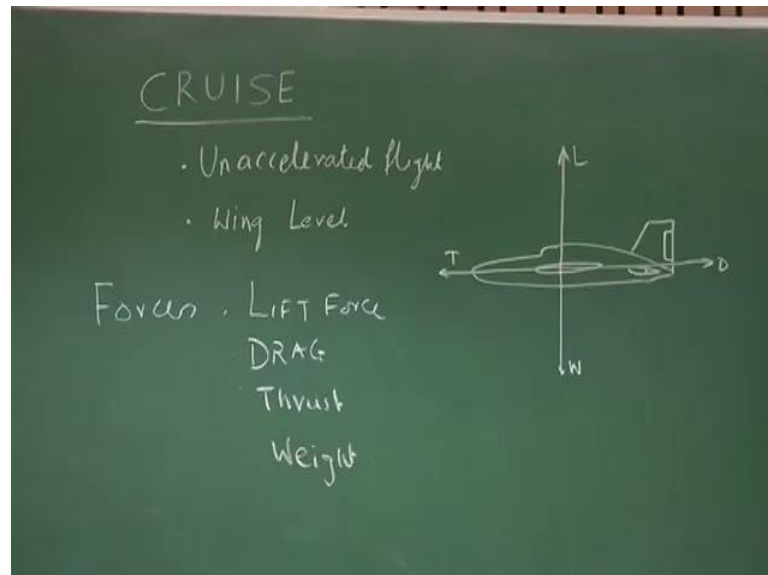


NOC: Introduction to Airplane Performance
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Lecture - 11
Thrust Required, Power Required: Cruise.

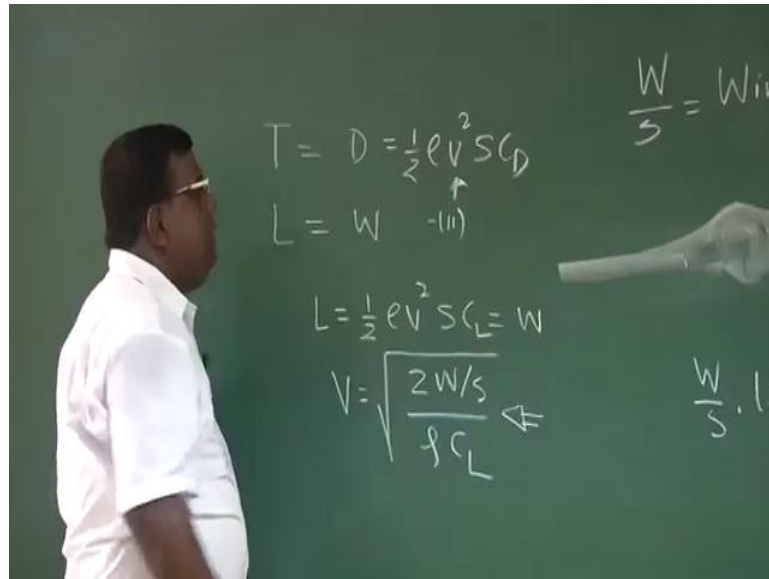
Hello dear friends, now today we will be discussing on one of the important phase, flight phase of airplane the cruise. When I say cruise, so airplane will be going this straight lines like this and it will be unaccelerated, all the forces and moments will be balanced and wings level that is, ((Refer Time: 00:32)) if the wings will not have any orientation like this, this straight and level and this is a straight flight, that is typically a cruise flight.

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And, when I say the accelerated and unaccelerated flight, we mean that, the net forces are balanced and what are the forces acting on it, the forces acting are one is lift force, then drag, then the thrust and the weight, whole performance exercise. Understanding the performance of a airplane is focused towards this whole process lift, drag, thrust and weight.

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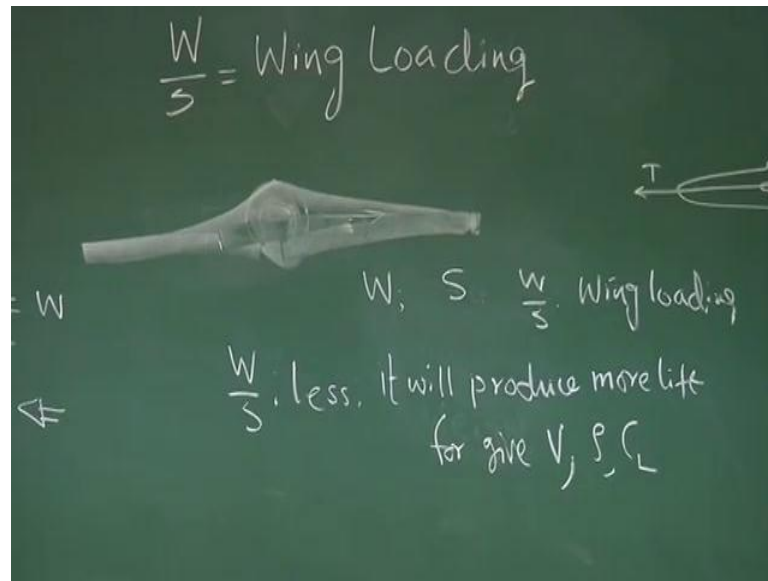


If I now, come back to this diagram if really this aircraft is going in an unaccelerated flight, then I can write thrust equal to drag and lift equal to weight. Let us, also remember one thing a designer will design an aircraft, but who will be flying it, a human being a pilot and the pilot may not be an engineer and may not be a person, who understands lift, drag all the theory that has gone into designing an airplane.

So, it is a role of a designer to ensure that he develops a language, which pilot can understand and mostly this is done through instrumentation, we will be... I am folding this understanding step by step. Let us, come back to first on this force balance, which is thrust equal to drag and lift equal to weight. If I take the second equation, the lift is we know by now, lift is half rho V square S C L, so I can write V is equal to under root 2 W by S by rho C L.

Because, the lift is half rho V squares C L, which is equal to the weight, as per the equation number 2. So, let us focus on this equation, the two very important parameters here one is C L another is W by S and W by S we will be referring very frequently as wing loading, this W by S wing loading.

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Let us, develop some field for this wing loading, suppose we want to lift this mass m by giving a forward velocity. By now, we have understood that I can generate a lift using a wing a fixed wing, so I translate this to a diagram like this, where I call this part wing. And, let us say for a given weight of a airplane the area is S is the area of the wing, then the ratio W by S is called wing loading. What is the physical significance of it?

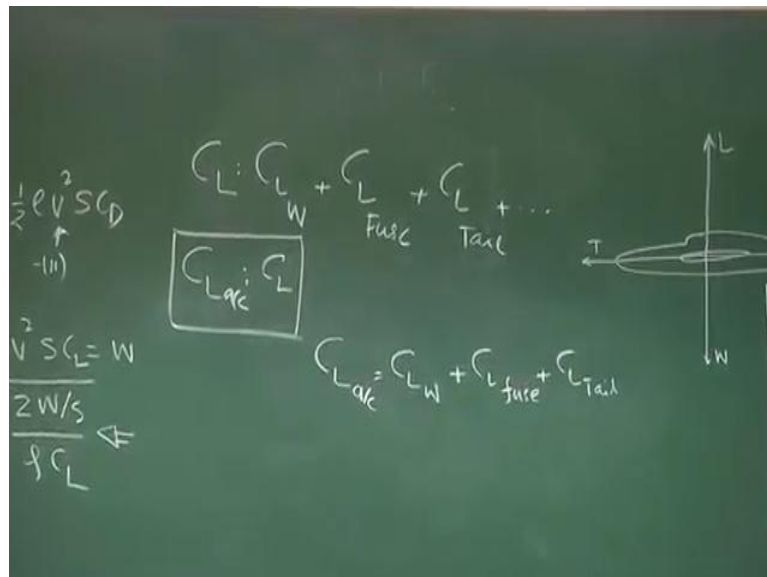
What is our aim? Our aim is to leave the airplane, leave the airplane how, by generating the lift through the wing, primarily from the wing. So, if the wing area is large, then I will be able to produce more lift, you can come here this equation $\frac{1}{2} \rho V^2 S$, more area means I will be able to produce lift for a given velocity, if area is more I will have more lift. So, in a sense if I now translate to W by S , which wing loading if S is large it will have more lifting characteristics it will produce more lift.

So, the wing loading will be less, so W by S less means produce more lift for a given V ρ and C_L . What is the more lift for a given, yes that is right. So, what is the utility of this wing loading ratio? Think typically of a glider, the gliders will have a larger wing, larger wing area. So, if larger wing area, a small larger wing area, so small wing loading I repeat the glider will have larger wing area for a given weight it is wing loading will be less; that means, the velocity required to balance lift equal to weight at a given altitude will be less.

So, what is the implication of that? If velocity is less; that means ((Refer Time: 06:10)), your drag required come to this equation half rho V square S C D. If, velocity is less your drag experience by the airplane will be less, in turn the thrust required also will be less and in turn the engine size will be less and the weight also will be less. This is very, very important parameter and we must always try to see these two equations together.

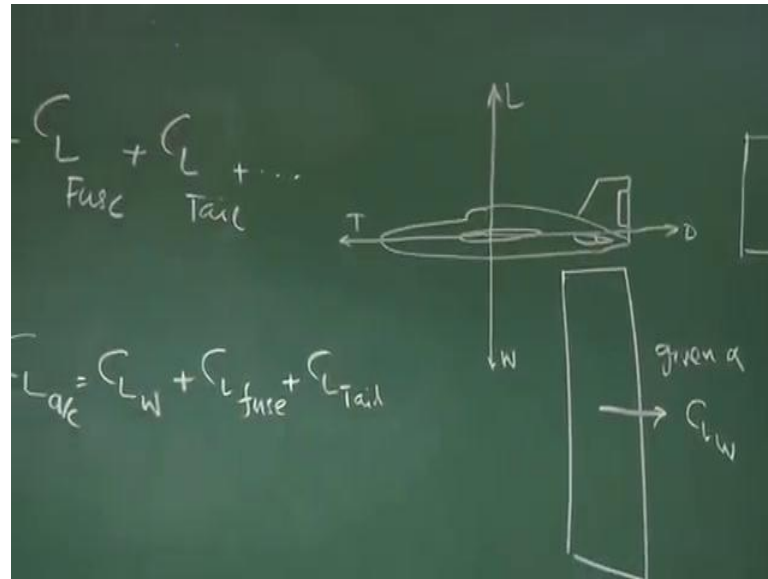
So, that is about the wing loading, some first exposure. This another important turn, if you see in this equation is C L. What is this C L? This is not just C L of the wing, it is C L of the whole airplane.

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So, I will write, I will be very clear in my mind this is C L, because of wing this is C L, because of fuse large this is C L, because of tail and if any other components are there that they cannot there could be any other surfaces. So, C L is for the C L aircraft. But, C L we are talking about the C L of the whole aircraft, not just C L of the wing that should be very, very clear in mind. Now, there is a word of equation, when I write C L of the aircraft as C L of the wing plus C L of the fuse large plus C L of the tail, it should not give a wrong impression that C L of the wing.

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If, I want to estimate C_L of the wing I take the wings separately and find for a given alpha, I find C_L of the wing in isolation. Similarly, I find C_L of the fuselage in isolation, so will I find C_L tail in isolation and then add it up, that will be a mistake that is not correct. Because, see the point, point is C_L of the wing in isolation and C_L of the wing, when it is attached to the body have a different geometry conditions.

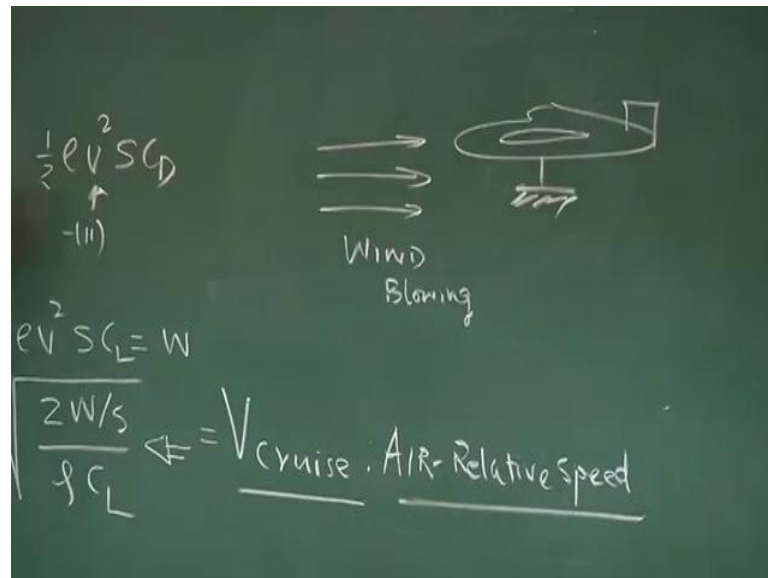
For example, the moment I put this wing over this aircraft there will be an influence of body on the wing and the wing on the body. So, there will be an interference on body to wing and wing to body, so C_L get modified, similarly when I put the tail, if I take tail in isolation, I should be very careful that the value of C_L computed in isolation will not be the exact, what airplane will be generating. Because, this tail will be near the vicinity of the body, fuselage near antennas and there will be interference.

So, when you find out C_L of the aircraft through a wing analog, through analytical method all these things are taken into account and finally, we say it is the C_L of the whole aircraft. It is true that, if I see the numbers the C_L of the aircraft is predominantly decided by C_L of the wing, because and that is true also, because the wing has a unique role, wings role is to generate lift, and what is the role of tail the role of tail is to generate moment.

So, lift is not that important for tail, because the tail moment are the distance between aerodynamic center of the tail to c_g of the airplane, that distance into the small force of

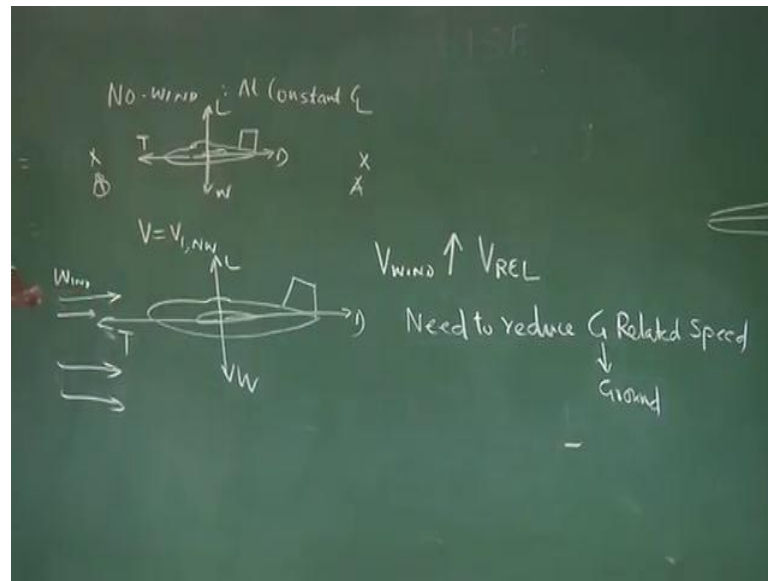
the tail gives the moment, which is required to control the airplane. Now, this V when I use this equation ((Refer Time: 10:03)), this V is actually is V cruise that is, what is the interpretation of this, it is the cruise velocity at a given attitude, where you are maintaining lift equal to weight. And this is another central point about this V cruise, if I ask, is it really the velocity of the airplane with respect to the ground, the answer is no.

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Because, the lift depends upon the velocity, which is air relative velocity, that is meaning thereby, suppose the airplane is stationary on the ground. Suppose, I have fixed it on the ground and there is a wing and there is a wind blowing, it is possible there is a wind blowing. So, what is the speed of the airplane with respect to the ground? It is zero, it is fixed, how it will produce lift, because there is an air relative speed. So, V cruise we should very clear this is AIR relative speed.

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Now, let us go little bit deep into this, let us do an example, suppose I am reaching I want to reach from point A to point B and I am flying like this. I am assuming that there is no wind and also I am assuming that, I will be flying at a constant C_L , no wind and at constant C_L . So, let us say for this configuration the velocity is V_1 or I say V_1 no wind. What will happen? If I, if suddenly similar situation everything remaining same and certainly, there is a head wind blowing like this say wind and, still I want to fly at this constant C_L .

To maintain lift equal to weight my question is should I increase the speed or decrease the speed. Transitively it is obvious, because the lift depends upon the air relative speed since the wind is coming towards the airplane, the air relative speed will increase. So, actually I have to reduce the speed with respect to the ground, so as V_{wind} increases $V_{relative}$. So, I need to reduce ground related speed, this is ground related speed.

So, that the air relative speed remains same as in the first case and lift is equally balanced with the weight, otherwise the airplane will lift up. So, this concept will be used, you will see that, when we are moving in a airplane into the wind or there is a tail wind, how the work performance of the airplane will change, sometimes to advantage, sometimes to disadvantage.

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The image shows a chalkboard with the following handwritten text:

$$T = D$$
$$L = W$$

$$T = \frac{W}{L/D} = \frac{W}{\frac{C_L}{C_D}}$$

= Thrust Required

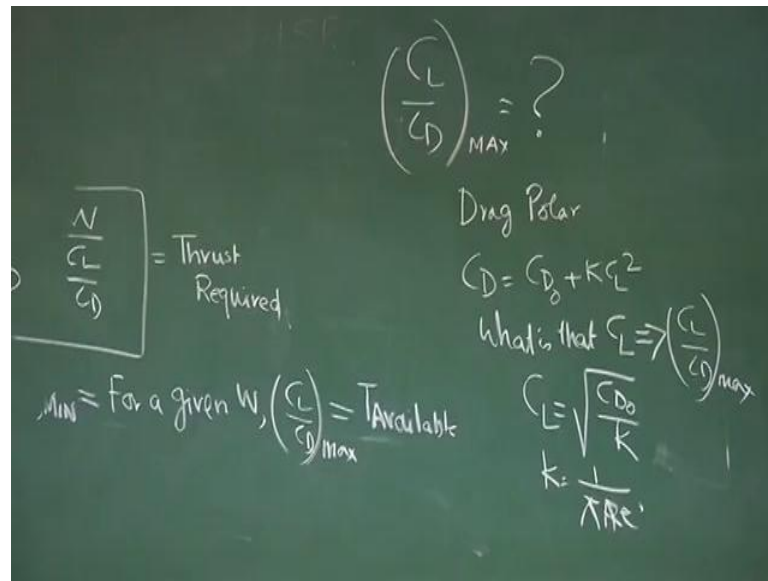
$$T_{\text{Required, Min}} = \text{For a given } W, \left(\frac{C_L}{C_D}\right)_{\text{MAX}} = T_{\text{Available}}$$

$\left(\frac{C_L}{C_D}\right)_{\text{MAX}}$

So, with this understanding let us come back to the cruise, we say thrust equal to drag and lift equal to weight, so I can manipulate this two equation and write thrust equal to W by L by D or this is W by C_L by C_D , what is this thrust, this thrust is thrust required, this is required, required for what this thrust, this thrust required for ensuring it is equal to the drag and there is a sufficient appropriate value of C_L , at that given speed, which maintain lift equal to the weight, what is our aim our aim should be I should fly the machine in such a way, that thrust required is always minimum most of the time the cruise will try to look for thrust required minimum.

So, I will look for thrust required minimum and if I see here, what does mean, it means for a given weight, C_L by C_D should be maximum. It means for a given weight C_L by C_D should be maximum. Now, what is the meaning of C_L by C_D to maximum, that also should now, ask our self, what is the meaning of C_L by C_D maximum or I put the question differently, I want the pilot to fly the machine.

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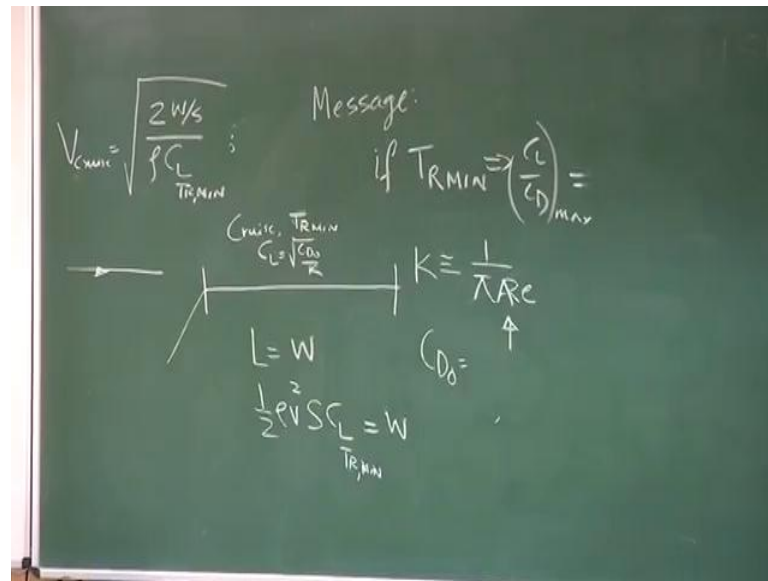


So, the thrust required is minimum and this thrust required minimum will be compensated by thrust available, which comes to the engine. So, if thrust required is minimum, then thrust to be delivered by the engine also minimum, that is an efficient way of flying. But, if you tell the pilot please fly the machine such that C L like C D is maximum pilot will be understand, what the meaning of this. So, let us try to decode information, what is the meaning of C L by C D maximum for a pilot.

So, we have to go back to drag polar a typical drag polar form C D equal to C D naught, plus K C L square for low speed almost, what we say 100 meter per second less, then 0.3 and smooth surface mostly it is fine this will be and I almost exactly representation. The question is, what is that C L, what is that C L, which will ensure C L by C D is maximum, very straightforward this will be one of the assignment problems. You have to find out the condition through differentiation and you can show that answer is C L should be equal to C D naught by K, where K is K you know is roughly it is 1 by pi aspect ratio into e.

So, what is a message? Messages is if we want to fly at thrust required minimum ((Refer Time: 18:36)), then I need to fly at C L by maximum, which means I need to fly at a fix C L. Because, C D naught values is fixed typically value will be 0.02, 2.23 K for a given aspect ratio K is fix, so C L is fixed.

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So, what is a message, message to pilot is message is, if I want to fly as thrust require minimum, then fly at C L by C D maximum and that is fly at C L is equal to C L fixed, which is given by C D naught by K for a different airplane this value will be different. Because, you know the K is 1 by pi aspect ratio e, so depending upon an aspect ratio depending upon the wing layout value of V will change aspect ratio will change. Then, C D naught also will change depending, what is the geometrical counter, what are the positions, what are the profile of the wing, so for a different airplane this value C L fixed will change.

However, if I want to fly at thrust required minimum, where your engine be minimum thrust, which I call thrust available I need to fly to fix C L, which is C D naught by K. Let us say, this value is around 0.2 let us say, so what is the meaning suppose you are telling pilot you cruise, you cruise here, please cruise such that thrust required is minimum, that is a statements you are given or as a designer as given to the pilot it means he is to fly C L equal to C D naught by K again pilot will not be comfortable.

If, you want to calculate C D naught value and K he is not aware, what is C D naught, what is K mostly. So, what is the way to handle this situation C D lift equal to weight. So, you know that half rho V square S C L for thrust required minimum, will be will be equal to weight. So, from here I can write V cruise will be equal to 2 W by S by rho C L for thrust required meaning. Let us see here, W is fixed if I assume I am assuming that

there's not much difference in weight, because of fuel consumption S is fixed C_L thrust required minimum is fixed, because it is given by C_D naught by K , which are fixed.

So, for a given altitude for a given value of a ρ there is again fixed value of V cruise, so, pilot will be flying around that value. If, you can generate a table for him for at this altitude maintain this velocity at this altitude by this velocity and that is all the guide guideline for him to do the flying a such required minimum and of course, with the feel he may not be require explicate those numbers, but here an idea, then through instruments he ensures he is flying at a thrust requirement minimum, so this part clear.

So, I will just repeat this and decode it point by point, so that it goes into your mind I always remember, whenever you are talking about an airplane finally, it will be a pilot in command flying this machine, he is not the designer he did not be aerospace engineer. So, we need to translate always understanding into a language with pilot to be able to easily understand. And, that is what makes the difference between a good engineer and bad engineer.

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The image shows handwritten equations on a chalkboard. At the top, it states $T_{R,MIN} \Rightarrow \left(\frac{C_L}{C_D}\right)_{MAX} \Rightarrow C_L = \sqrt{\frac{C_{D0}}{K}}$. Below this, a boxed equation shows $V_{T_{R,MIN}} = \sqrt{\frac{2W/S}{\rho \sqrt{\frac{C_{D0}}{K}}}}$.

So, what we say, we know that for thrust required minimum I must fly at C_L by C_D maximum; that means, I have to fly at a C_L , which is C_D naught by K ; that means, the velocity for thrust required minimum will be nothing but, $2W$ by S by ρ into C_D naught by K . So, that is what the velocity for thrust required minimum, should be maintained by the pilot at a given altitude, please understand also here this value will

change depending upon the altitude. As I go higher and higher the value of rho is decreases, so this value also will change it will increase, so far we are discussing about the thrust required for cruise.

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Power Required: Cruise flight

$$P_R = T_R \cdot V = \left(\frac{W}{\frac{C_L}{\rho}} \right) \cdot \sqrt{\frac{2W/\rho}{C_L}} = \frac{W^{3/2} \cdot \sqrt{2/\rho}}{\left(\frac{C_L^{3/2}}{\rho} \right)}$$

P_R for any V, C_L, ρ .

$P_{R \min} \Rightarrow \frac{C_L^{3/2}}{\rho} \Big|_{\text{maximum}}, T_{R \min} = \left(\frac{C_L}{\rho} \right)_{\text{max}}, C_L = \sqrt{\frac{C_D}{K}}$

Now, will be discussing about power required for cruise before, I start talking about power required and trust required. Let us also understand we are talking about two types of engine one is i c engine with proper air combination with this engine inserted in terms of power and another one is jet engine this engines are rated in terms of thrust written of thrust. So, far we are talking about thrust required, now we are talking about power required and power required for what, power required for cruise flight.

Remember, power required for cruise flight is one important mission, then power required to climb is another important mission. Similarly, thrust required for climb is another important machine. So, we are only parts of a segment of whole flight envelope you talking about, which is cruise, in cruise is, what we know there airplane goes on accelerated with the constant speed like this in a almost in a straight line. So, very simple if I know thrust required verses T R, then power required will be T R into V force into velocity, what is this V, V is nothing, but velocity required at that altitude to maintain lift equal to weight.

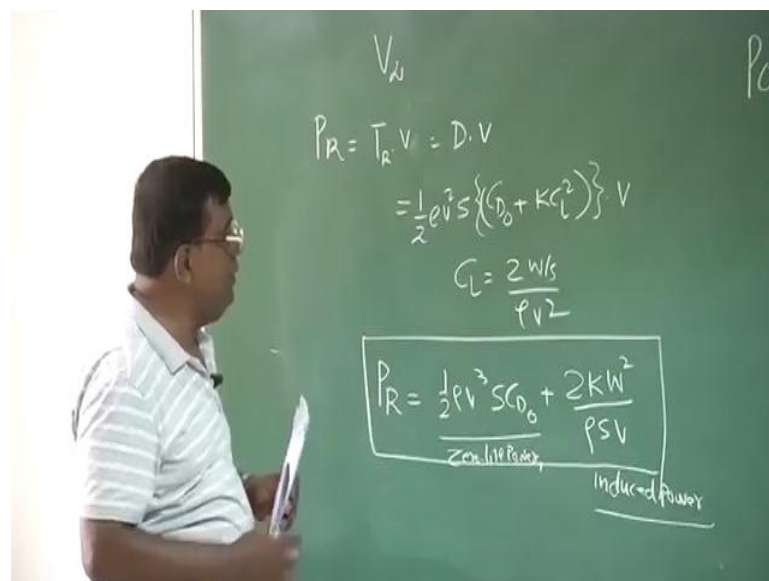
So, I can write this as W by C L by C D, which I know thrust required expression and for V, I can write 2 W by S by rho C L. So, this is now my power required expression. So,

what we can see from here, this I can finally, also write like this $W = \frac{1}{2} \rho V^3 S C_L$, then this is $C_L = \frac{2W}{\rho V^3 S}$, here I write $C_L = \frac{2W}{\rho V^3 S}$ by C_D here, again I write $C_D = K C_L^2$, somewhere this route of S also will come. So, I write $C_L = \frac{2W}{\rho V^3 S}$ here, let us check W and $W = \frac{1}{2} \rho V^3 S C_L$ is $W = \frac{1}{2} \rho V^3 S C_L$ root 2 is here, and S , I put it here and $C_L = \frac{2W}{\rho V^3 S}$ and $C_L = \frac{2W}{\rho V^3 S}$ half $C_L = \frac{2W}{\rho V^3 S}$ by C_D is here and ρ .

This simple expression tells us one thing that for power required for any V or C_L at a given altitude can be computed, if I know, what is the weight during the cruise, what is the $C_L = \frac{2W}{\rho V^3 S}$ by C_D and what is the density of air at that altitude. Also, it tells me one more information that, if I want if I am looking power required minimum, for a given weight given altitude there in the condition is C_L , $C_L = \frac{2W}{\rho V^3 S}$ by C_D should be maximum, what was thrust required minimum condition. For, thrust required minimum, we call the condition was $C_L = \frac{2W}{\rho V^3 S}$ by C_D should be maximum and we have demonstrated that it means to fly the machine at $C_L = \frac{2W}{\rho V^3 S}$, which is given by $C_D = K C_L^2$.

Similar question I will be asking here, what is that C_L I should fly, so that $C_L = \frac{2W}{\rho V^3 S}$ by C_D is maximum or in turned the power required will be minimum. So, that is a question we are going to adjust, also in adjusting this question, we also try to see two components of powers like, we have thrust required zero lift thrust plus induced thrust. Here, also will try to see, what is zero lift power and induced rangers power that is our objective, let us go step by step.

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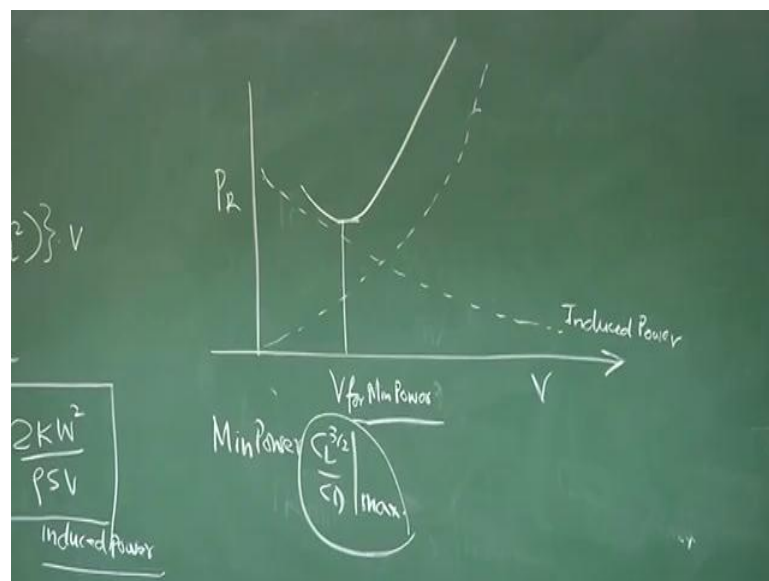


So, now I do a little bit of algebra or simple expression T into V infinity is equal to D I do not write infinity V into V many text book, they follow the nomenclature for V it is V infinity it is called free stream velocity. That is the velocity for away from their aircraft, what is the significance of V infinity, will be talking in as the post develop, but I am using V .

So, this is equal to what half $\rho V^2 S$, then C_D naught plus $K C_L^2$, into V and I can write C_L equals to $2W$ by S by ρV^2 . And finally, I can write an expression for pop power required as off $\rho V Q S C_G$ naught plus $2KW^2$ by $\rho S V$ this is straight forward. Message you have to just plugging this expression here and do some arrangements to get this expression, what is important for us is, what is this term this is as the speed increases power required also will increase as far as this term is concerned and that is called zero lift power zero lift power.

The physical interpretation is if you make the airplane aerodynamically efficient in a sense that the shape is such its parasite drag is minimum, then you have this value also less. And, second one goes inversely V this is the induced power, it is very clear as I am increasing the speed the C_L require is less C_L require less means drag should be less drag require less means here in this power, also will reduce this. Basically, coming from the C_L cord line as C_L reduces your induce drag also reduces, so in this power also go on reducing with speed.

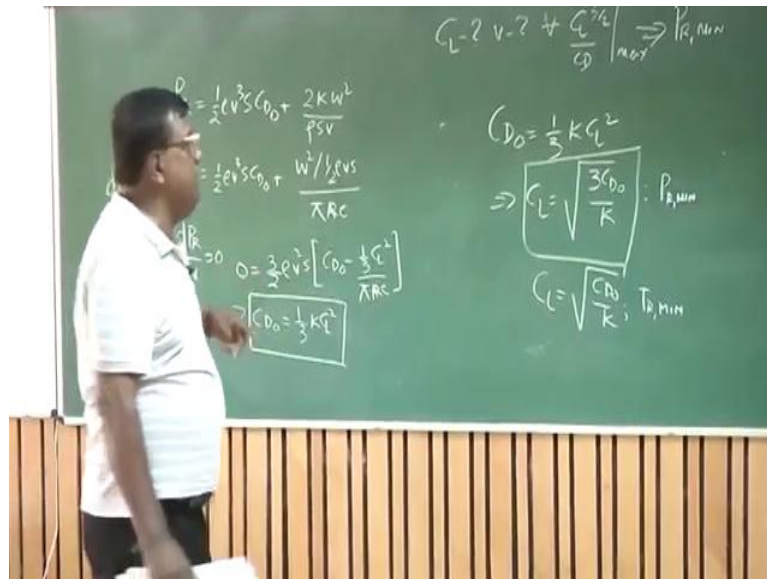
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So, if I now plot this, this is the induced power and this is the zero lift power and you find that net power will be something like this. And, this is the point, where this is V for minimum power, please note that this point is not here, this is less than this point and I will explain this.

Let us, analytically find out, what will be this point, but one thing we understand for minimum power, we are seen that $C_L^{3/2}$ by C_D should be maximum I was the meaning, if I want to fly at minimum power I should fly corresponding to that C_L , for which this is maximum and naturally for the that C_L there is a fixed velocity, because it is a cruise fir a given altitude. So, we know find out analytically what is that C_L , for which I should fly, so that I get minimum power.

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So, the question I am asking is, what is that C_L or V , for which $C_L^{3/2}$ by C_D is maximum, that will ensure I am flying at power required minimum.

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$$P_R = \frac{1}{2} \rho V^3 S C_{D0} + \frac{2KW^2}{\rho S V}$$

$$P_R = \frac{1}{2} \rho V^3 S C_{D0} + \frac{W^2 / \frac{1}{2} \rho V S}{\pi A R e}$$

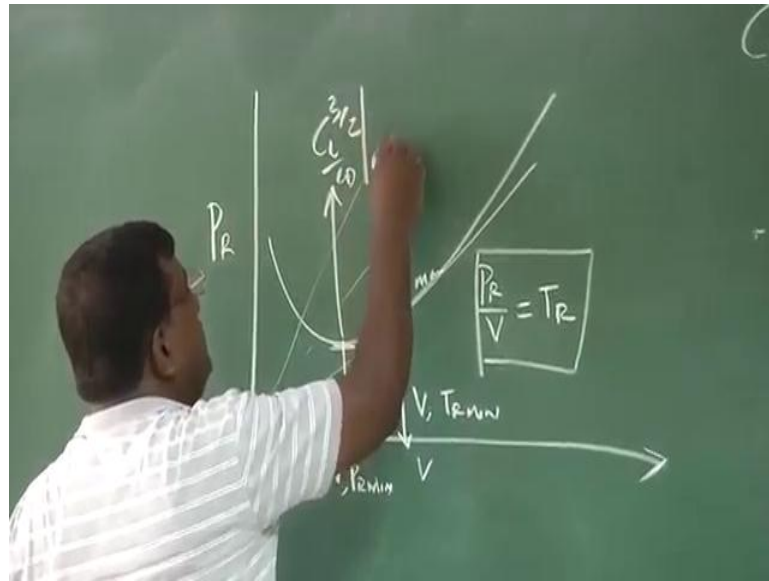
$$\frac{dP_R}{dV} = 0 \Rightarrow 0 = \frac{3}{2} \rho V^2 S \left[C_{D0} - \frac{1}{3} \frac{K}{\pi A R e} \right]$$

$$\Rightarrow C_{D0} = \frac{1}{3} K \frac{C_L^2}{\pi A R e}$$

So, I will now take derivative power required by dV and V equal to 0 and that will tell me will give me expression, which you can check 0 is equal to 3 by $2 \rho V^2 S$, C_{D0} naught minus $\frac{1}{3}$, C_L square by π aspect ratio e , which implies C_{D0} naught equal to $\frac{1}{3}$, $K C_L$ square, ((Refer Time: 34:20)) C_{D0} naught is equal to $\frac{1}{3} K C_L$ square it means C_L equal to under root $3 C_{D0}$ naught by K .

Now, I have to fly the machine at a C_L , which is given by $\sqrt{3 C_{D0}$ naught by K to get power required, power required minimum, what was for thrust required minimum for thrust required minimum C_L was under root C_{D0} naught by K such required minimum. Now, for cruise flight if I am having same altitude, then which case I need more velocity that will be decided by, if I am required flying at a power required minimum my C_L will be higher. Then, for C_L thrust required minimum since velocity is higher the velocity will be less in this case will be for power required minimum, then the less than V for thrust required minimum.

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This is another interesting see from power required verses velocity graph, if I draw a tangent, then this point will thrust point V for thrust required minimum and of course, this point is V for P R minimum, why this point for thrust required minimum, you know the slope is P R by V, which is nothing, but thrust required and that is a minimum thrust required will go on writing this slop. So, from power required from velocity graph I can find out what is the velocity I should fly. So, the thrust required is minimum, and this will corresponds to C L by C D maximum this will corresponds C L 3 by 2 by C D maximum.