

NOC: Introduction to Airplane Performance
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Lecture - 43
Control: Elevator

Very good morning dear friends, I am sure your eyes must be tired, seeing so many expressions, very dirty looking expressions generally. But we have practice of feedback from the student, we called a reaction survey in IIT, Kanpur. And for many times, when I got the reactions from the student, there was one common observation ((Refer Time: 00:35)) board work is very dirty, immediately in next semester I will try to improve it. But, suddenly again the board work will become dirty.

So, few of my students again ask sir, again I said I am statically stable as far as board work is concerned. So, that is why we are trying to see, how I can fly a statically stable airplane. Why I am saying statically stable airplane? Because, I know that airplane, which is statically stable, it will not allow or oppose any change from the equilibrium. We are trying to understand, how can you fly a statically stable airplane; what is important to observe, when I am talking about statically stable airplane; that statically stable airplane means, it has an inherent initial tendency to oppose any change.

That means, if I am flying a 2 degrees, if I want to fly a 3 degrees, angle of attack C_L corresponding to 3 degree angle of attack, it will oppose. So, I have to hold the control and take the airplane to 3 degree angle of attack and balance my forces and moments. About that equilibrium, again it is statically stable, so it will try to stay in 3 degrees, even you want to change it to 4 degrees angle of attack, again it will oppose.

So, you have to put a control through the elevator, give a moment, come on, hold on, balance all the forces of moment, again that becomes a new equilibrium, again about that equilibrium, it is statically stable. That is how we ensure that, I can fly, can design a control, which should know very clearly it statically stable, it has inherent tendency to come back to the original equilibrium. That is the preamble of our discussion today and to understand this in a manner through modelling, we wrote all these equations, lift and the wing, tail, try to find out moment.

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CONTROL: Elevator $C_{m_{0W}} = C_{m_{0FW}} + C_{L_{\alpha}} \left(\frac{x}{\bar{c}} \right)$

$$C_{m_0} = C_{m_{0W}} + C_{m_{0fs}} + \eta V_H C_{L_{\alpha t}} (\epsilon_0 + i_w - i_t)$$

$$C_{m_{\alpha}} = C_{L_{\alpha W}} \left(\frac{x_{cg_{aircraft}}}{\bar{c}} - \frac{x_{cg_{wing}}}{\bar{c}} \right) + C_{m_{\alpha t}} - \eta V_H C_{L_{\alpha t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$\frac{x_{np}}{\bar{c}} = \frac{x_{cg_{wing}}}{\bar{c}} - \frac{C_{m_{\alpha t}}}{C_{L_{\alpha W}}} + \eta V_H \frac{C_{L_{\alpha t}}}{C_{L_{\alpha W}}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \frac{\sum_{i=1}^n h_i}{\sum_{i=1}^n C_{L_{\alpha i}}}$$

And we got neat expressions, one was C_{m_0} as C_{m_0} wing plus C_{m_0} fuselage plus $\eta V_H C_{L_{\alpha}}$ tail into $\epsilon_0 + i_w - i_t$. And we got $C_{m_{\alpha}}$ as $C_{L_{\alpha}}$ wing into $x_{cg_{aircraft}} - x_{cg_{wing}}$ by \bar{c} plus $C_{m_{\alpha}}$ fuselage minus $\eta V_H C_{L_{\alpha}}$ tail into $1 - \frac{\partial \epsilon}{\partial \alpha}$. And also we find out neutral point location as $x_{cg_{wing}} - \frac{C_{m_{\alpha t}}}{C_{L_{\alpha W}}} + \eta V_H \frac{C_{L_{\alpha t}}}{C_{L_{\alpha W}}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) \frac{\sum_{i=1}^n h_i}{\sum_{i=1}^n C_{L_{\alpha i}}}$. This looks much neater, whatever effort we made yesterday, ((Refer Time: 03:58)) was this three expression, let us visit them as a designer.

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What is $C_{m, \alpha}$? We know that, if I want to fly at some C_L or α , I need a particular value of C_m . We also agreed, if I have designed the airplane efficiently, I should automatically get this C_m without giving any elevator deflection. By giving elevator deflection, suppose this is a tail, by giving elevator deflection, I can generate moment C_m even at α equal to 0. So, this graph can be shifted here, here, depending upon what deflection we have given.

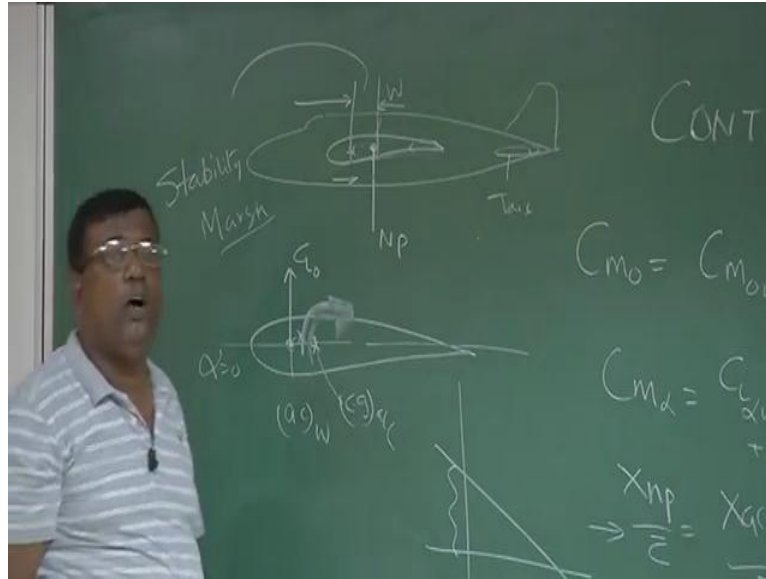
But, for efficient design, I will ensure that, I should be able to trim my aircraft by this C_L or α without giving elevator deflection. That means, this C_m contribution should come from $C_{m, \alpha}$, because of wings and $C_{m, \alpha}$, because of fuselage and then C_m , because of tail. And we have not done any explicit expression for C_m fuselage, which is generally very low and those, who are interested, they can read books, they are very, very low. So, we are more focused on $C_{m, \alpha}$ wing and C_m tail.

And in second was $C_{m, \alpha}$, the message was what, that C_L wing; that is wing that there, they should be placed, the history of the wing should be placed in a manner. The tail should be placed in a manner that overall $C_{m, \alpha}$ should be less than 0, thus slope should be negative, total contribution. And then, you try to find out, what is the limit at which I can lay out my C_g or I can take C_g backward.

At that point, the aircraft become neutrally stable or at that point $C_{m, \alpha}$ will become 0. The message is you could take C_g beyond that, then it will become statically unstable, these are the three interpretations. If you see in $C_{m, \alpha}$ wing, you have seen this is $C_{m, \alpha}$ wing plus $C_{m, \alpha}$ into X bar; that is the difference between, I just say if we write X by C bar and we try to understand that one first from designer perspective.

What a designer should do to ensure that, $C_{m, \alpha}$, because of the wing is finally is positive. We not be able to compensate for whatever C_m is required, but it will ensure that, it is not negative.

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Because, we are using cambered aerofoil, for a cambered aerofoil $C_{m_{\alpha}}$ wing is less than 0, so I have to do something with the second term. What is a second term? Second term is, if this is the $a.c$ of the wing, I will ensure that $C.G$ of the aircraft is behind the $a.c$ of the wing. So, that α equal to 0, I know there is a lift force or C_L naught and this distance is X . So, this will give you nose up moment or it will give C_m not positive and you should be sufficient enough to counter this negative value.

At least this should becomes 0, very good, if you can get some positive value, because finally, you have to get so much of $C_{m_{\alpha}}$ naught in combination with tail and fuselage. So, message is very simple if the cambered airfoil, try to put the $a.c$ of the wing ahead of $C.G$, although this will give a destabilizing contribution that you could from here the moment I put $C.G$ behind $a.c$ of the wing, this term is positive, this is positive. So, whole wing contribution to $C_{m_{\alpha}}$ is positive, which is destabilizing.

But, you should do not worry, what is the problem our friend is here. So, what is the problem, the stability part will be taken care by our friend tail, horizontal tail. So, this is always negative. So, I can go on changing the value of V_H , which is tail volume ratio and try to compensate for stability loss, static stability loss, because of wing $a.c$ being a head of $C.G$.

So, messages is clear, do not worry about wing in terms of stability, static stability, give some considerations, so that it can help us in giving $C_{m_{\alpha}}$ naught. Because, anyway

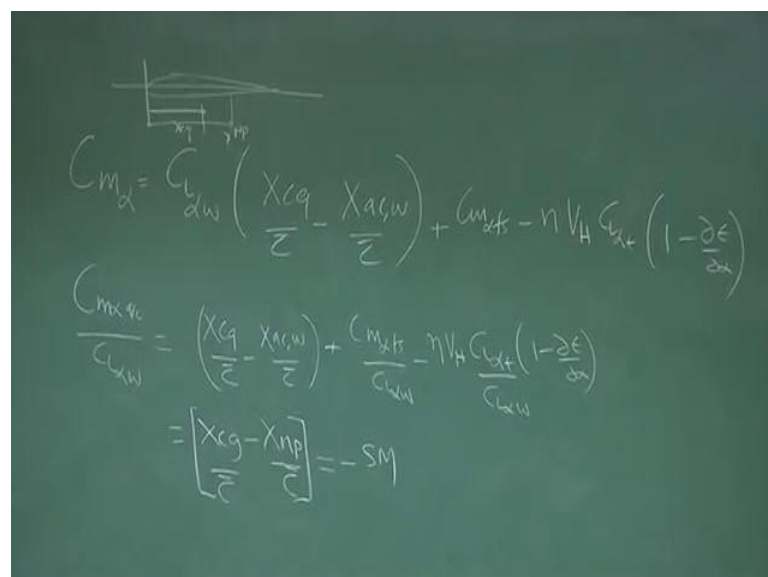
cambered airfoil, if we put the a c of the wings behind the C g, C m naught from the wing will be further negative. So, difficulty in trimming, you put a c of the wing ahead of C g, it will have a destabilizing component, it does not matter. Tail will take care will appropriately design the volume ratio, which is nothing but, tail area into the tail momentum term divided by wing area and mean aerodynamic chord of the wing.

Typically, this value I have to can start 0.5, 0.6 good enough and this man tells you what, as you are designing the airplane; that is a configuration. We have selected some wing, some tail, it is a tail, the wing and depending upon it is location of a C, Location of tail, because tail volume ratio contains l t. All this things you can get a fixed number for neutral point. So, let us say neutral point is somewhere here.

The message is, when you layout in the C g, you will not move the C g beyond this line, beyond this point and this gap between the C g and the neutral point, we call it stability margin. A stability margin and it could be between 5 to 15 percent of the chord, this is, that is an initial estimate. So, once I understand this, now I ask a question, if I want to change the equilibrium of the airplane from one flight condition to another flight condition.

How do I deflect my elevator; that is, what we say, elevator control or will be talking about elevator control power or final aim to find out, how much elevator should I deflect to trim a aircraft at a particular C L.

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The image shows a chalkboard with a diagram of an aircraft and several equations. The diagram at the top shows the center of gravity (CG) and the neutral point (NP) relative to the mean aerodynamic chord (MAC). The equations are as follows:

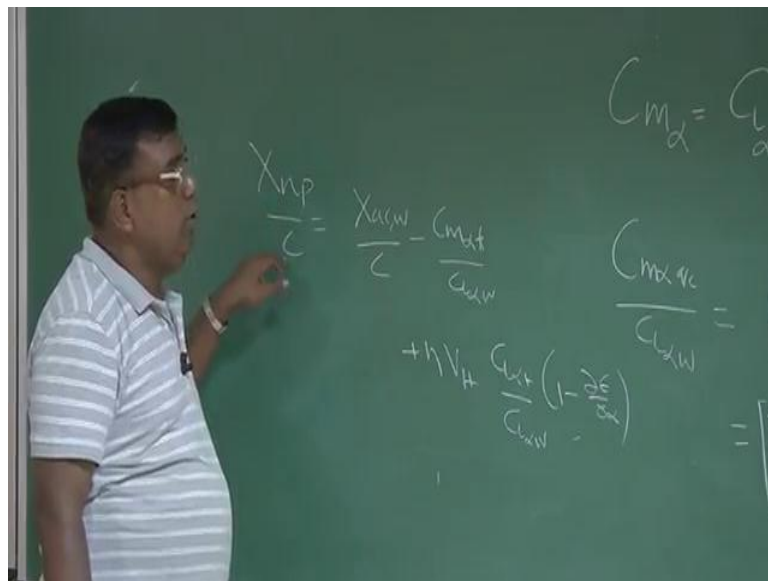
$$C_{m_{\alpha}} = C_{L_{\alpha}} \left(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACW}}{\bar{c}} \right) + C_{m_{\alpha T}} - \eta V_H \frac{C_{L_{\alpha T}}}{C_{L_{\alpha}}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$\frac{C_{m_{\alpha}}}{C_{L_{\alpha}}} = \left(\frac{X_{CG}}{\bar{c}} - \frac{X_{ACW}}{\bar{c}} \right) + \frac{C_{m_{\alpha T}}}{C_{L_{\alpha}}} - \eta V_H \frac{C_{L_{\alpha T}}}{C_{L_{\alpha}}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$= \left[\frac{X_{CG}}{\bar{c}} - \frac{X_{NP}}{\bar{c}} \right] = -SM$$

So, to understand that, let us do little bit of reappreciation of the expressions and we try to get an additional designers perspective. Note this $C_{m\alpha}$ wing is X_{cg} by $C_{L\alpha}$ wing minus X_{ac} wing by $C_{L\alpha}$ wing plus $C_{m\alpha}$ fuselage minus $\eta V_H C_{L\alpha}$ tail and to $1 - \epsilon$ by $d\alpha$, I divide by left hand side and right hand side by $C_{L\alpha}$ wing. So, what will have, I have $C_{m\alpha}$ of the whole aircraft by $C_{L\alpha}$ wing is equal to X_{cg} by $C_{L\alpha}$ wing minus X_{ac} wing $C_{L\alpha}$ wing plus $C_{m\alpha}$ fuselage by $C_{L\alpha}$ wing minus $\eta V_H C_{L\alpha}$ tail into $1 - \epsilon$ by $d\alpha$ divided by $C_{L\alpha}$ wing.

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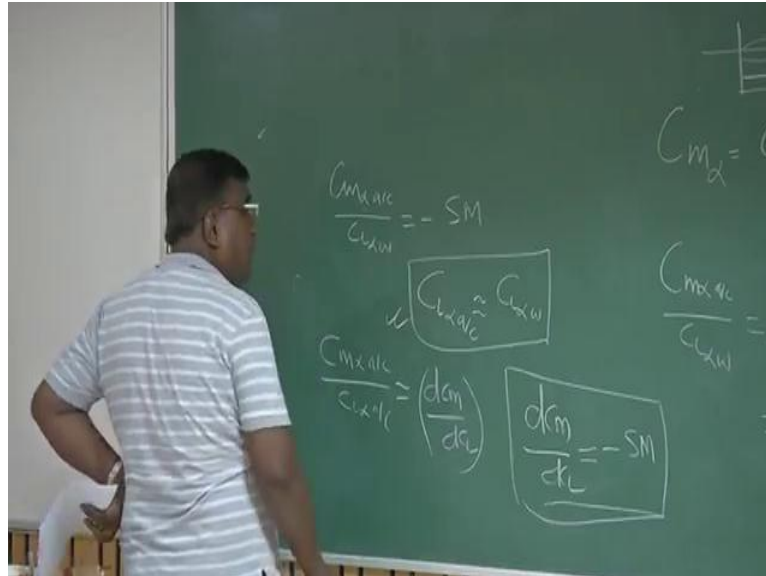


Now, what was the expression for neutral point, see X_{np} by C , if C it was X_{ac} wing by C minus $C_{m\alpha}$ fuselage by $C_{L\alpha}$ wing plus $\eta V_H C_{L\alpha}$ tail by $C_{L\alpha}$ wing into $1 - \epsilon$ by $d\alpha$. This are expression already you have, now carefully see the right hand side, if I just manipulate this I can right, this equal to X_{cg} minus X_{np} . You could see, if I take minus sign common here, then X_{ac} by wing minus $C_{L\alpha}$ id large by $C_{L\alpha}$ wing plus ηV_H wing alpha by tail by $C_{L\alpha}$ wing, $1 - \epsilon$ by $d\alpha$.

And that is nothing but, X_{np} by C Location of neutral point. So, this ratio is nothing but X_{cg} minus X_{np} ; that is, if be the aircraft, if I am measuring the X_{cg} , this is X_{cg} and this is a neutral points X_{np} . Then, this difference as I told you a static margin, so this is

equal to minus of static margin, because static margin is define that $X_n p$ minus $X_c g$, so minus of static margin.

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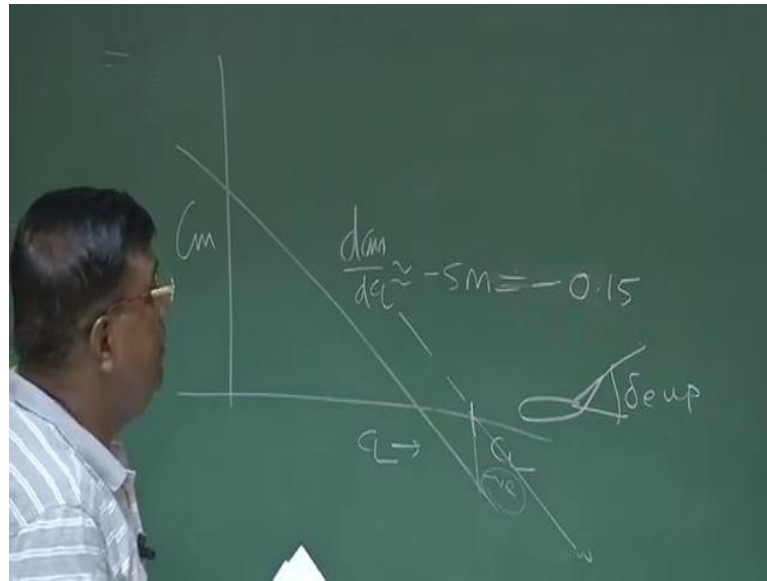


So, what we have got in a simpler term, I got C_m alpha of the aircraft by C_L alpha wing is minus static margin. Now, if I do an approximation, this is what I designer a place important role, because we are trying to get the initial estimates. You know, what is the primary role of the wing, given that later, so the obvious the C_L alpha of the wing based on the reference area is the predominant C alpha of the whole airplane. That is, I can assume with a 10 percent in accuracy that, C_L of the aircraft is nothing but, C_L of the wing.

That is C_L alpha of the aircraft is proximately equal to C_L alpha of the wing, this is an approximation. This is I am doing because I understand the C_L alpha of the tail based on the wing reference area will very small compared to C_L aircraft of the wing. So, this is what the whole tricked to get a designers field, then I can write C_m alpha of the aircraft by C_L alpha of the aircraft and this is nothing but, $d C_m$ by $d C_L$ of the aircraft.

Hence, I can write with this approximation $d C_m$ by $d C_L$ of the aircraft is approximately equals to minus static margin, this is a wonderful for designer to design an airplane. That is why I am stressing at this point. You see, how beautifully I use this for designing an airplane.

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So, the problem what you are looking for is, suppose this is your C_m versus C_L graph on the slope is $\frac{dC_m}{dC_L}$ and that is, we have agreed that $\frac{dC_m}{dC_L}$ approximately minus static margin. And if it is 15 percent so minus 0.15 $C_{\bar{L}}$ or minus 0.15 by it is 15 percent static margin, which is as per distance is concerned 15 percent of chord, mean aerodynamic chord. This is the separation neutral point and the C_{g} , C_{g} is the head of neutral point and it is statically stable minus 0.15.

Now, what we are trying to do is, I want to know, if I am flying at this C_L or if I want to fly at this C_L , another C_L , I know by virtue of it being statically stable, it will generate a negative moment, nose down moment. So, I have to counter that and make sure that, this C_L versus C_L graph is passes like this from this equilibrium. So, question is how much moment, positive moment I should give to compensate this negative moment; that will be given by C_L versus C_L graph.

How do I generate this? I generate this positive moment now to counter this negative moment by giving what an elevator up deflection. So, the elevator up by the elevator up deflection should to generate enough moment, pitching moment, positive to counter this negative moment. So, that at this point, again the moment is 0. So, there at trim or there are equilibrium. So, if this understanding is there, then we will formulate.

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$$C_L = f(\alpha)$$
$$C_L = C_{L_0} + C_{L_\alpha} \alpha$$
$$C_L = f(\alpha, \delta e)$$
$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta e}} \delta e$$

$\alpha = 0$
 $C_L = C_{L_0}$

Let us see so far we are talking about C_L is function of α for a given Reynolds number Mach number etcetera. So, we wrote C_L as C_{L_0} plus C_{L_α} into α . Now, if C_L is also you could see, not only function of α , it is also function of δe , do you agree with me or not.

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$$C_m$$
$$\frac{dC_m}{dC_L} \approx -5m$$
$$C_L = C_{L_0} + C_{L_\alpha} \alpha$$
$$C_L = f(\alpha, \delta e)$$

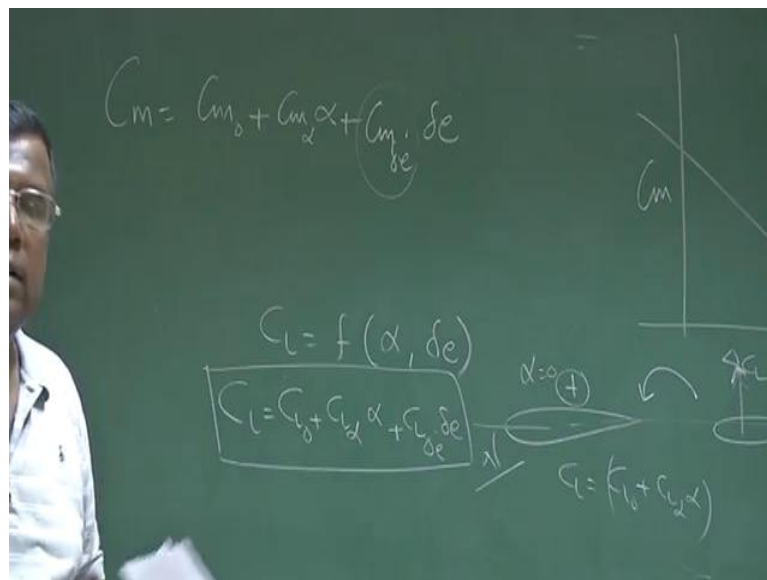
Suppose, this is the wing, this is the tail right and total C_L some of this α , then C_L is C_{L_0} plus C_{L_α} into α for the whole aircraft, C_{L_α} is for the whole aircraft. But, if I now give deflection of the elevator like this, which is positive

deflection, then when the alpha is equal to 0 this will produce a lift. So, now, my C L will also become function of delta e.

So, in a partial derivatives sense, we are assuming it to be linear, I say the C L, I can right it as C L naught, when both alpha delta, you are 0. Then, find out what is C L alpha; that is C L alpha, if the per unit change in C L, because of the alpha, holding elevator and other thing constant. Then, I say at C L delta e into delta e, C L delta e means is, how much change in C L, because of deflection of delta e, holding other the other is constant, no change in alpha.

So, this will give me a description of C L and please understand all this value C L naught, C L alpha C L delta, they can be estimated depending upon geometry and flight condition. So, you assume that, these are available for a given geometry and given flight condition. So, if this is true the moment I have deflecting by delta, if not only given some C L or delta C L, it also gives the moment. So, now, I must modify the moment equation also.

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So, far how I was writing moment, moment I was writing, moment coefficient I was writing as C m naught plus C m alpha into alpha. Now, what does happened because of delta e deflection of elevator, there is an increase in C L or there is a change in C L, change in the correct word, it may increase, it may decrease. The overall C L of the

airplane depending which way it deflected not only it will change C_L by this force we create moment about centre of gravity.

So, it will also give pitching movement depending upon, if elevator is up, then it will give a pitch up moment, elevator is down, it will give pitch down movement. In this case, it will give pitch down moment or nose down moment, if it was up like this, it will give pitch up or nose up moment. So, I need to add $C_m \delta e$ into δe , where $C_m \delta e$ where I say the additional change in the pitching moment coefficient, because of δe or in terms of the dimensional quantity, I say additional pitching moment generated, because of elevator deflection, all about centre of gravity

Thank you.