Lecture - 12
Excess Thrust Power: Climb Angle and Rate of Climb

Yes, good morning friends, in the last lecture, we were discussing about the Thrust required, Power required for a cruise flight and you also found out a condition. If I want to fly the machine with thrust required minimum, then what should be the C L, which I should fly. And we also found on an expression, if I want to fly the airplane in cruise mode, in power required minimum mode, then what should be the C L, which I should fly.

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If I write those things V for thrust required minimum, the expression we got was something like this, 2 W by S and rho C D naught by K and V for power required minimum was 2 W by S rho under root 3 C D naught by K. And please remember, these are all pertaining to cruise flight, this we should not miss cruise. And more pictorially, we know that these are the basically schematic of different flight path and airplane will be taking and we are concentrating here, which is the cruise phase.

So, the message is very clear, for a designer if I want to fly the machine for the cruise and I want to ensure that, thrust required is minimum that means drag is minimum. Then,

I need to fly at a C L, thrust required minimum is basically drag minimum, I need to fly at a C L, such that C L equal to C D naught by K, which actually means, K C L square is equal to C D naught. This is K C L square; this is the induced drag component, why this induced drag is there; that is because of lift.

So, you call it lift induced drag and at a speed where this induced drag component and parasite drag component in terms of coefficients, if they are same and at that point, if I flying the machine, which actually means I flying at a C L equal to C D naught by K, which also means I am flying at a velocity 2 W by S by rho C D naught by K under root, I will be getting thrust required minimum. If I have a closer look here, I could see that, if W by S if the wing loading is less, then velocity or speed required for thrust required minimum also will be less.

Similarly, for speed for power required minimum, if wing loading is less, the speed for power required minimum also will be less. And when I say speed will be less, it has a direct impact on the engine; that means I will be requiring a lesser rated engine. So, the engine weight will be less and engine complexities will be less. So, designer will depending upon the different conflicting requirement he has, he will actually select W by S.

And at the end of this course, you will realise that how do I select a wing loading for a particular airplane, which still satisfy best of all those flights phases or sometimes or many times, the designers has to give a weightage, what is the primary role of this machine, if it is for a cruise for a transporter plane, he may give more weightage here. If it is a fighter airplane, he may give more weightage to as a maneuver or take of distance or landing distance. So, all these things will come. By before we take educate ourselves to take a decision on this, we are trying to build the basic understanding.
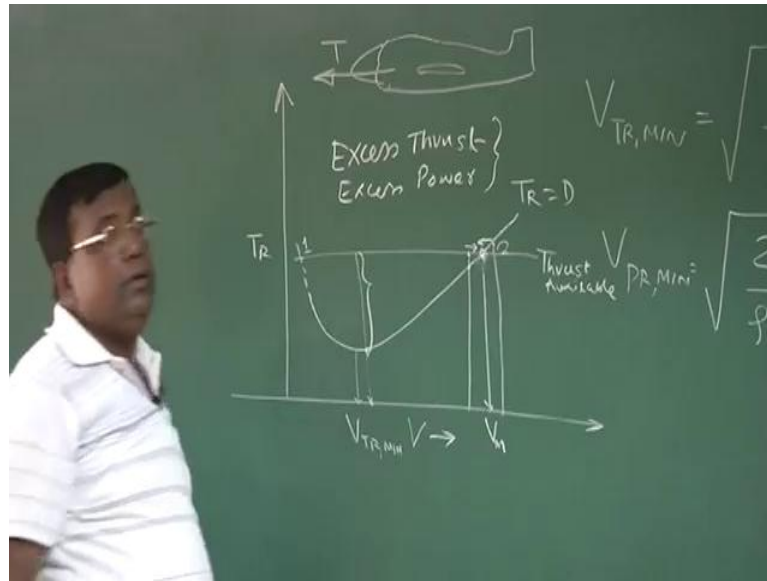
Now, let us see, we have also seen this thrust required versus speed or velocity, the graph typically goes like this and this is the point, where the V for thrust required minimum. Please remember, thrust required is equal to drag. So, when I say thrust requirement minimum, I am implicitly say this is telling that, the drag required minimum. Now, come back to the engine, the airplane has an engine, it will be delivering some thrust.

Typically, let us say I am drawing typical representation of trust from a jet engine, turbo jet type, I can clearly assume, it is constant and it remain almost constant. Though in practice, you will find, there are secondary influences on the speed as well. Now, see there is a two interesting point, suppose I allow me to extend it like this. So, I have got one point here, I have got one second point here.

Let us try to understand, what are these points, what is this point, at this point, you could see that, the drag required which is given by this graph, because this is thrust required and we know thrust required equal to drag. So, at this point, whatever engine is applying; that is we call it thrust available, at this point, they are same; that means, if I am flying at this speed, this is a maximum speed and I will be going with without any acceleration, constant speed.

But, if I want to fly at some point here, you could see here, here I see that, if I am flying at this speed, there is an excess thrust, this excess thrust can accelerate the airplane, it can lift of the airplane. So, many things it could do. So, this is a one concept of excess thrust.

Similarly, we will see excess power, these two are very important concept and we see how this concept can be utilized to analyze an airplane performance in terms of rate of climb, in terms of acceleration, without changing the altitude, all those things will be discussing.

But, before you go to that let us ask a question, let us say the airplane was flying at this point 2 and because of certain reason, the velocity has increased. Is this is clear? Suppose it was flying at speed, let us say I call it V M and because of certain reason, the velocity has suddenly increased. What this graph tells you? The moment the velocity increases, you could see that the drag is more than the thrust, available for the engine. So, it will automatically decelerate.

So, it will have a tendency to come back to this point and suppose for some reason, it is now flying at a lower speed, then what happens, here the thrust is more than the drag. So, it will accelerate the airplane towards this point 2. So, we say as far as velocity stability is concerned; that is statically stable at point 2. I repeat again, suppose I am flying at point 2, if for some reason the velocity has increased. From this graph you can see, now at this point, this drag experience by the airplane would be more than the thrust given by the engine.

So, it will start decelerating, so it will have a tendency to come back to this point. Similarly, if it is flying at a speed less than V M, the thrust will be more than the drag experience by the airplane. So, it will have a tendency to go back to this point 2. So, we say at point 2, the aircraft is statically stable in terms of speed is concerned. But, if I try to see here at point 1, suppose if I am flying at point 1, here also trusted and drag are equal.

If for some reason, there is an increment in the speed, what will happen, see any point beyond this, the moment there is an increment, the trust is always more than the drag. So, the airplane will go on accelerating and this point will never come back to this point 1. So, at this point, it is not statically stable in relation to the speed. Also, there should be another point, it should come to our mind, for any plane there is something called a speed called V stall which we have discussed.

V stall there is a speed, the lowest speed with which, it can manage lift equal to weight; that means, it is actually putting C L to C L max, he has put all the fledge down and angle of a tag maximum.

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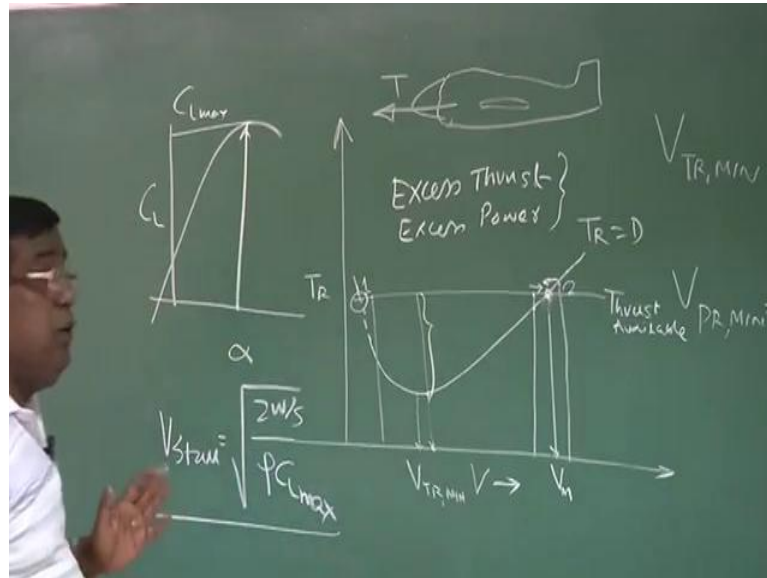
If we recall, this is C L versus alpha if something like this, if you have flying somewhere here. So, this value C L max and V stall will be equal to 2 W by S, rho C L max. So, this decides your minimum speed limit that, if the airplane goes below this speed, it is of no worthy as far as flying is concerned. That means, for all practical purpose, for us the relevant speed or velocity will be which are greater than V stall or greater than equal to V stall.
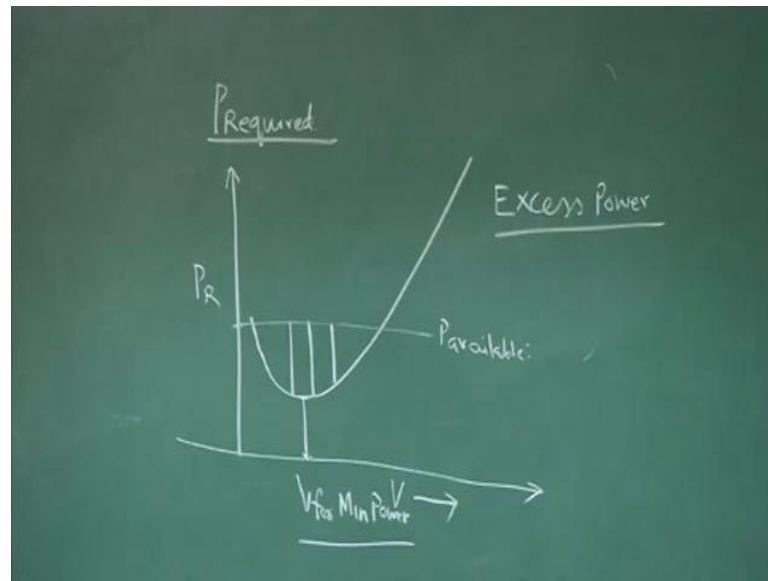
Now, if you again revisit power required versus speed or velocity, graph is also looks like this. And let us say, this is the power available, typically, a proper layer I C engine combination, where the power available almost remain constant for practical purpose, as far as understanding the basic physics behind it. In actual practice, please remember, there are secondary effects, we will be talking later.

Now, let see here, here this gap, by now we know that, this is the V for minimum power. But, what are these lines? This gap is as I was mentioning this is excess power. And let us see, how can we utilise this excess power in adding value to the performance of an airplane, when I say power available, please also understand. When, I say power available or thrust available, they all will go on changing with altitude.

So, whenever I draw a diagram it is for a fixed altitude, for all our convenience in your discussion, you can take this as a C Level condition and as a lecture progress, we will give you a correction to it as we gain the altitude, the physics does not change.
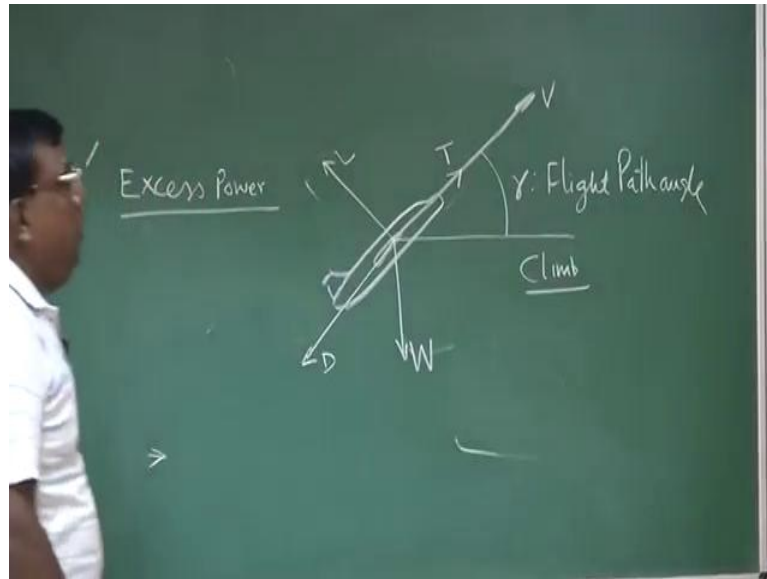
Let us draw this diagram, this is a typical schematic representation of an airplane, which is not cursing, it is now climbing, you say it is climbing or flight phrase is climb. Let us see here, if I now try to write the force balance along the velocity direction, along this direction, what do I get? I get of course, there is a thrust.
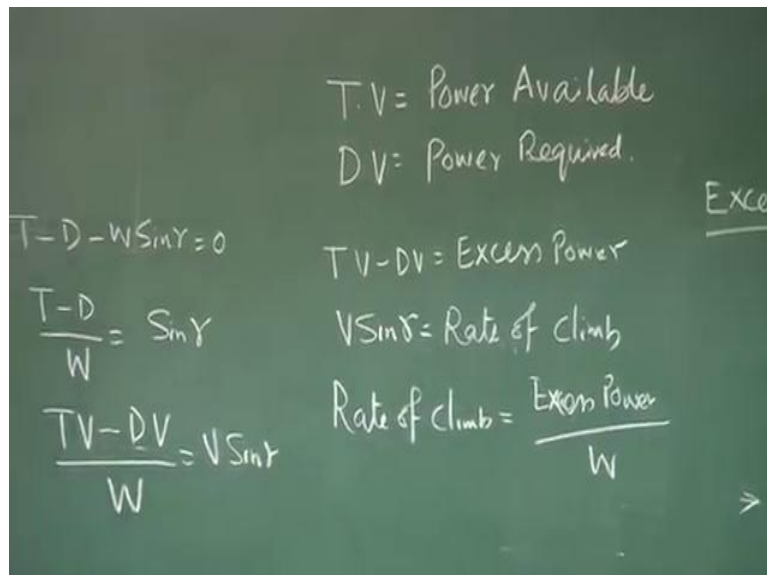
So, I guess T minus D minus W sin gamma, this is going to m d v by d t. What are the assumptions here, when I written this equation? First of all, I am assuming that thrust and velocity in same direction. Number 2, I am assuming that, the masses remain constant

and of course, alpha is very small. So, if this is the equation and if I am discussing only static performance, which actually means that steady state performance, so we are not taking any acceleration. Then, I can simplify this equation from T minus D minus W sin gamma is equal to 0.

There is no acceleration, this is a static performance. So, what happens now? T minus D by W is sin gamma. What is gamma? Gamma is a, we know very well, gamma is a flight path angle; that is the velocity vector mixed with the horizontal plane. So, how it is climbing? The velocity vector with the horizontal is the flight path angle or climb angle. So, what is a message from here? If I want to fly the airplane at a particular climb angle, I should ensure that it has enough thrust to compress a drag and this relation should be valid.

If I now multiply both sides by V, what happens? This will be T into V minus D into V by W is equal to V sin gamma, I multiplied both side by V. What is this T V by D V by W, let us understand this. T into V is what is basically thrust into velocity or speed, but this is a power available; that is power available from the engine at a given altitude. What is D into V; that is the power required. So, what is T V minus D V? So, T V minus D V is an excess power.

Remember, we are talking about excess power. So, now, I and what is V sin gamma? See here, if this is V, I can resolve this into components, V sin gamma, it is a vertical component. It is the speed at which the airplane is going up and that is called rate of climb. So, now if I, in this equation, I can write rate of climb equal to excess power divided by W.

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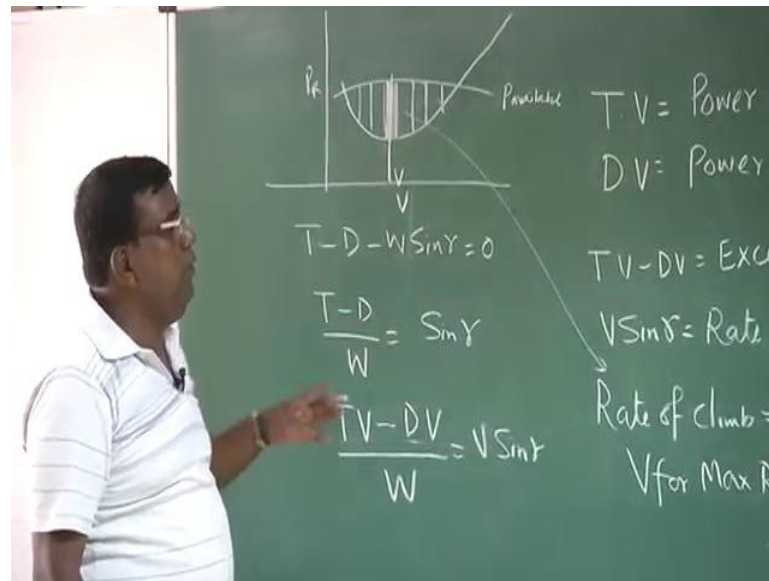
Now, if you again go back and recapitulate, what was the diagram, this was power required. So, speed and let us say, this was the power available, then this gap, they are the excess power, this is what is this and this excess power is linked to rate of climb. So, now, understanding understand this diagram, can you answer a questions at what speed should I fry to get maximum rate of climb, you could see here the excess power goes on increasing and comes to a maximum value here.

And then, again goes on decreasing, that as you know rate of climb is linked to an excess power. So, I know that V, where the excess power is maximum is that speed, I should fly to get rate of climb maximum. So, could you understand the linking of excess power to rate of climb, this concept will be utilized in defining a few more parameters, which are very, very important for aircraft performance.
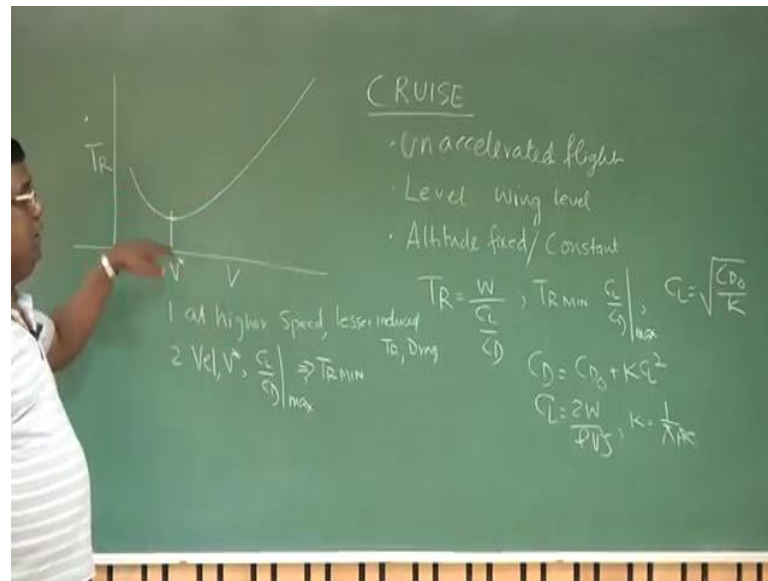
Yes, welcome back to this additional session. Let us revised, what we have covered so far, we were focusing on cruise flight. And what do you understand by cruise flight, cruise is, it is an accelerated flight. Number 1; un accelerated flight and when you say level cruise, level means wings level; that typically it is like this. If I am doing a cruise, I am going the constant speed, wings level not like this, wings level like this and also I am ensuring that, there is no change in the altitude.

When, I say fix point the altitude fixed or remain constant, this we have understood and once we understand, what is a cruise light, we try to understand, what is the thrust required to maintain the cruise light. And in that, we realise that, thrust required can be model as C L by C D, W divided by C L by C D and we also understood that to have thrust required minimum C L by C D should be maximum. And for that, we realise that to have C L by C D maximum, to have thrust requirement minimum, we need to fly a particular C L, which is given by C D naught by K.

Then, we try to understand, how can I plot the variation of thrust required verses speed and we realise that thrust required will follow this trend and we also understood at higher speed. Since, C L required for maintaining lift equal to weight is less, it will have lesser induced thrust, thrust required or drag, which are same. And typically there is a point, which is if I fly at a velocity as given V star or which C L by C D is maximum, then I

have thrust required minimum. I also understood how to calculate thrust, because we know thrust is W by C L by C D.

And from drag polar, I know C D equal to C D naught plus K C L square and for 11 flight C L is nothing, but 2 W by rho V square S. So, given weight, given speed, given density; that is that what altitude is flying. Even, the wing area, I know C L k I know because K is 1 by pi aspect ratio e. So, I can calculated C D as I also know the C D naught is fixed for an airplane for at low speed. So, once I know C D, once I know C L, I know C L by C D, I can find thrust required at different speeds and I have plotted the thrust required different speed in this figure.

And in that we found out, if there is a particular speed at which thrust required is minimum and that corresponds to a case, where C L by C D is maximum and for that C L is C D naught by K. Once, we are through with thrust required, we try to also understand, if this is the engine, which you supply first from the engine say for typically a jet engine, then the point of intersection here will give me the maximum velocity.

That is a time, when I have put the throttle of the engine full and at that point, the thrust available and drag, the thrust required same. So, this is my V max, this also we have understood.

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After that we went for power required, it was straight forward, because this is a power required at a cruise; that means, power required with the constant velocity for that power required is thrust required into V. And we already know thrust required as W by C L by C D into V and V, I also know, because it is a cruise. So, lift equal to weight. So, V is nothing but, 2 W by S by rho C L.

So, once I know V, once I know C L by C D, I can find the power required and in this, when we substitute a V here, we realise power required will be proportional to C L 3 by 2 by C D and for power required minimum, we realize that C L 3 by 2 by C D should be maximum. Unlike, for thrust required minimum, it was C L by C D maximum and like for thrust requirement minimum, this translated into C L equal to C D naught by K under root for power required minimum, we found this translated into C N equal to under root 3 C D naught by K.

We also realise that, if I want to fly at C L equal to C D naught by K; that is such requirement minimum. Then, the C L require to fly a thrust require minimum is lesser compare to C L required to fly at power require minimum. Therefore, the velocity require to fly at some thrust required minimum will be higher compare to velocity require to fly a power required minimum. So, V power required minimum is less than V thrust required minimum.

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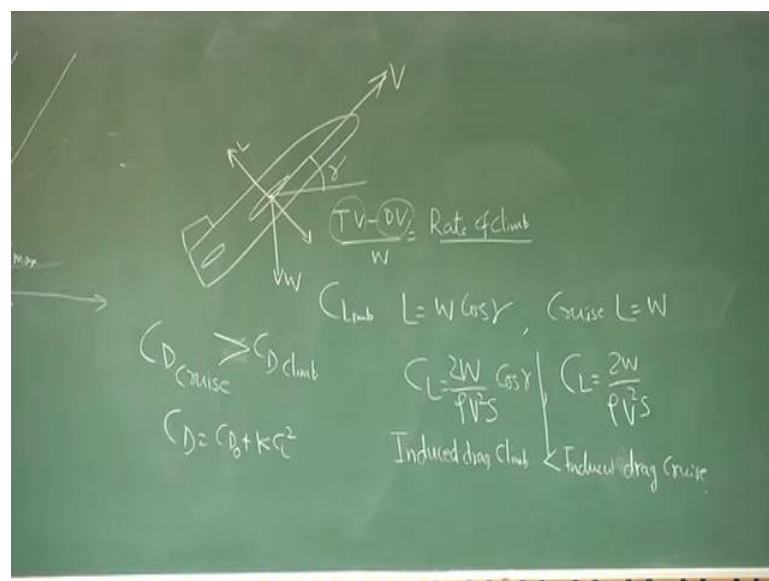
Once, we understood this, we also try to see, what happens if an airplane goes on climb and there, we also realize that, that T V minus D V by W as rate of climb. This T V minus D V is nothing but, the excess power. What is T V, T V is the thrust available into V to power available and what is D V, D V is the power required to overcome the drag. But, there is a catch, when I am doing a climb, let us understand, when I am doing a climb, then lift is W cos of gamma or gamma is the fight path angel, where to the cruise then lift equal to weight.

So, you could see that C L is 2 W by rho V square S into cos gamma and for C L in this case is 2 W by rho V square S. So, which one is greater, since here is the cos gamma, you could see clearly that C L during climb is less compare to C L during cruise. That means, the induce drag for climb is less than induced drag cruise; that in totality means C D cruise is less than is greater than C D climb.

So, when I interpret this, this you understand, because C D is nothing but, C D naught plus K C L square. So, that means, if I come back here, I need to understand that D into V in this case is not the power required for cruise, it is power required for climb. This distinction we must have, although for gamma less than 15 degrees, for all practical propose, they are same or comparable.

So, you do not make such of a mistake, but fundamentally, we need to understand that while we climb D into V is the power require in climb, which is less compared to power require in cruise, keeping other thing same.
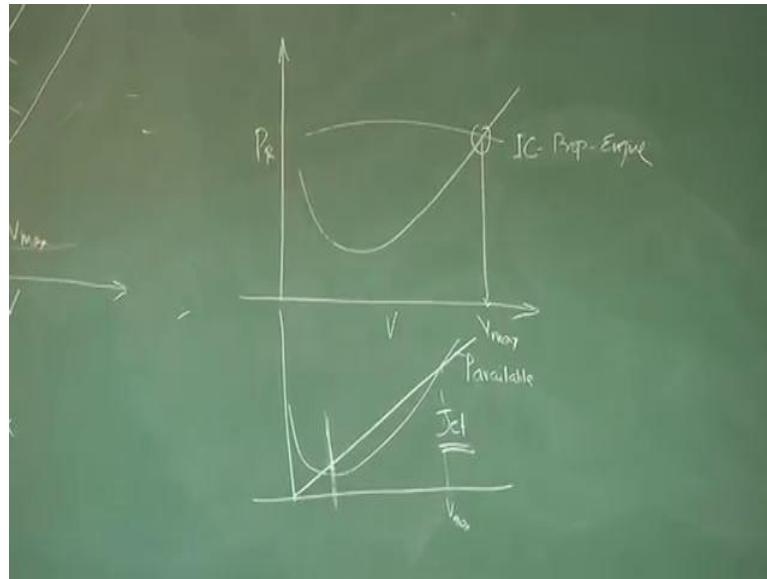
We also realize that, this power required verses V, if I draw like this and if this is typically for a I C Driven popular combination engine, this is the V max and for jet engine, if you want to draw it like this. Again, this is the power required, the jet engine, this is the power available; this is for jet engine to see this point is the V max. But, as a designer, we need to also that if I am coming for a landing, I will be operating somewhere here. So, I have got enough excess power.

So, in case of any eventuality, I can use that excess power to come out of it. However, when I am using a jet engine, you could see that this excess power near the stall or nearby landing that time the excess power relatively is less compare to for such engine. So, this is one distinction a designer need to keep in mind.
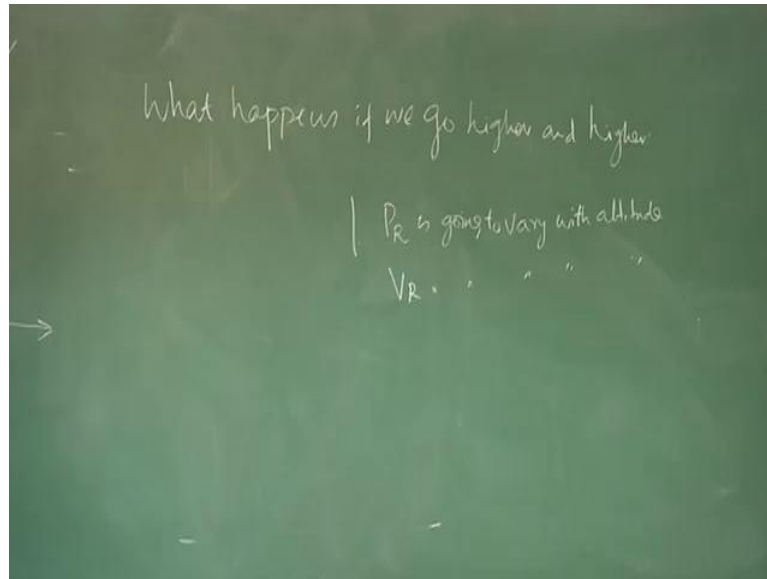
What happens, if we go higher and higher; that is more specifically, how the power required is going to vary with altitude, how the velocity required is going to vary with altitude.
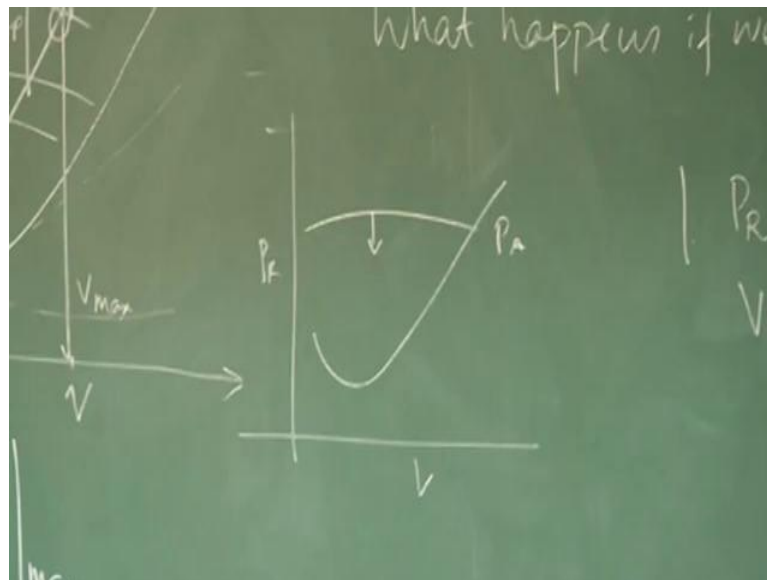
So, this is important, because please understand, we are writing power required verses V plotting it like this, this is friction altitude. As I am going higher and higher, the density of air is reducing, so that, will have an effect on this. Second thing for evaluable again as we going higher and higher density is going to reduce. So, this also will have an effect on

this, this will try to come like this and this will also try to shift. Let us see how I can we can guess, we can estimate how this variation means, change with altitude; that is exactly will be doing.

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Now, we know for cruise is 2 W by S by rho C L. So, let us say at C level, we denoted as V naught as 2 W by S rho naught C L. So, this V naught this represent C level condition, rho naught is C level density. Similarly, power required at C level will be, we know this expression 2 W cube C D square by rho naught S C L cube. Now, let us assume that we are still flying at same C L, but where going higher.

So, define altitude, so what does it mean physically, also if I am coming same C L, let say 1 kilometre, I am having lift equal to width. Now, I am fly at 2 kilometre, so lift still equal to width. So, for that since density has reduce, I must compensate that lots of lift through velocity increasing the speed if I am keeping the C L same. Because, C L is achieved by the orientation of airplane, whatever C L is required, I have orient the airplane in such a way, I have to get such an angle of tag. So, gets C L is what C L design.

Now, we are keeping the same C L. So, we have compensate through velocity. So, now we can do for same C L, we can right V altitude is equal to 2 W by S by rho C L. So, I am not changing C L, because we are trying to see that, what is the effect if you keep the C L same, but you can notice that, now rho is not rho naught, rho is rho at some altitude.

Similarly, power required at altitude will become 2 W cube C D square by rho S C L cube, again note that is rho is rho naught.

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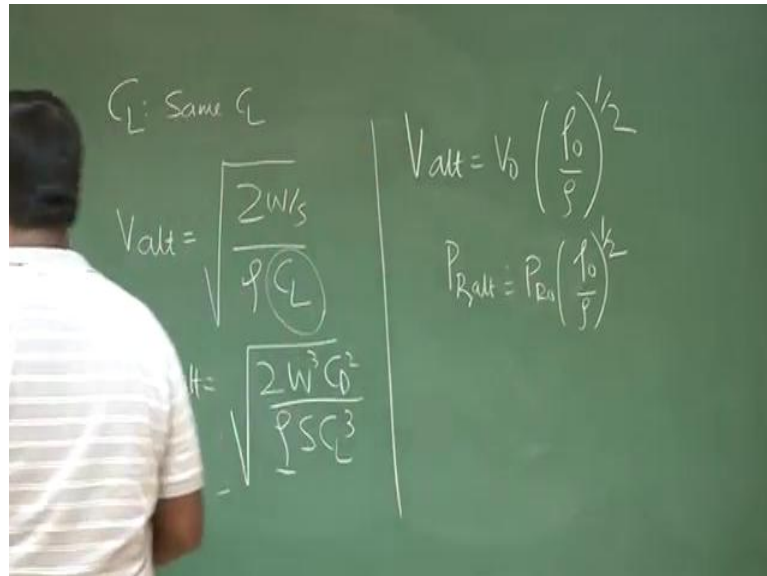
So, using these two equations, I can right in a combined way V altitude equal to V naught, rho naught by rho to the power half and power required at altitude is equal to P R naught into rho naught by rho to the power half. What is the meaning of that, it is very clear as I am going higher and higher, this rho value is going become lesser and lesser. So, this ratio will go on increasing I need higher velocity to fly at same C L to maintained lift equal to width, similar story also here.
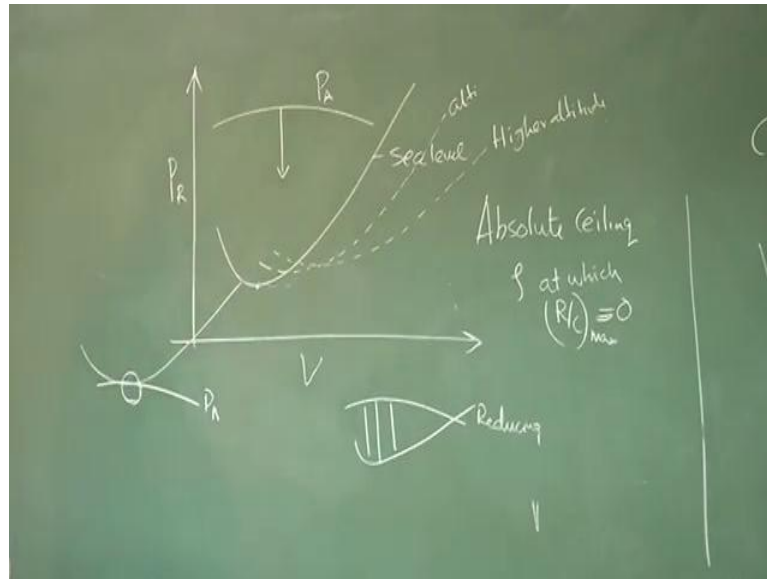
So, the power required suppose this was C Level as I write C Level condition, I can see from there, as I am going to fly at a higher altitude keeping same C L, the velocity which was here, now I require higher velocity. So, you will find the graph, this is altitude, then further, this is higher altitude. What happens to thrust or power available, the power available let us say here, typically for I C engine, popular combination.

As I go higher and higher, this man goes down, this man rotate upwards. So, there is as more and more, you this gap will go reducing. So, there will be an altitude, where will find if this is a power required the power available is just matching here and this is the point, where you have absolutely no excess power for rate of climb. And that is define in our aerospace industry as well as absolute sealing; that is we called absolute sealing that is reach that altitude used at which the rate of climb, maximum rate of climb is 0.
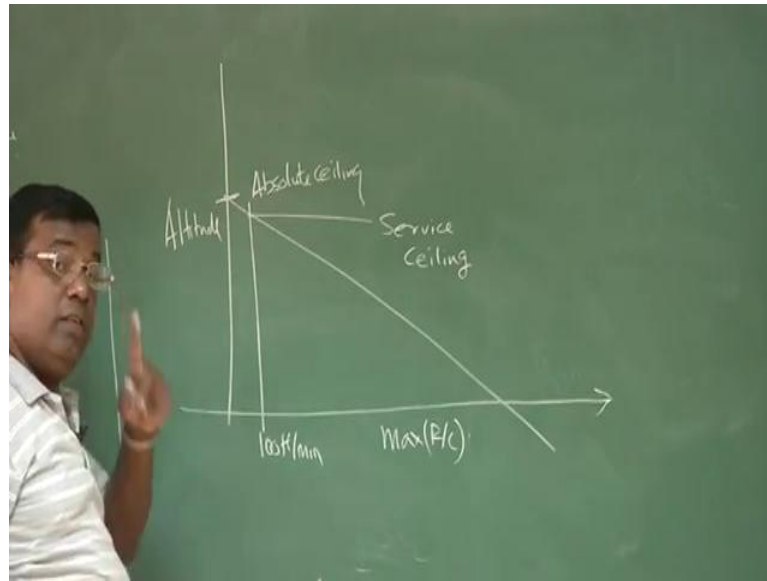
So, how it is generated, if I draw a graph, this is altitude variation can be and this is maximum rate of climb, graph is something like this, here this altitude is called absolute sealing and we for practical propose. We define an altitude, which is called service ceiling is called service ceiling service ceiling is an altitude at which the maximum rate of climb is not 0. By it is 100 fit per minute for all operation propose, we use service ceiling not absolute ceiling.