NOC: Introduction to Airplane Performance
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Lecture – 30
Take off Performance: Continued...

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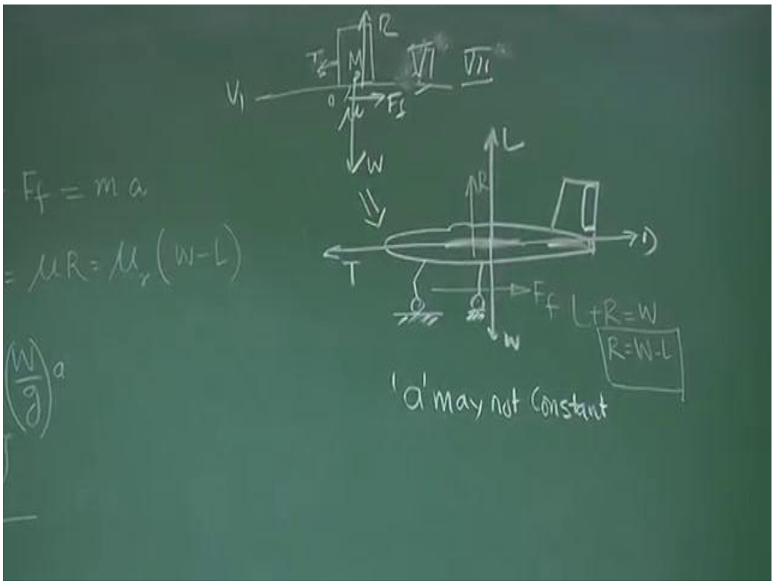

Now, what is happening here? Let us see this, what is happening here. So, let me first write the equation the way I wrote for that, this is T minus D minus because of the wing in contact with the surface, the landing gears. I say net frictional force is F f and let see the ground reaction is R and weight is W. So, one thing I know that here let me complete this L plus R equal to W or R equal to W minus L, naturally if lift is equal to weight then R will be 0, this I know.

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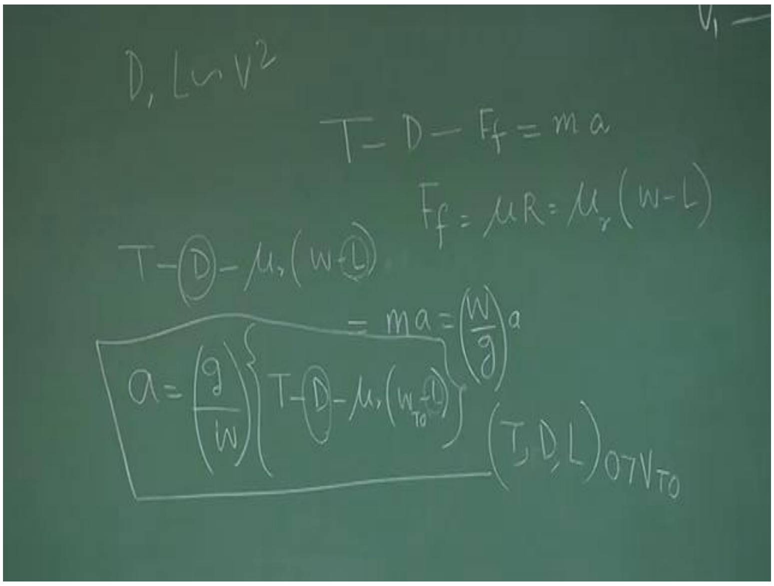

So, if I now write this, complete this equation I have T minus D minus F f is equal to m into a and for F f, I can write equal to mu into R that is mu R. I am purposely introducing R for rolling friction coefficient and for R, I am writing W minus L. So, now, what is this equation transform into, T minus D minus mu r into W minus L equal to, let me write it here equal to m a or I can write W by g into a. We could see here, this term the drag and L both drag and L changes with V square and this airplane is accelerating from 0 to V, which is V take off.

That means each point see the velocity or the speed is changing, this net force acting on the aircraft is also changing, it is not constant as it was here, that is the problem now. Now, the acceleration if we see if I write acceleration, it will be g by W T minus D minus mu r W minus L that is all. What is the W here? This W is nothing but, W take off, because we are on the ground. As I am climbing and climbing after one hour or a finite time half hour even, you will find the vehicle reduce, because the fuel is consumed. So, this W take off is the net gross weight.

So, if this is the acceleration now how do I solve this problem, but we have learnt from class 6 or 7th standard, if the acceleration is somehow constant, somehow if I can assume it is a average acceleration you will work for this, then I have to only recall my 6, 7th standard knowledge to solve this problem and we will be doing exactly that. What is found is, if I compute this thrust we changes it with speed, drag, lift. All this term if I compute at 0.7 times V take off… Is it clear? If I compute this term thrust which changes with speed we have be very appreciable, but it does change, drag you know S V square lift V square.

So, actually as the speed increases this term will go on changing, hence the acceleration will not remain constant. But, what is found is, if I compute drag and lift and try to give a correction for thrust at 0.7 V take off that is, as if it is moving with an average acceleration which can be assumed to be constant, evaluated at 0.7 V take off. That means what, I will calculate drag at 0.7 take off I will calculate lift at 0.7 take off and I will do thrust correction for 0.7 V take off, if it is required.

You can for all practical purpose you can assume thrust is not varying, drag and lift that is the primary thing here as for as causing the acceleration is conserved. So, what is learnt? Very simple, that though this acceleration is changing; however, if I calculate this lift and drag at 0.7 V take off I can assume this is the average acceleration and good enough to calculate this S G.

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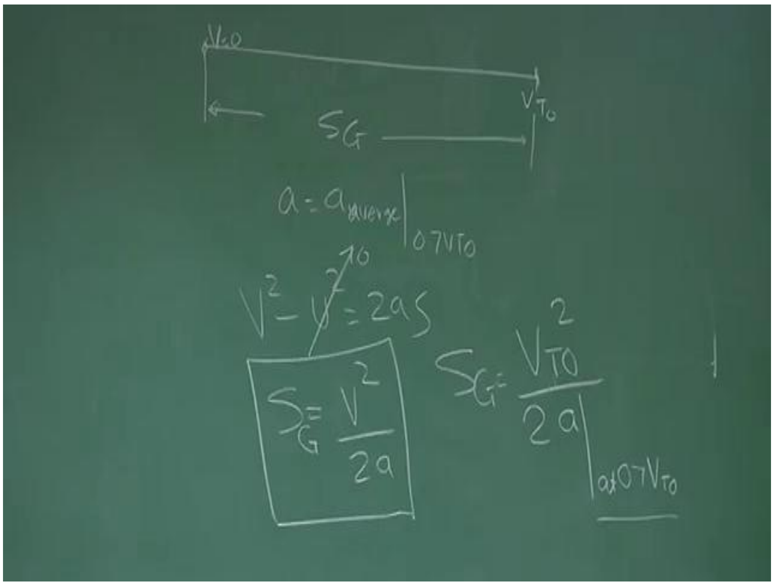

V equal to 0, V equal to V take off, I call this distance S G. So, now, tell me if a is a average evaluated at 0.7 V take off, then where is the problem, then simply I go for this V square minus U square equal 2 a S, I know initially velocity is 0. So, I know S square equal to V square by 2 a. So, I know S equal to V square by 2 a and that is nothing but, my S G. To be more precise, S G I can write as V take off square by 2 a evaluated at 0.7 V take off, very simple you see how good it is if your concepts are little bit clear in class 6, 7 or 8.

Now, you will put the expression and see what additional information we get out of this.

Please remember, whenever you write anything in the black board, whatever equations you are writing finally, our design I is back in your mind. We try to extract maximum information, so that I can incorporate to your understanding in designing an aircraft; otherwise, it is just like writing some equations.

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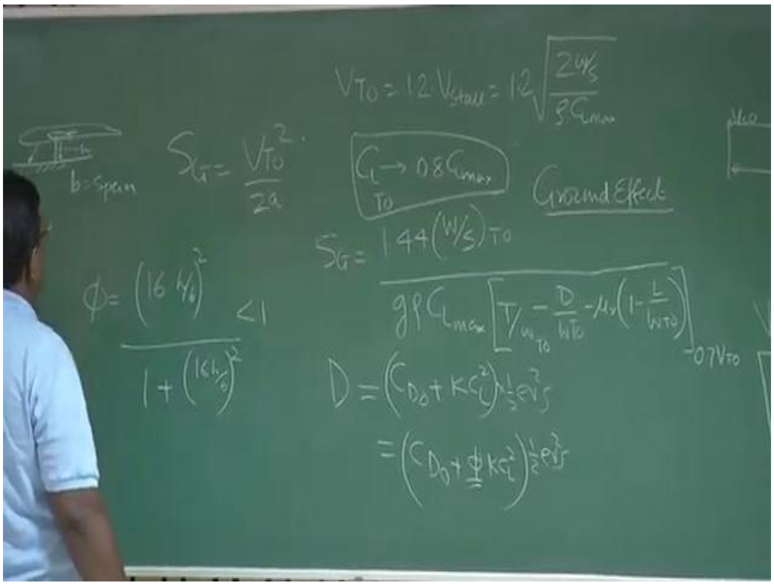

So, S G now once I substitute V take off square by 2 a and what was V take off, it was 1.2 times V stall, so this was 1.2 times V stall was 2, so W by S rho C L max. So, if I now put it here what will happen, I will just get an expression S G equal to let me write the expression 1.44 W by S take off by g rho C L max T by W take off minus D by W take off minus mu r into 1 minus L by W take off. So, this is W minus L, so that is divided by W, so it has come like this.

I am sure you would be able to derive this expression, we have to just substitute for V take off these expression and for a, we have to put the average acceleration. So, I will write this evaluated at point V take off, I will check that whether you are doing this small, small derivations or not through assignment. Please do that, because this will give you insight. As I would mention, do you think the whole effort for calculating S G is over here, let us see or visualize one thing.

So, for whatever aerodynamic characterization we have done that is estimation of C L, estimation of drag polar, understanding about down wash, all those things where when the aircraft was flying in air fall away from the ground. Now, imagine when this airplane is coming is taking off from the ground, so these wings are very close to the ground. So, what will happen those wing tip what is say which is being generated while it is touching or accelerating on the run way, this wing tip what it say is, we will now go and interfere with the ground and their strength will be reduced.

For knowing the details about why this is being reduced, you can move, read any book on aerodynamics. But, please understand from flight mechanics point of view, near the ground when the airplane is going like this for takeoff, because of presence of ground this wing tip what it says, their strength gets reduced and the induced drag K C L square component also get reduced, for that the down wash also go to little lesser in magnitude.

So, let me see, if want to compute the drag I need C D naught plus K C L square, the drag polar that to be used, then I multiply this with half rho V square S that will be my drag, when it is going for a takeoff. But, I know because the wing is too close to the ground, the wingtip what it says will interfere with the ground and they are very reflection and the wing tip what it says we loss their strength. So, the induced drag component will be less, so I cannot really use C D naught plus K C L square.

So, what is used is C D naught plus phi K C L square and this is half rho V square S. So, this phi number wise if you take induced drag is around 80 percent 70 to 80 percent good enough. That means, 20 to 30 percent reduction is there, 20 percent is fairly a good initial design approximation. But, there are many researches effort given this value of phi, given an expression to estimate phi for incompressible flow and that is modeled as 16 h by b, I will explain what is this plus 1 plus 16 h by b whole square and this value is less than 1.

What is h? H is the distance of the wing from the ground which is the h and b is the span, b you know is the span, if it has flow 2 to 3 times the span vertically, then ground effect should not be taken at all. So, what is the new addition thing we have done? We know because of ground effect I am calling that as ground effects, again I repeat very loosely or very what you will call, I am not going deep into it, the ground effect of the flight mechanism I will tell, in this case is the wingtip what it says which were generated when the airplane is accelerating and generating lift.

This wingtip what it says will interfere with the ground, there were reflection and the strength of wing what it says will reduces, reducing the induced drag and that is modeled through phi, this is one change you need to have. Another question comes, when airplane is going like this and starts taking a turn like this, we call rotate like this, how much time it will take, how much time is allowed to rotate the airplane. For all practical purpose, it is not more than 3 seconds.

So, rotate then transition, this whole some times for a high T by W aircraft we may find they happening one time, you are going like this, you would not want to distinguish this, but we need to cater for those. So, coming back here again, once I know this how to calculate and some time you will find, many literature there is a advice when you calculating the lift do not take the C L max, you take point a time the C L max.

So, for C L you take it C L take off, you take it as 0.8 time C L max. All this ((Refer Time: 14:54)) will be clear once you solve a numerical problem, but now this session is to understand the physics behind it. These are all the numbers have come, not necessarily because of well understood physics, but it is from the regulatory angle also to ensure that everything is safe. Now, we will come back here, I try to give an interpretation to this S G and from the designers I will try to get some interference.

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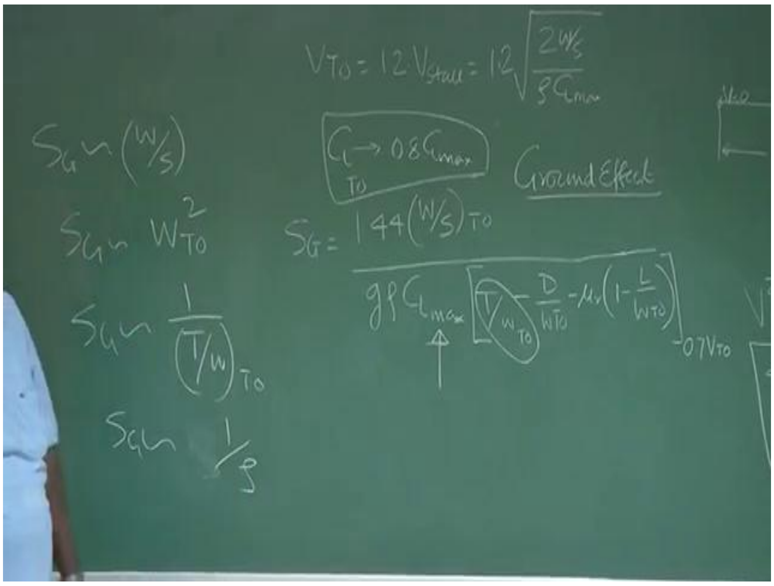

What do I see? I see S G is proportional to W by S that is, if reloading is less S G is less. Wing loading is less means large wing, but the probably with large wing is large wing mean large drag, so there is a conflict. So, designer has to see what, how do I handle this and you will see that, that is why high speed airplane they do not want very high span wings, they are low span wings, low aspect ratio wings by now you understand.

Then, also you see that S G if you carefully see if I take this W T o W take off on the top here, then actually S G will be proportional to W take off square. What is the message?

Message is, if the weight is increased heavy aircraft will have larger S G; that mean they require larger ground load distance to achieve V take off. So, in the moment you want to design a heavier aircraft, we should be careful it will demand larger ground load distance.

But, as a designer we cannot keep quiet, we have to handle the situation, larger aircraft, but not so large take off distance or ground load distance loosely. What is the option? Option is increase C L max, use all high tech innovative flaps, let the drag increase will handle it. But, less C L max increase, more than C L max if you see T by W is S G if you have an engine whose T by W is very high that will help in reducing the S G distance that is ground load distance which is also obvious.

If I have very thrust efficient engine, the acceleration will be faster, because T by W is high, so S G will be less. Then finally, I come to density you see, it is inversely proportional to density of air from where you are taking off. As I was mentioning, from here we could see that very clearly as density decreases S G increases that is, if you are going higher altitude airport, this is a Jammu Kashmir let say Leh Ladakh, but density of air will be less.

So, you will require larger ground load distance, this is extremely important also you understand from here that density not only changes with attitude, density also changes with whether it is a winter season or a summer season. So, let us see for winter or summer season how S G will get affected. Now, we will try to have a closer look on this density aspect.

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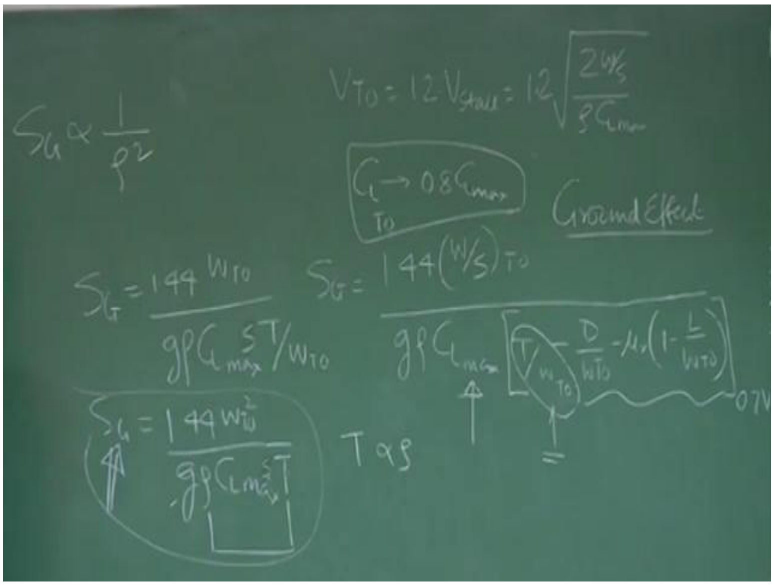

Let us see for simplicity we assume that this term is more dominating than the rest of this term, which is not a very bad assumption. So, I neglect this term compare to this term, after doing this approximation I can write S G as 1.44 W take off square by g rho C L max into T, we also know the thrust varies with density. So, in an effect here the density variation, here is the density variation, so I can write S G varies inversely with rho square.

Just not rho, earlier we have taking about rho density, it is actually more than that, that is why you will find if you are trying to take off in half summer days, then the density of air will be less. So, you will be requiring larger S g, in a winter season for density of air is more compared to summer season we will find S G will be less and reverse happens in the winter season for the same aircraft of the same location of the airport from where we are going to take off.

This is very important understanding that if I want to reduce S G message is increase C L max reduce wing loading increase T by W. This is the primary parameters which a designer will keep in mind.

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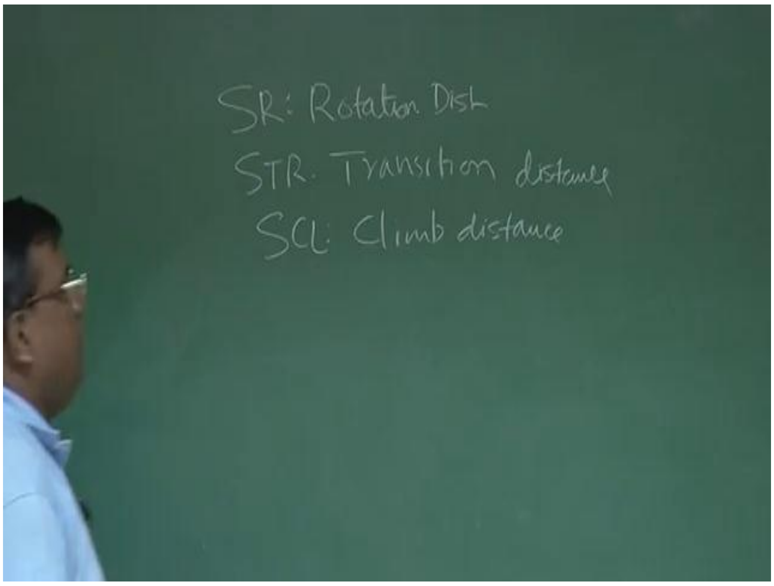

Now, S R which is rotation distance and S T R is transition distance and S C L which is climb distance complete here distance what are all this.

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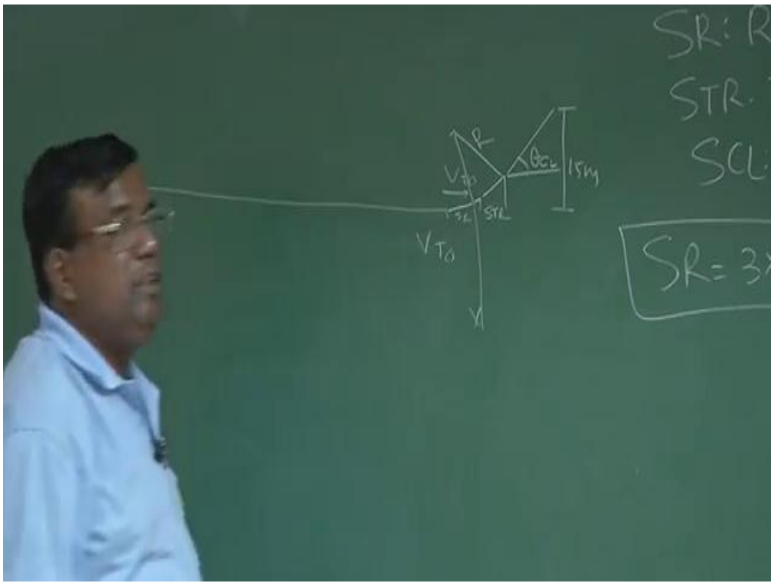

We know that airplane start from here come to stage that is we take off then rotate the airplane for 3 seconds then there is a transition were he try to arrange for climb. So, this is S R, this is transition S T R and then there is a climb which is 15 meter. We have trying to find out S R first, since we know it is at the most for 3 seconds and this time also it is moving with v take off S R is very simple, S R is 3 seconds into V take off job is done.

Now, do it transition what happens bring the rotational face, the nose will goes up still moves V take off like this, then there is a pull up to type things, slight pull up and during this pull up the pilot also adjust himself all the controls for a climb ready for a climb if the adjust the controls. So, this part is best module as a pull up which radius of R and this is a climb, so this is theta climb angle C L in this part clear, this will rotation aero plane was going like this, as it attains V take off it rotates that is nose we will leave now leave the ground still moving forward and started going up and there is a transition like this during which the pilot adjust itself for a climb preparation.

So, this part is for 3 seconds, so S R is 3 into V take off and this part is part of a pull up manoeuvre, we find out and this is a straight climb. With this standing, it is a little bit of geometric we will find out this distance.

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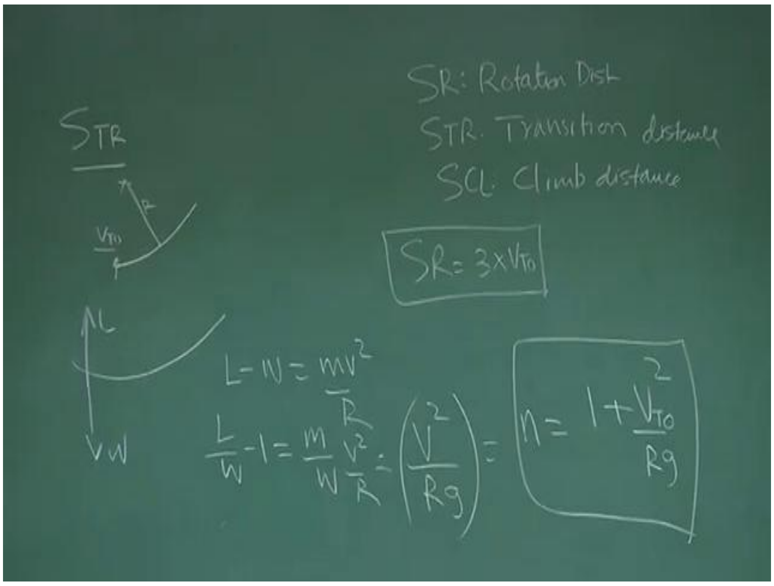

To calculate S T R will again see here assumption is here it is V take off and it is following a pull up with radius R and we know by now that for a pull up this is L, this is W then L minus W equal to m v square upon R and if I divide by the w I get L by w minus 1 equal to m by w v square upon R and this is equal to v square upon R g. So, I get an expression n equal to 1 plus v square upon R g.

What is the message? Message is if I know sum assessment of the value of load factor n during which it was just doing is pull up and seems I know v as V take off if I know g. So, I should be able get the value of R and then use geometric I should be able get the value of S T R, so to have an assessment of n.

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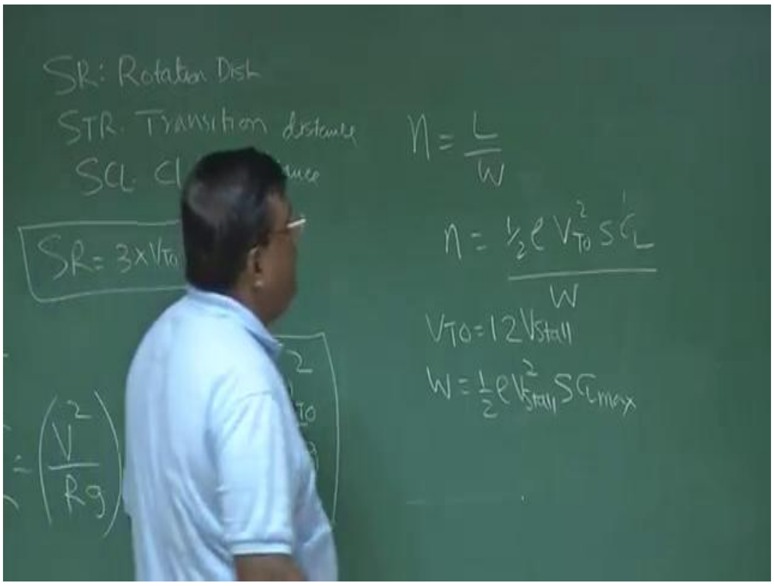

What is n? N is lift by w, why it is going for takeoff what we know a lift is half rho v take off square S C L and w I write here. What is V take off? V take off is 1.2 times v stall and what is w, w I can write as half rho v stall square s into C L max w is by definition of v stall is half rho v stall square S C L max equal to w V take off I know 1.2 v stall and this velocity are speed is V take off square and here I write C L and now try to find n as we have assume that during the take off the C L is pointed C L max.

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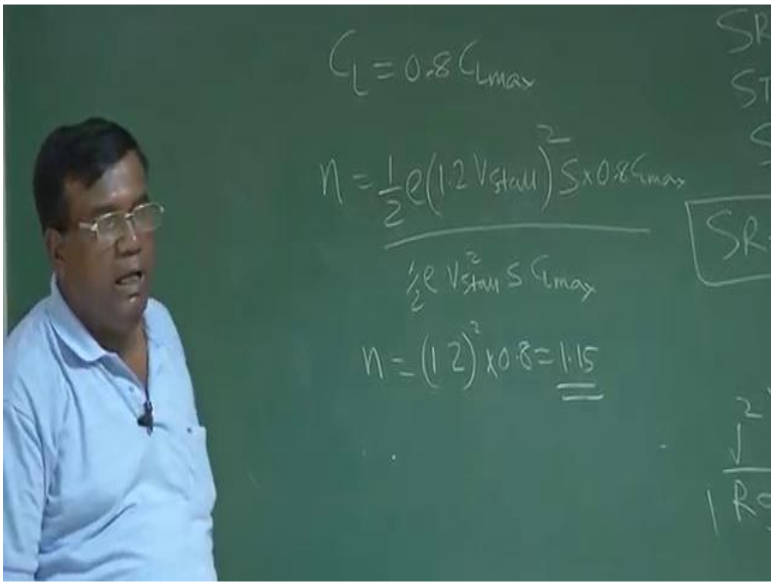

C L take off is 0.8 C L max then second that expression I get n equal to half rho v take off is 1.2 into v stall square S C L is 0.8 C L max and divided by w, w I know half rho v stall square S the C L max. Because, that is how v stall is defined if I seen of this ratio I will get n equal to 1.2 square into 0.8 which is roughly equal to 1.15 just 15 percent more than situation n equal to 1 to what is the message, message is now you know what is n we know what is v take off find out R. So, what will be R?

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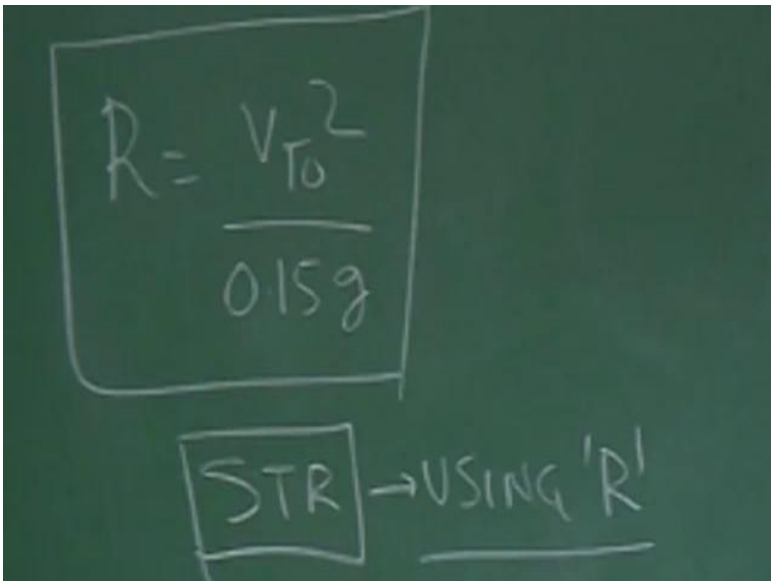

Now, R will be v take off square by 0.15 g, so please see that by doing is approximation we know the value of R. Now, I have to calculate S T R this is important using this expression of R to get the value of S T R, we will now do a little explanation use the help of geometric.

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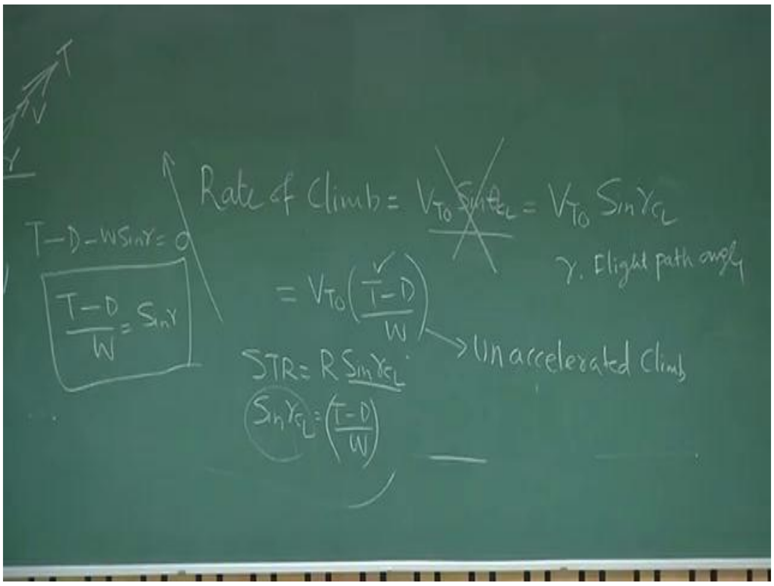

You know by now you are all expect rate of climb it was define as V take off sin theta C

L actually I am using as theta C L to be more precise I will prefer to write it has V take off into sin gamma C L. Because, for us gamma is the flight path angle, if you seen text book they sometime use theta C L I am not writing theta C L I am cutting it here I will write V take off into sin gamma C L to give a stress that for flight path angle, we are using notation gamma not theta, theta is for the pitch orientation I teach at the airplane respect to the ground, gamma is angle between the velocity vector on the origin.

So, this will be equal to V take off by now sin gamma serially T minus D by w for a un accelerated climb, this is for do you remember that do you want me to repeat it climb. How it came? Let us revisit and so that you do not flip further files. Remember this is airplane which is going with the velocity v, this is w, this is the gamma flight path angle not theta, gamma flight path angle. So, what it can write T minus D is here D minus w sin gamma is equal to 0 for an un accelerated climb from here I write T minus D by w is equal to sin gamma.

So, for sin gamma climb are will to this expression our aims to find out S T R and S T R is nothing but, R sin gamma C L and from this expression you can find out sin gamma C L is equal to T minus D by w. So, I put sin gamma C L value computed familiar this expression R I have already found out. So, I can find out what is S T R.

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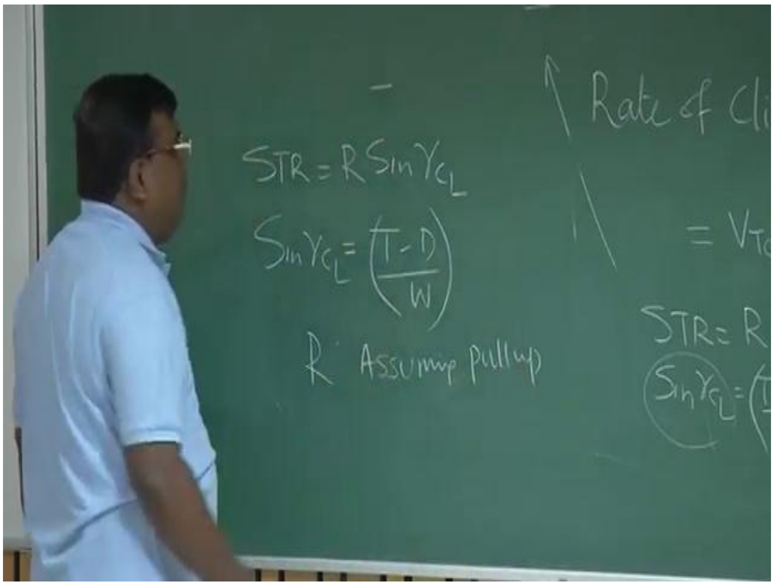

If I repeat again, the last part we will understand that S T R is approximated as R sin gamma C L and sin gamma C L I have calculated using T minus D by w, R we have calculated assuming pull up already I shown that once I know this number I have know this is.

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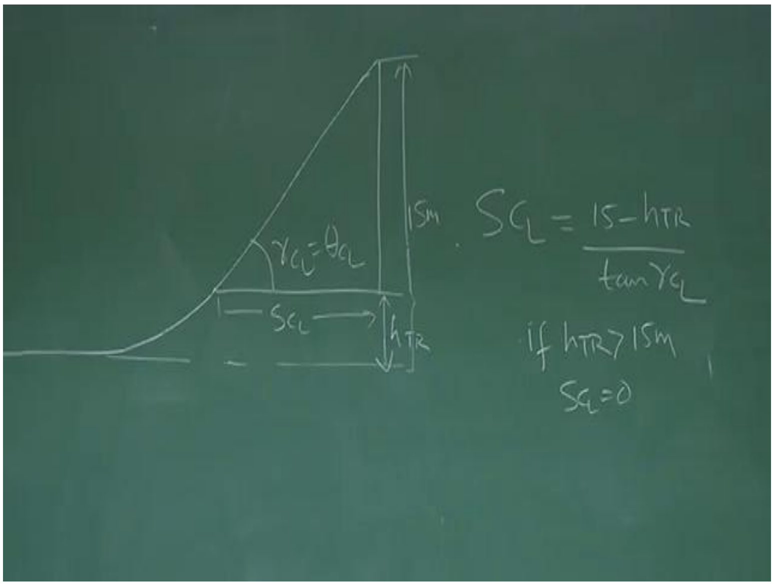

Finally, we are left with which is gamma C L many book will I theta, theta C L and find out this distance which is S C L and you know there was the pull up, where the role and like this and let say this distance is from the ground, let the distance the h T R. So, I get easily find out S C L by using this relation S C L equal to 15 minus h T R by tan gamma C L if h T R greater than 15 meter then S C L equal to 0 this is by simply by this geometric.

Because, you know this is the total height is 15 meter, we are simple as that I hope you should be able to do this yourself. So, once you calculate all this segments you add it up we will get the take off distance. For the question comes will you allocate that much of distance? Answer is; obviously, known because mid way they aero plane can failed, the engine may stopped so you have to give margin for that. So, generally you keep 100 percent margin, if take off length is 500 meter you will keep around 1 kilo meter runway. Please keep this in mind, if land is available, if land is available then you would more stringing condition.

Thank you very much.