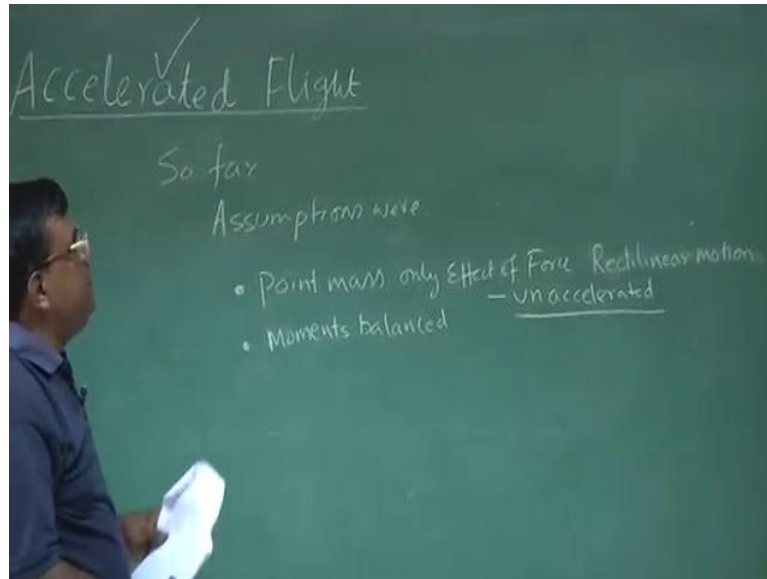


NOC: Introduction to Airplane Performance
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Lecture - 23
Accelerated Flight

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Welcome back, we will be discussing now, today different type of flight regime. So, far if I recall, we were having an assumptions of course, we are assuming the airplane to be a point mass, we are not bother about the moment of inertia of the airplane. Because, we are not discussing anything about the rotation property of the airplane, we are talking about cruise, talking about climb, distance in a rectilinear motion.

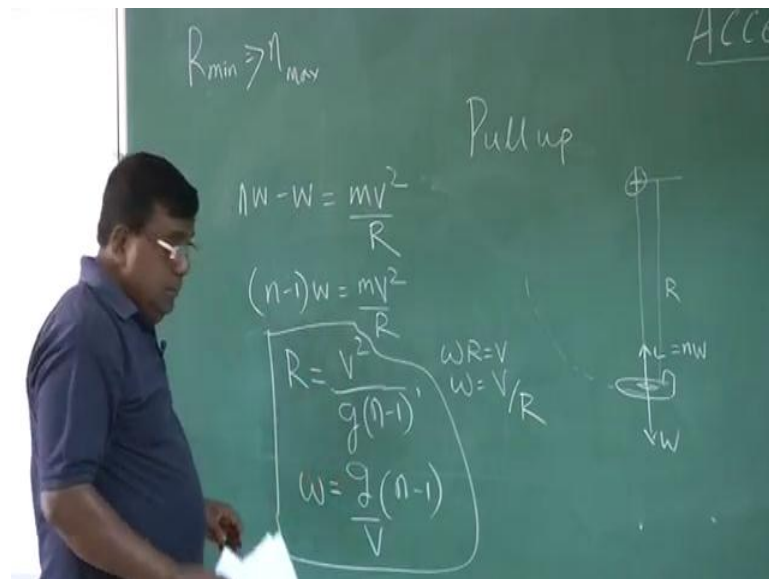
We also had an assumption, there are un accelerated; that is the forces are balanced. Of course, there is implicit assumption that, moments are also balanced; that is I am not bothered about this, I am saying the airplane is going like this, no such rotations are happening, because moments are implicitly balanced. In the second course, you will know how the moments are balanced; that is the different matter all together, but assumptions are this.

Now, we are going to relax one of these conditions and that is, we still believe that we will be following point mass. However, this time we will say, it is accelerated flight, this assumption that rectilinear motion un accelerated; that is relaxed. So, we will be terming

this session under the title accelerated flight. You know, one over we think of an airplane, the young man will always think, airplane I should do this, I should do this, turn, I should do roll like this.

Like, when you buy a motor cycle, you never think of only going ((Refer Time: 01:49)) like this, you think of I should be able to take a sharp turn, I should be able to accelerate very fast and that is what is the significance of a youth. And in aerospace, the adventure will always remain as long as aerospace remains growing. So, this part of youth, part of that adventure is an individual requirement to be a good aerospace engineer.

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So, now, we will be talking about accelerated flight. Let us first discuss about pull up; what is typically a pull up, assuming I am going like this, say for your clarity, I am going like this lift equal to weight, thrust equal to drag and I suddenly increase the lift. So, what will happen, if I increase the lift; that is the lift is more than the weight, what will happen? Please recall you have agreed that lift is perpendicular to velocity; that is how I define lift.

So, now from here you think, if a body is moving in this direction and you are pulling it by a string. So, tension, what happens, it goes into a curved path and this force is perpendicular to velocity, this reminds you about centripetal acceleration, so we will be using that understanding to model a pull up maneuver. So, let me draw, this is an airplane, this is a lift, this is a weight. If you want it to go for a curved path and since, it

is going in a curved path, so there it is no more unaccelerated flight, reason accelerated flight.

So, if I draw this diagram like this and then, I try to write equation of motion, again assuming, this is a point mass ((Refer Time: 04:02)) So, I have not relax this and of course, we are assuming moments are balanced. So, then I can write see lift, which is now not equal to weight, if it is going like this. So, either lift equal to nW , where n is the load factor, which is nothing but, ratio of lift to weight. So, now, lift is not equal to weight, lift equal to nW , so this is nW .

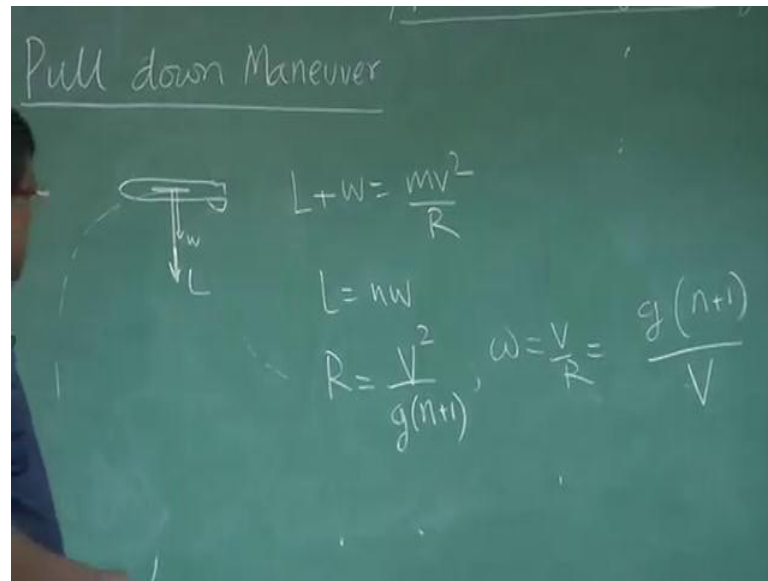
So, if I now write the equation, what will happen nW minus W will be equal to mV^2 upon R , where R is the radius of turn. You see the vertical plane like the pull up. So, how do you understand from a class 11, 12 understanding, this is the net force perpendicular to the velocity vector. More importantly nW is perpendicular to velocity vector and if I see this diagram, see this force, which is responsible for giving the centripetal acceleration.

And now if I do little manipulation, I can write $n - 1 W$ is equal to mV^2 upon R or I can write R equal to V^2 by $g(n - 1)$. This is a radius and if I want to know, what is the angular rate ω , you know ωR equal to V or ω equal to V by R , if I used relationship, then I can write ω is equal to g by $V(n - 1)$. Let me check, R equal to V^2 by $g(n - 1)$ and ω equal to g by $V(n - 1)$.

What is the message you are getting out of this? Yes, let us come back here, if you want R to be minimum, I want to turn like this R minimum, how can I achieve R minimum or you can achieve minimum, if for a given V and given value of g , if n is maximum. What is R minimum, n is maximum? That is you having high load factor. How do we get a high load factor? You should have a very effective wing loading, very effective lift distribution on the wing.

So, all these things will get coupled, when I think in terms of R minimum, R minimum is n maximum. Similarly, you could see for a larger angular rate, again for a given V , n has to be large, which is by physics it is fantastic. Because, unless we have the lift, who will do the pull up, it is a lift only, same time you can see that, R is also depending on V , ω is also inversely depending on V , this is or say observation you must have.

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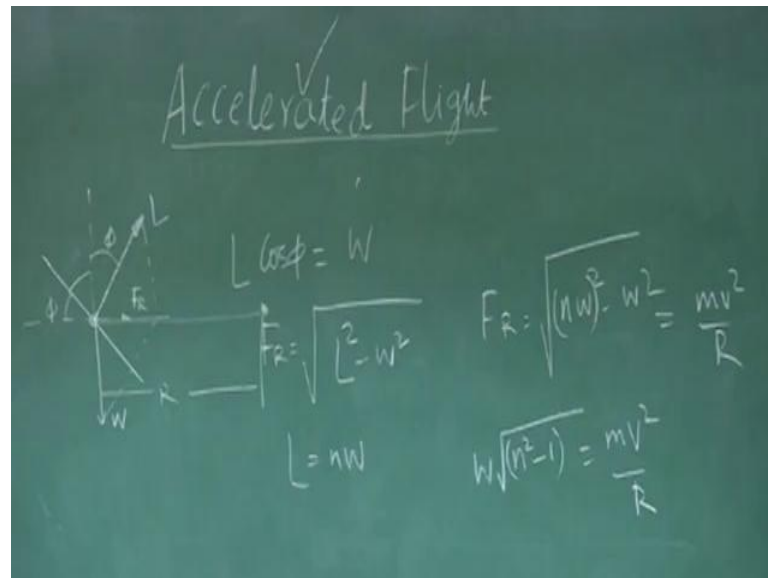
Here, is another maneuver called pull down maneuver, where I can like an inverted flight, here the lift acts downward and also the weight is acting and it is going in a circle in a vertical plane. If I now write, same equation I write L plus W will be responsible for giving centripetal acceleration and I know L equal to n W. So, by manipulating this two equations, I can write R equal to V square by g n plus 1 and omega is equal to V by R; that is equal to g n plus 1 by V.

Here, also you could see, if I want R maximum, R minimum, then n has to be maximum for a given speed. If I want omega to be very high, n has to be a maximum, naturally this is true, because if I want to very sharp turn, this is lift; that is going to do all these things and lift is more, then the lower factor is more. So, it is consistent with the understanding, how that whole mechanics is going about.

Once, I cover this pull on maneuver, now automatically it comes to our mind, why we are restricting to vertical plane, rigorously used maneuver, all the maneuver is the turning flight, you turn like this. Now, think of ((Refer Time: 10:00)) if this is the airplane, this flying like this and it wants to turn in horizontal circle; that means, I need to have a force which should be towards the center of the circle. To, where from I get this force, if the wing is like this, I do not get, because lift is like this, but if I bang this airplane, then the lift is now in climb. And one of the components, horizontal component, this will be responsible for giving me a circle in the horizontal plane.

I repeat, if I am flying like this, this is the lift, if I want to take a circular turn in the horizontal plane; I need a force in this direction. So, what I do, I bang the airplane, the moment I bang, this is the horizontal components will be responsible to give a force in this directions. And because of that, I will be taking a circular turn or we say turning flight. If this is understood the modelling becomes simple, let me draw it.

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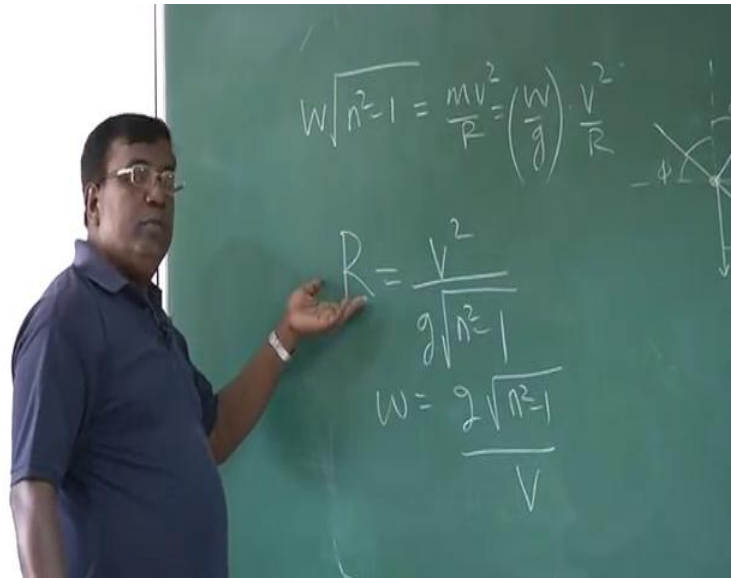
See, we have bang the airplane by an angle phi, this is the lift, this is the weight, this is again phi and this is I call it a alpha that horizontal force, which will be responsible for giving me a motion in the horizontal plane, a circular motion in the horizontal plane, a curved motion in a circular plane. Now, from this force diagram I can write $L \cos \phi$ equal to W and F_R equal to under root of L square minus W square from this triangle to this.

If the lift, this is the lift, this is F_R , this is the weight, so I can reserve the weight, one along this, one along this, so I can write this simple. Once, I write that, now again I know lift equal to nW , is it a load factor. So, I can write F_R as under root nW square minus W square. So, after doing some maculation I can write W under root n squared minus 1 is equal to mV square upon R , because this F_R is only giving me the centripetal acceleration. So, I write this F_R is equal to mV square upon R .

What is this R ? R is the radius of turn in the horizontal plane; that is, if I am turning like this. So, what is the radius of the turn; that is R and what is this F_R , F_R is the horizontal

component, which gives me enough force. So, that I have equisetive centripetal acceleration to ensure a horizontal turn.

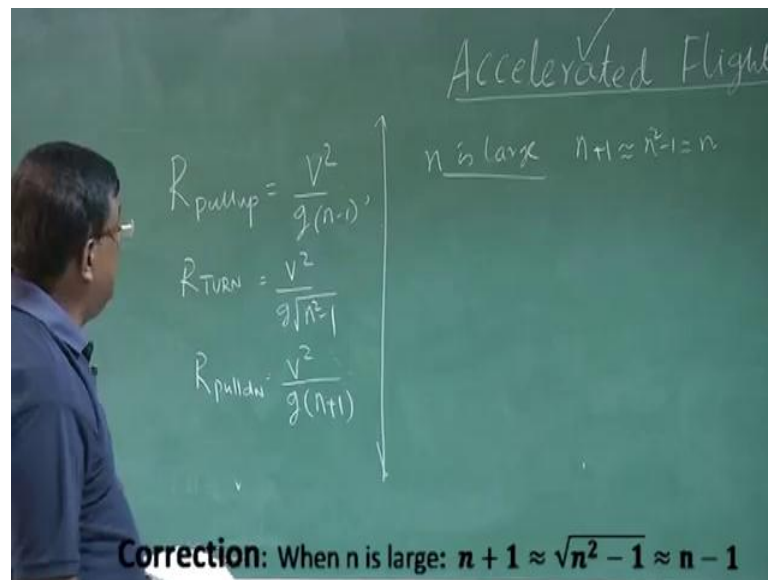
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Now, if I write this $m V$ square upon R , for m , for your completion, I am writing m is W by g into V square upon R . And now, if I do little bit of adjustment, I can find out an expression R as V square by g under root n squared minus 1 and omega as g under root n square minus 1 by V . So, again you see that, if I want radius to be small or minimum for a turning flight, I need n to be large, which is fair enough, because of after all n is nothing but, lift divide by weight.

In large weight, lift is more than the weight and lift is the all this activity. So, we are happy about it, do not forget here, the some component of weight is also coming in this case. Is a $F R$, if we see is L square minus W square. So, let us summarise and try to get a genetic feeling about this turning flight.

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We are since R for pull up as g to V square by g minus 1, let say R for turning flight as V square by g under root n square minus 1, R for pull down V square by g n plus 1. Now, a think of a situation, let us say n is large, when we talk about acceleration flight, our aim is always to go higher and otherwise, why do you want acceleration to flight. Natural tendency, I should fly a higher load factor.

Suppose, n is large and we do an approximation n plus 1 approximate equal to n minus 1 equal to n or I can promote precious, I can write n square minus 1 and this approximation that n is large. If n is 6, then it is one is neglected compared to 6, just a get an idea about some very important understanding for a designer, because all this things have from the mechanics.

But, the designer should translate this to it is configuration of an airplane, we should be capable of generating requisite forces to achieve all this parameters, performance parameters.

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$$R_{\text{pullup}} = \frac{V^2}{g n}$$

$$R_{\text{TURN}} = \frac{V^2}{g n}$$

$$R_{\text{pulldown}} = \frac{V^2}{g n}$$

So, if I apply this approximation here, here I get R pull up is equal to V square by g n, again R turn also I get as V square by g n and R pull down also I get as V square by g.

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$$R = \frac{V^2}{g n} \text{ if } n \text{ is large}$$

$$w = g n$$

$$L = \frac{1}{2} \rho v^2 S C_L$$

$$V^2 = \frac{2L}{\rho S C_L}$$

$$R = \frac{V^2}{g n} = \frac{2L}{\rho S C_L g n}$$

$$\boxed{n = \frac{L}{w}} \Rightarrow R = \frac{2L}{\rho S C_L g}$$

Accelerated F
n is large $n+1 \approx n^2-1$

So, we have seen that, if n is large, then I can express R as V square by g n, whether it is a pull up, pull down or a turning flight. Similarly, you can show omega equal to g n by V. Now, we want to extract some information for a designer; that is the important question is, if I want to make an aircraft, which is capable of generating radius of turn, let us say a 1, some value.

Then, I must ensure that the design layout is such and design layout of what, design layout of primarily of wing, because wing is giving this force. So, how do I connect one of the wing important properties like wing loading with this R or W, angular rate; that is our aim and keep this in mind R equal to V square by g n, if n is large. What we are doing, first we are edit L in the nothing but, half row V square S C L, from there I am getting a expression of V square, V square will be 2 L by row S C L.

No issues, now since R is equal to V square plus by g n, I substitute V square by this expression, so what I will get, for V square, I will get 2 L row S C L and g and n is here g into n, where n is a load factor. What is n, it is a load factor, so n equal to L by W.

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Accelerated Flight

$R_{min} = \left(\frac{W}{S}\right)_{min}$

$n+1 \approx R-1 = n$

$R = \frac{2L}{\rho S C_L} \frac{1}{g \left(\frac{L}{W}\right)} \Rightarrow \frac{2}{\rho} \left(\frac{W}{S}\right) \frac{1}{C_L g}$

$R = \frac{2}{\rho} \frac{1}{C_L g} \left(\frac{W}{S}\right)_{min}$

$W = g \sqrt{\frac{\rho C_L L}{2 \left(\frac{W}{S}\right)}}$

So, I will replacing n by L by W in the next expression, you see 2 L row S C L, 1 by g instead of n, I am writing L by W. After that, I realise this L gets cancel and W come on the numerator. So, I can simplify this expression as 2 by row w by S, 1 by C L into g or the final expression come radius as 2 by row L by c l g a W by S. As I was telling you after all these forces come in, because of wing and or the important property of the wing in relation to lifting characteristics is wing loading.

Because, if wing loading is small; that means, in a large area wing area. Now, see here all n is to get R minimum and what does it say, if for R minimum I must also ensure W by S minimum. Keeping other thing constant, all needs also tells that if I want a particular R, what should W by S or if you want particular angular rate, what will be W

by S. Please, also realize here if I want larger angular rate very fast turning, again it is says that wing loading should be very, very low; that means, larger wing area, larger wing area means more lift and very consistent.

So, you see simple manipulation rearrangement of whatever we have learnt or in some realistic approximation, how wonderful results we get which are so useful for a designer; that is the purpose of this course. That whatever performance we are talking about finally, will ask a question, am I need position to give the designer the right inside in terms of laying out this component.

So, here at least one thing we have done understood for ourselves; that if I want particular radius or particular omega, what should be my wing loading. Please, understand; when you select the wing loading, it may be selected because of many other criteria's, where because of range, because of cruise mission, because of climb mission there are so money requirements will come.

But as a good designer, we need to know for every mission, what is wing loading required and find a new take out freeze a wing loading, which try to satisfy most of them or satisfy most of the parameters most of the mission requirements in a waited manner. Like, if you are design a transporter plane, then you give more voltage for wing loading for cruise performances, like that you have to be smart enough to get an optimal configuration

Thank you.