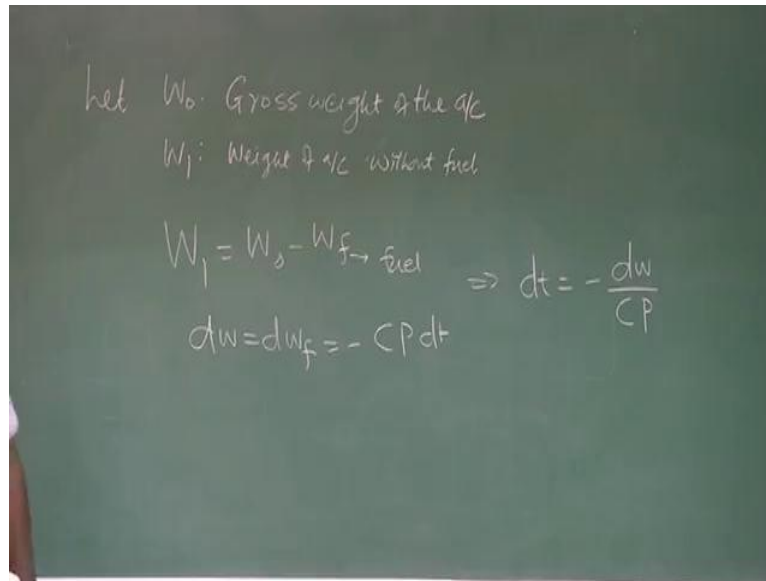


NOC: Introduction to Airplane Performance
Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kanpur

Lecture - 21
Range and Endurance: Continued

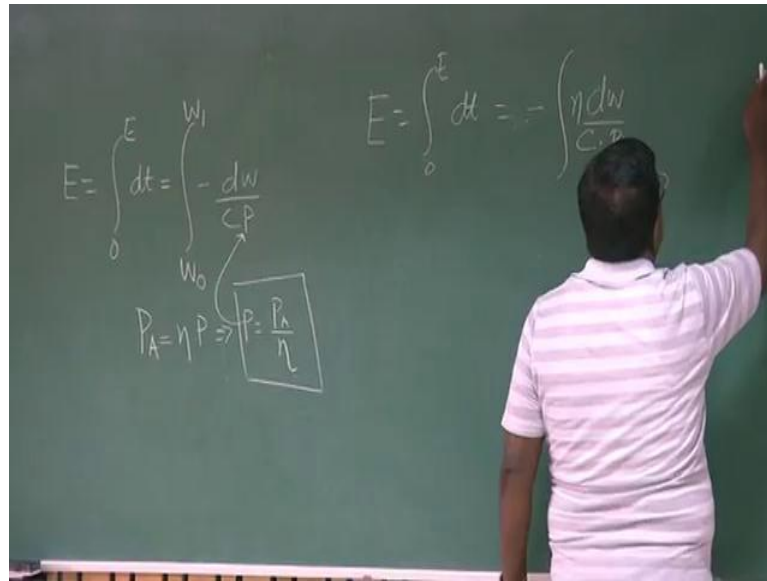
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Let us say, w_0 is the gross weight of the airplane or the aircraft; that means, it has structure weight, payload, fuel, everything and let W_1 is the weight of aircraft without fuel. Please understand, in the gross weight the payload weight, structure weight, fuel weight everything is included. In W_1 structure weight, payload weight, any other weight is included, except the fuel weight. So, I can write W_1 is equal to W_0 minus W_f , where W_f means fuel weight.

So, from here I can write dw is equal to dw_f , because I am pretty sure that I am not dropping anything out of the aircraft. The change in the weight of the aircraft is because of consumption of fuel, fuel is burnt, so it has reduced, so this is equal to minus $CP dt$. From here I can simply write dt equal to minus dw by CP , again I want to restate this p is the power available at the brake. Not the power available that is, not the power extracted by the propeller.

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So, if I want to now find out endurance, endurance will be 0 to E, total time dt and during the total time, the gross weight which is W_0 and when the fuel weight is consumed that is, W_1 . W_1 is the weight of the aircraft when no fuel; that means, whole fuel is consumed, so I write minus dw by $C.P$ and we know that p is the power available at the brake. But, power available for the airplane is η into p , so I can write p is equal to p available by η , this p I will be using here.

So, now my expression for E becomes 0 to E dt equal to E , E is already written, is equal to minus dw by C into power available and the η goes on top, η here. Now, we know since we are talking about cruise and we know that power available equal to drag.

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The image shows a chalkboard with the following handwritten equations:

$$E = \int_0^E dh = - \int_{W_0}^{W_1} \frac{\eta dw}{C \cdot P_A}$$

$$P_A = D \cdot V$$

$$L = W$$

$$V = \sqrt{\frac{2W}{\rho S C_L}}$$

$$= - \int_{W_0}^{W_1} \left(\frac{\eta}{C} \right) \frac{dw}{D \cdot V}$$

$$= - \int_{W_0}^{W_1} \left(\frac{\eta}{C} \right) \frac{dw}{\left(\frac{D}{\sqrt{\rho S C_L}} \right) \left(\frac{L}{D} \right) \sqrt{\frac{2W}{\rho S C_L}}}$$

So, this I can into v, power available is drag into v, so I can substitute power available, then I get an expression minus. This is W naught to W 1, again I put here W naught to W 1 eta by C d w by D into V. Now, what is v? From lift equal to weight, I know V equal to 2 W by rho S C L. What is rho? The altitude at which the airplane is flying, C L is the C L cruise. Typical value of C L cruise for most efficient airplane will be around 0.2, 0.3, but if you are flying at a different other altitude for different conditions, the C L may change.

So, now I will substitute here value of V, so I will get minus W naught W 1 eta by C into d w by D into 2 W by rho S C L under root. For cruise, lift equal to weight, so what I will do? I will write this as minus eta by C W naught to w 1, I will write d w by W, since W equal to lift, I compensate this W by putting L and already D is here. So, and then, I have under root 2 W by rho S C L. What I have done?

Since, lift equal to weight, what I have done from here I have added one W in the denominator and since lift equal to weight I have multiplied by lift. So, expression remains same, but this has given me an advantage, I have got an expression for L by D and here, also there is a C L sitting here.

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$$E = 2 \left(\frac{\eta}{c} \right) \left(\frac{C}{L} \right)^{3/2} \left(\frac{\rho S}{2} \right)^{1/2} \left[W_1^{-1/2} - W_0^{-1/2} \right]$$

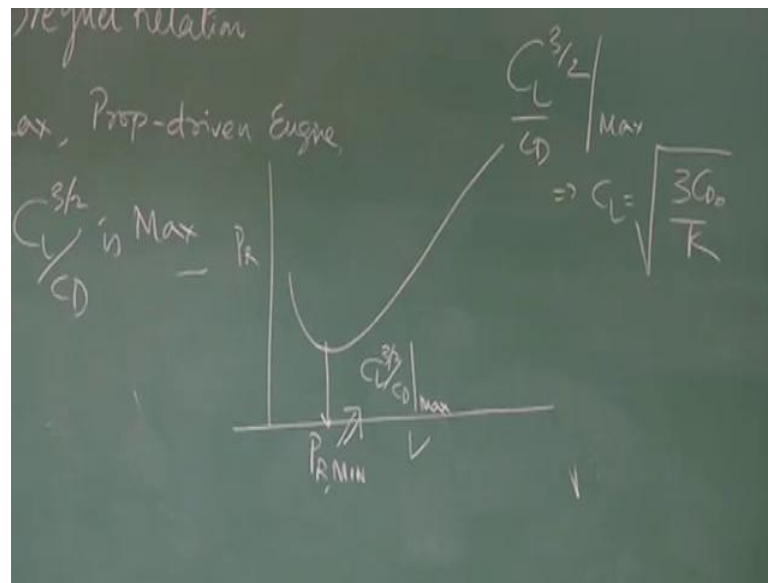
$$E = \left(\frac{\eta}{c} \right) \frac{C^{3/2}}{L} \left(2 \rho S \right)^{1/2} \left[W_1^{-1/2} - W_0^{-1/2} \right]$$

$(E)_{\max} = \frac{C^{3/2}}{L} \rho S$ is Maximum

Now, if I write further if I further simplify, I can write E as 2η by C, $C L$ 3 by 2 by C $D \rho S$ by 2 to the power half W minus half W 1 to W naught. Please note, that the order of integration has been changed to absorb this minus sign, it was W naught to W 1, now minus sign is absorbed, it is W 1 to W naught. So, if I expand it I get an expression E is equal to η by C, $C L$ 3 by 2 by C D 2 rho S to the power half W minus half W 1 minus half W naught minus half.

Please note that, when we arrived at this expression we have taken out η by C outside this integration. We have assumed that there is no effect on C, because of change in weight. Also you should remember that C does change with the altitude, but here for simplicity we have assume it to be constant. What is more important here? See, what we are seeing here, seeing that E will be maximum for a given altitude, given initial weight and final weight or fuel weight. E will be maximum, when $C L C$ by 2 by C D is maximum.

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Please understand, it is very important result which is also popularly known as Breguet relation. It is very handy in the design process of an aircraft. What we are seeing here, that E is maximum for propeller driven engine provided. You fly the airplane, such that C_L/C_D is maximum. Are you familiar with this term, C_L/C_D ? Are you familiar with this term? This C_L/C_D is maximum and this is propeller driven engine.

Let us go back to power required versus speed graph, this was the point, power required minimum and what was the condition. Condition was C_L/C_D is maximum. So, what is the understanding, that if I want to fly a propeller driven IC engine aircraft and if I want to have larger time in air or maximum endurance, I should fly such that C_L/C_D is minimum. This C_L/C_D is maximum or in turn, it means C_L/C_D maximum, it means you are flying at C_L equal to $\sqrt{3C_{D0}/K}$.

So, for a pilot he will be now flying at a C_L , which is a fixed number, which depends upon the C_{D0} value of the airplane and K of the airplane. This is clear, again I come back here. If I want to have E max, I need to fly such that C_L/C_D is maximum that in turn mean, I am flying at C_L equal $\sqrt{3C_{D0}/K}$ and this is strictly for propeller driven IC back of engine aircraft.

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$$ds = V \cdot dt \quad dt = -\frac{dw}{CP}$$

$$\int_0^R ds = \int_{W_0}^{W_1} V \frac{dw}{CP} = - \int_{W_0}^{W_1} V \frac{dw}{CP}$$

$$R = - \int_{W_0}^{W_1} V \frac{dw}{CP} = \int_{W_0}^{W_1} V \frac{L}{W \left(\frac{DV}{\eta} \right)}$$

$$P_A = P \eta$$

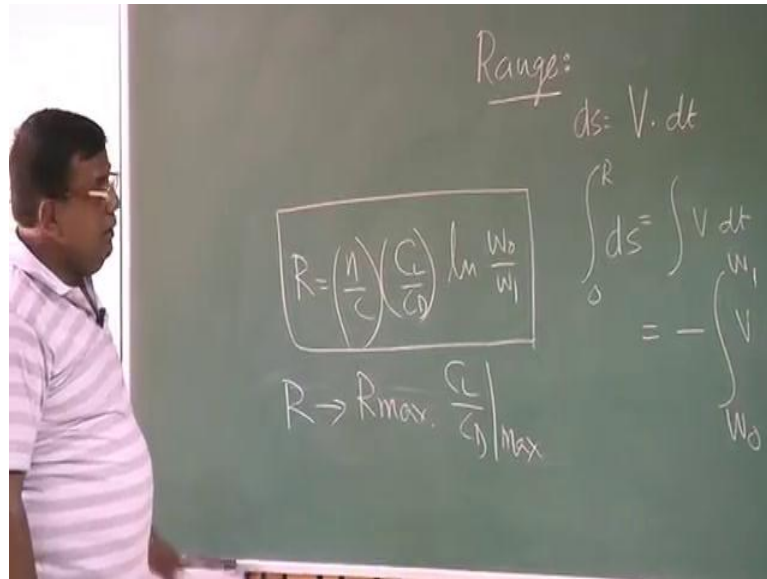
$$P = \frac{P_A}{\eta} = \frac{D \cdot V}{\eta}$$

Now, from endurance now we go to range. This is again range for which type of aircraft? This is piston driven propeller driven IC engine aircraft. Fundamentally, what is range? Please understand, typically range means the total range from takeoff to landing, but here we are focused mostly on the range during the cruise. So, this is that way approximate, but very handy for designing an airplane. So, for if it is a cruise, then range I can find out small distance travelled will be V into dt as a distance.

So, if I integrate this ds from 0 to range, it is equal to $V dt$. Now, question is what is dt ? Now, we have already seen dt is minus dw by CP , so I put it here, so this becomes minus $V dw$ by CP , again the weight from W_0 to W_1 . Here I can do little manipulation; first I will change the order of integration to absorb this minus sign.

So, I write W_1 to W_0 , then V for dw I will divide by W and multiply by lift, because lift equal to weight and then, here the power that is DV by η correct. Why this term? We know that P available is equal to P_A into η and here this term, this is the P which is at the brake. So, this P is equal to P_A by η and P_A is nothing but, D into V , so D into V by η . These V , V gets cancelled. So, what we are getting?

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We get range equal to eta by C, see there will be a C here, there is another term C here, it is C into P. So, C was missing here, so I put this C, so eta goes on the top, so eta by C integral n C L by C D and ln W naught by W 1. Let us repeat here, d s is minus V d w by C P that is equal to W 1 to W naught. We have changed the order of integration and V and C V by C d w by W, I have put one W multiplied by L, because lift equal to weight.

It does not change, P we know d v by eta and this V and this V gets cancelled, so this eta goes on the numerator. So, I have got eta by C and here I have got L by D, which I can write C L by C D. So, this is d w by W and the integration of W by W is ln, when I put the limit it comes to ln W naught by W 1, clear. Now, see for range, what is the message. For range message is, for a given eta and C, range will be maximum, when C L by C D is maximum.

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The image shows a chalkboard with several handwritten mathematical expressions. At the top left, there is the expression $-\frac{dw}{C_P}$. In the center, there are two equations: $P_A = P \eta$ and $P = \frac{P_A}{\eta} = \frac{D \cdot V}{\eta}$. To the right, there is a derivation showing $\frac{C_L}{C_D} \Big|_{max} \Rightarrow C_L = \sqrt{\frac{C_{D0}}{K}}$, with an arrow pointing down to the underlined text T_{RMIN}. On the left side, there is a circled expression $\frac{dw}{C} \frac{L}{W} \frac{1}{\eta}$.

What is the meaning of that? We have already seen C_L by C_D maximum means, it is C_L equal to under root C_{D0} by K or this is also a condition for thrust required minimum. So, if I want to fly for higher range for a given tank of fuel, message is very clear, that for maximum range I should fly, such that C_L by C_D is maximum or in turn I should fly at a C_L which is fixed given by C_{D0} by K under root.

And the pilot will be flying attain a speed at this C_L , so that lift equal to weight and he is having a cruise flight. So, this covers our discussion on range and endurance for propeller driven IC engine. Next, we will be talking about jet engine.