NOC: Introduction to Airplane Performance
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Lecture – 08
Standard Atmosphere: Description and Modeling

See, you were trying to understand how the lift is generated through a reaction between body and the medium. So, we have been talking about body in terms of C L, C D in terms of shape, in terms the area. But, since it is the interaction between body and medium, we must also know characterize the medium through which the airplane is flying.

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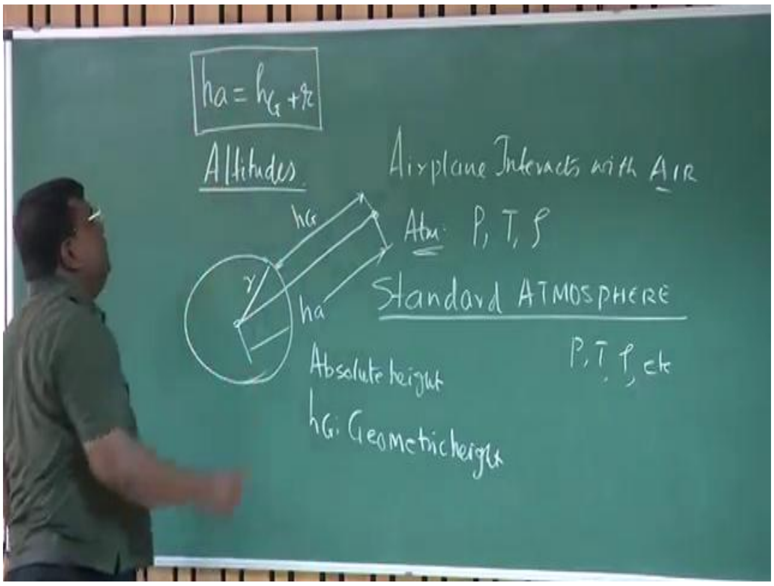

In this case, the airplane interacts with air and we call that the airplane is flying in atmosphere and that is where they are called atmospheric vehicle and like space vehicle, which travel most of his path in non atmosphere, in a space. But, we will be restricting ourselves to atmospheric vehicle and we are trying to know little bit about the atmosphere.

You know this atmosphere is characterized for the purpose of interaction between airplane viewing or airplane body with atmosphere through three thermo dynamic variables; that is pressure, temperature and density. After all, it is the force which comes because of the pressure. The temperature decides the density and also density by itself will decide, how much force actually the airplane will be experiencing for a given a speed of a given angle of attack.

So, we are trying to understand atmosphere through pressure, temperature and density and we also know that, we need additional information like, what are the wing conditions there or the wing is 20 knots or 40 knots, heavy wing, low wing. What sort of immunity is there or any other atmosphere, if attributes are there are not before you design and airplane for that flight corridor.

And why we need to define something called standard atmosphere, it is also very simple and relevant. When we design an airplane, we go for a wintering test, we go for fly test. Then, suppose we have decided two airplanes, we want to compare the performances. But, suppose I am evaluating the performance of an airplane in Delhi, atmosphere, suppose I am same airplane I am using to evaluate in performance in Srinagar.

So, I cannot expect they will generate, although will experience a same force for other conditions of speed, area being same. So, when I want to compare two airplanes performance, I know to compare them on a common platform or common atmospheric conditions and that is why it is important that we define standard atmosphere through pressure, temperature, density, etcetera.

So, that whatever design you have made, whatever airplane we have configured, if I want to compare them, I will compare them assuming all their airplane flying in standard atmosphere or what are their performance is, when I try to see them in the standard atmosphere. Let us again come back, I need to know little bit on the altitudes, when I use the term altitude, we always think about the height from the surface of earth. For all practical purpose; that is the height we refer to all the time.

We also know by now that there is a height which is relevant from the center of earth, for a space vehicle, etcetera, etcetera. We are more bothered about the height for the center of the earth, because we need to capture the gravitational force. So, we will define two types of height, one is h a, which we call absolute height and we will also define the height with respect to the earth surface h G and h G, we say geometric height. And if I call this r, small r as the radius of the earth, then it is very simple to see, h a is equal to h G plus radius of earth.

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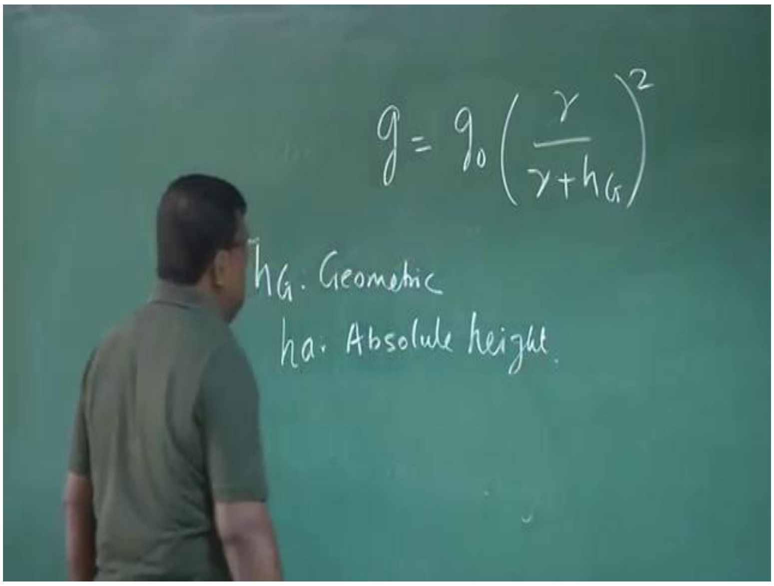

And we know that g at any session can be written as r by r plus h G squared. The message is, if I know radius of earth, if I know what geometric height I have, then if I know the local value of the g at the surface of the earth, which is g naught or technically we say C level g naught. Then, depending upon the h G, I can find out what is the value of g at a certain height, certain geometric height h G. So, that is not a problem.

So, we have define two altitudes, one is h G which is geometric, another is h a, which is absolute. Now, you see the problem, the atmospheric properties like pressure, temperature, densities, they are not only function of altitudes at what height you are or also function of, whether it is day, whether it is night, with seasonalities. And so many other factors may be sun activity, so it is a very dynamic in nature.

So, very difficult to define the standard atmosphere based on local measurements. So, what is done is, take measurements of different, different points, different, different places and then, make an average value, take an average value and internationally everybody will agree that yes, this is the sun atmosphere we will be the referring to when we are comparing two flight test or two performance test of two different airplane.

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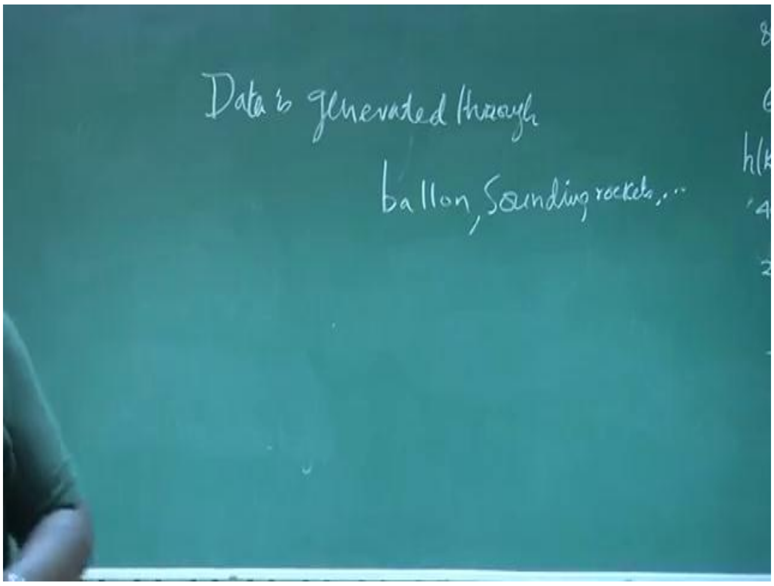

So, the process is like this, what is done, that this data is generated through balloon. You know, there are weather balloon they are launched and it goes on sensing thermodynamic variables, primarily temperature. Then, sounding rockets, etcetera.

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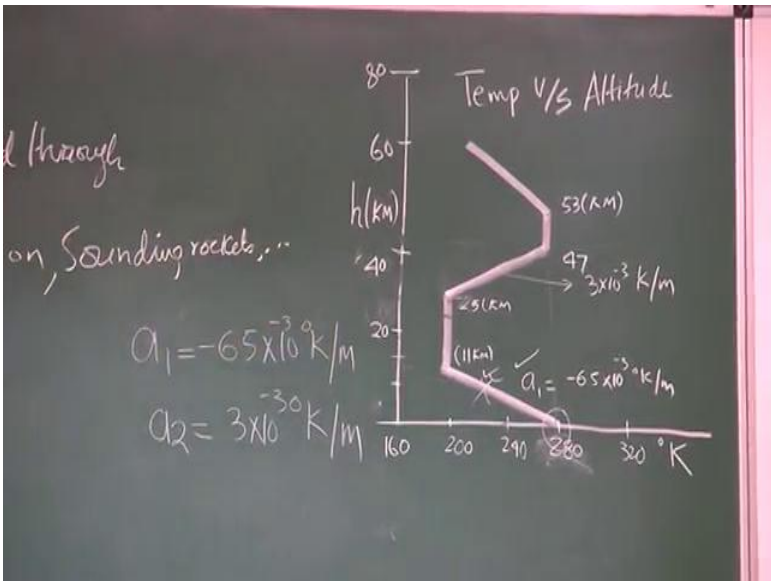

What do we see classically, the temperature versus altitude. Typically, if I take the average values of all parts, most of the parts, relevant parts and take the average, we will fine the temperature with altitude, it will vary like this. If I come here, see up to 11 kilometer, which is typically the trip of us, the most of us civilian airplane, jet engine airplane, they will fly at around at 11 kilo meter trip of us.

If I see here up to 11 kilometer, the variation is linear and the lace rate a 1 is approximately 6.5 into 10 to the power minus 3 degree Kelvin per meter. Similarly, the experimental observation shows that from 11 kilo meter to around 25 kilo meter, there is no change in temperature and we call it as isotherms. And again beyond 25 kilo meter to around 47 kilo meter, there is having the gradient.

But, unlike in this region, these are the positive slope and positive lace rate or we call it a 2 is plus 3 into 10 to the power minus 3 degree Kelvin per meter. So, this data or this trim is purely generated through experiments, through balloon, weather balloon, your sounding rockets at different places, different times and some average value has been agreed upon by international aviation society.

But, please come back to our original question once, once I want to characterize the atmosphere, I need to know it is thermodynamic variables, the pressure, temperature, densities, temperature profile is here.

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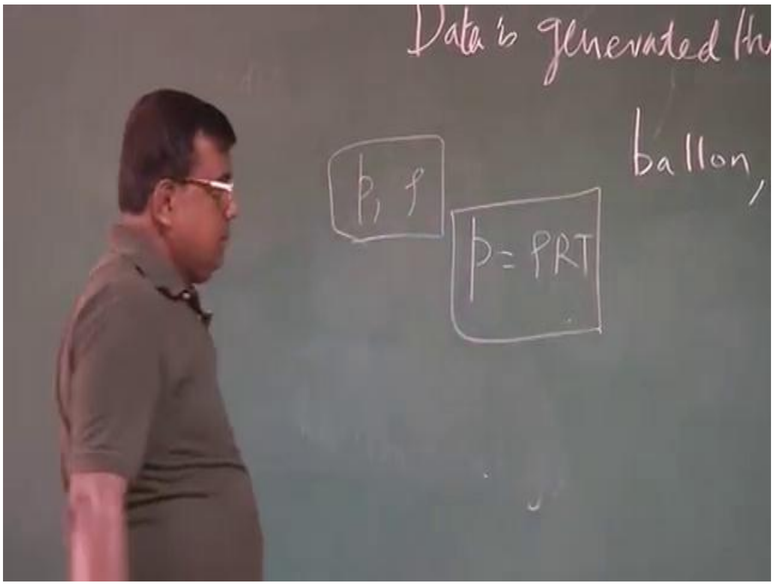

So, I need to know pressure and density that is extremely important, if I want to find out lift, where if I want to find out drag, these two thermodynamic variables will play important role. So, I need to create a standard atmospheric module by using this experimental data ((Refer Time: 10:57)) and how do I do that. That is done simply assuming that, the air follows perfect Gas law; that is P equal to rho R T.

That is we will be treating here following the perfect Gas laws and then, we will be try to find out, what will if I know temperature is here, pressure here, density here, then what is the pressure density at that point by using this in mind that, they are perfect gas.

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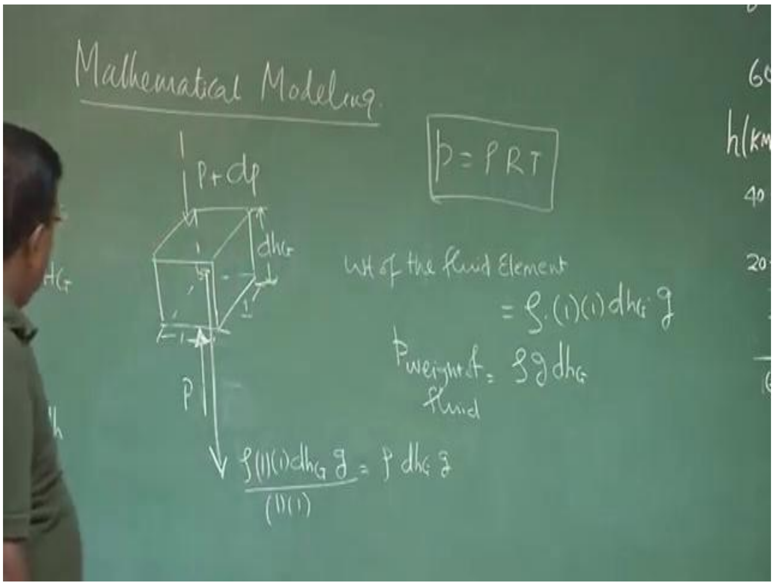

Now, we are trying to develop a mathematical model and we are assuming that, this is the element of air, which is cubical shape with length 1 unit, breadth 1 unit and the height d h G, why h G, because h G is relevant as well as atmospheric presence is concerned. So, if I try to do a static balance, we know there will be a pressure acting upward the bottom surface this is relative speed P.

At the top, it will be P plus d P and also note here, the weight of this fluid element. What will the weight of the fluid element? Weight of the fluid element will be mass into local value of g, mass is density into volumes. So, density is rho, volume is 1, length, breadth, d h G and into g, this is the weight of the fluid element. And this weight is acting on unit area, because this weight is acting on unit area 1 into 1. So, pressure because of this weight of fluids, in this case it is here that will be rho g d h G, because area is 1.

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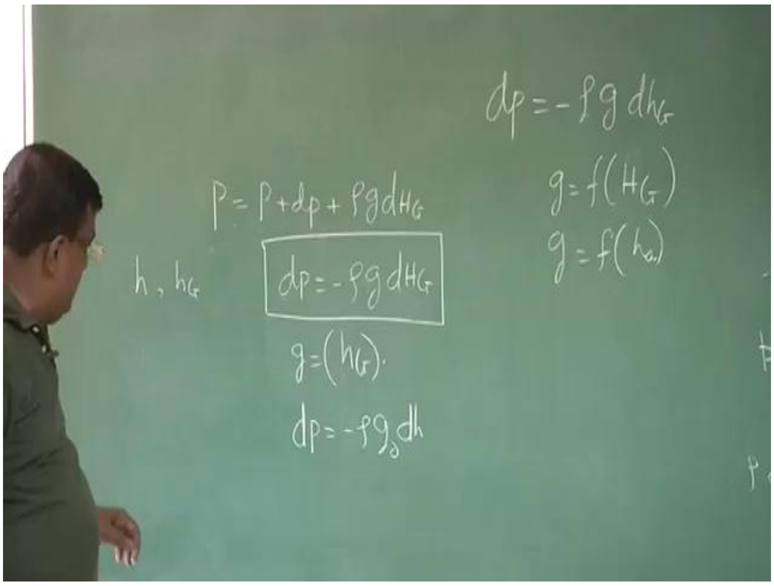

So, now if I simplify this I can write P is equal to P plus d P, P here, P plus d P acting down and pressure because of the way weight acting down. So, I can required them and find out that d P equal to minus rho g d delta h G. Now, see here, there is a catch as for as operational part is concerned. If I clearly see that, d P is minus rho g delta h G, G is function of h G. That is as I am going up and up; the h G that is the geometric height also increasing and g also will become function of h G.

So, if I want to integrate this to find pressure and an all, so I have to know explicitly, one thing that g is not going to remain constant. In fact to be more precise, the g should be also function of you know h, h a, which is the absolute height, because as for as g is concern, it is the references from the center of earth. But, since here d h G is there, it was g become function of h G, then integration become it is complex.

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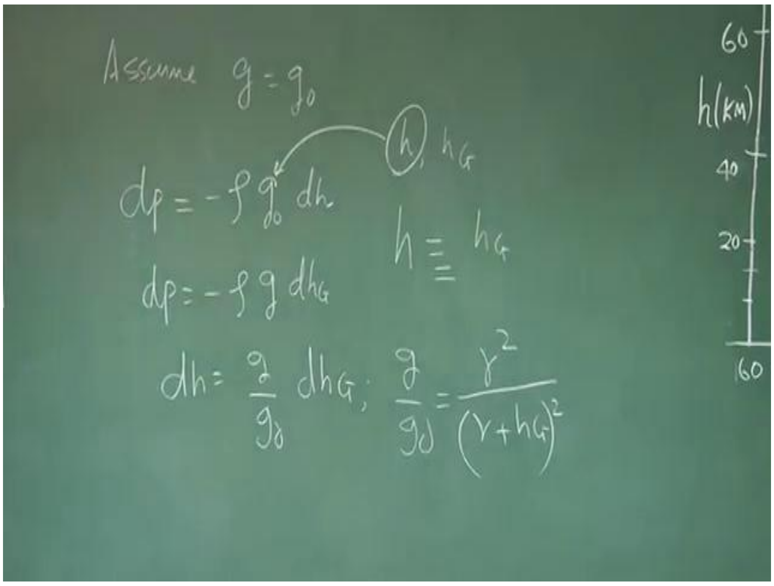

So, what is done for simplicity, assume g is equal to g naught; that is, we are assuming that value of g is, what the value of g at C level; that is constant. So, now, it could see for same differential pressure, I am writing minus rho g naught d h, for same differential pressure, I am writing minus rho g d h G. So, definitely from this relation I can see that, h and h G are not same and by geometry also, we know h and h G should not be same, h G because of consideration, because of the atmosphere.

Would this g potential by this h is defined vertices altitude, where I assume the value of g is not changing and you taking a g naught it is also vertices altitude and it call geopotential altitude. As long as I know, what is the relationship between h and h G, I have no problem, I can revived it back. So, do to that you could see for, if I use this two relationship, I find that d h equal to g by g naught and it d h G and since I know that g by g naught is nothing but, r squared by r plus h G square, where r the radius of earth.

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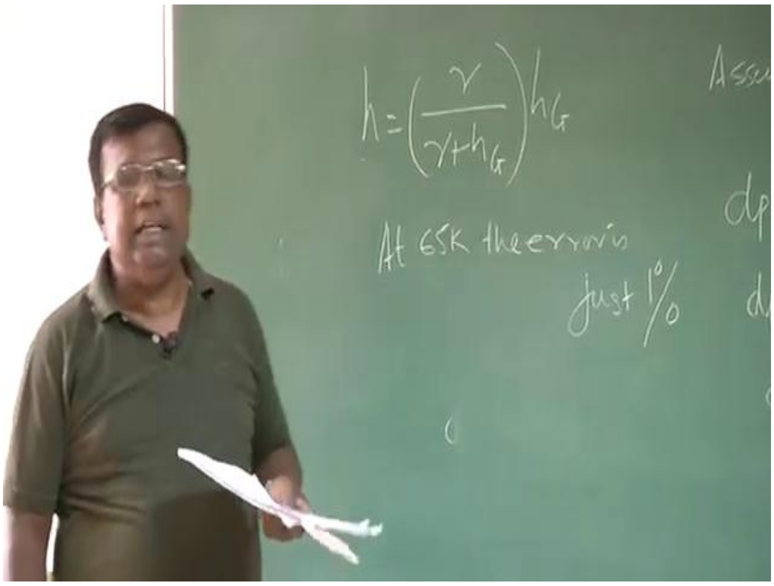

So, using this two relationship, I can write the h equal to r plus h G to h G. Now, what is the error I am taking when I assuming h G and h r same. Just to have an idea, if you plug in the value will define at 65 kilo meter, the error. The error is just 1 percent; we will understand assumption that h and h G are same that helps in making the analysis so simple. Still, you are getting the physics of the situation.

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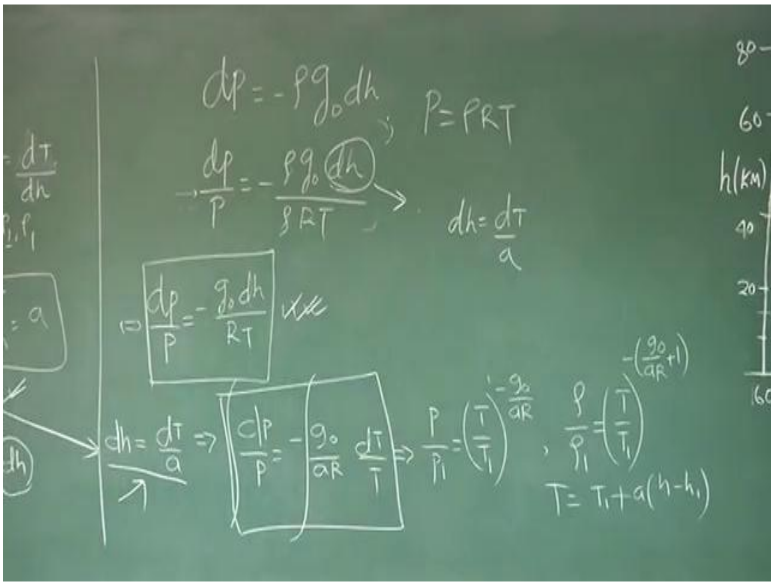

So, what is that important thing we have found out is, which will be reversely use to model atmosphere is minus rho g naught d h, not of back to this. What is our aim, the aim is the temperature profile is evaluable, I want to find out, what is the temperature which I though from here, what is the pressure, what are the density at this point or some other point here or some other point here.

And what is given to us, I know what is the condition here, from the static balance, what important relationship we got, we got d P equal to minus rho g naught d h and h, we have define a geopotential height, which is a vertices height. Assuming that, as soon as the gravities constant and we have also seen that h and h G hardly, they are differ. There are either 1 percent is away at 65 kilometers.

So, we carry forward with this relationship and also now, we use the perfect gas relationship P equal to rho R T. So, divide d P by t. So, I get rho 0 d h by rho R T. So, I get a very important relationship d P by P is equal to minus g 0 by R T in to d h and you see, this will be used for getting the values for pressure and temperature at different points.

Now, we are taking the first case, where this is the gradient region; that is the temperature is not constant. For example, here we are say is the isotherm; that is temperature is constant, but here temperature is changing. So, we will take how to find pressure and temperature and density for gradient region.

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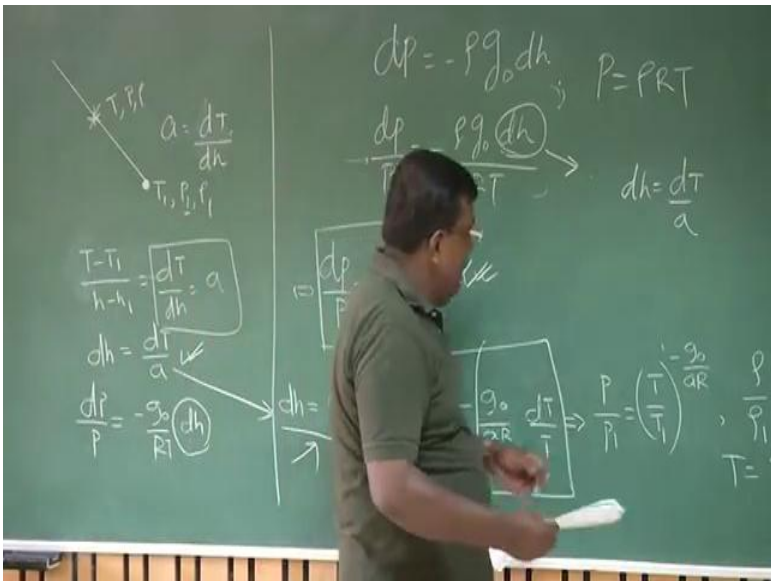

For example, is this condition is T 1, P 1, rho 1, if I know this and I know a as the lace rate d h by d t, it would see for this region this will lace rate a one. So, I can write T minus T 1 divided by h minus h 1, which is nothing but, d T by d h is equal to a is lace rate. So, d h I can write as d T divide by a, from this relationship, once a write that I can write d P by P, see here, d P by P is what, d P by is rho g 0 d h by rho R T.

So, now, I substitute this d h by d h equal to d T by a, by write that, then I get d P by P is equal to minus g 0 R T in to d h. So, d P by P is minus g 0 d h by R T, here I will be using this relationship that is d h is, from here I know by definition d h is d T by a, I will replace this d h by this value to get d P by P is equal to minus g 0 a R d T by T. Now, you see very neat expression, I can integrate both side and I know g 0 is constant.

So, I can write expression of P by P 1 is equal to T by T 1 to the minus g 0 by a R as will rho by rho 1 is equal to T by T 1 minus g 0 a R plus 1 and of course, we know T is T 1 plus a h minus h 1 full lace rate. So, simple I repeat again, I know this condition here, I use this relationship and I just substitute d h equal to d T by a, I get an expression like this. I integrate I get the pressure density and temperature for that point in the gradient region.

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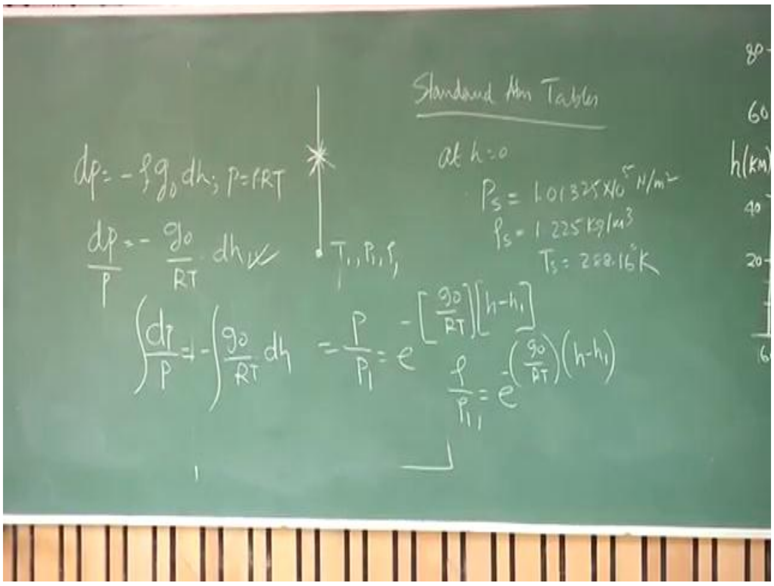

Now, if I want to find out for region whether temperature remain constant, which call isotherm; that is this region, let say this is the region I know here again T 1, P 1, rho 1 condition I know and find out this condition. What is the value of temperature, pressure and densities, we are working towards the method to find out, the thermodynamic variable, temperature, pressure and density.

The pressure and density is at different attitude, the given the initial condition T 1, P 1 rho 1. Again, use like this d P is equal to minus rho g 0 d h, this we have drive from the static balance, assuming fluid element in a static condition. And we have also drive is d p equal to minus g 0 R T into d h, because we have assume that, here is we have a perfect gas. So, p equal to rho R T and I divided d P by that. So, I got this relationship d P by P, P is rho R T that as be P equal to rho R T and I divide by d P by P 2 here, this is expression, this is done.

Now, if you see in the isotherm, I can simply integrate this, if I integrate both sides, because temperature is constant is isotherm it comes out. So, what do I get, I get P by P 1 is equal to e to the power minus g 0 by R T in to h minus h 1. Similarly, I can find rho by rho 1 is equal to e to the power minus g 0 by R T, h minus h 1. So, you could see in isotherm is a very straight forward.

So, what is the message, message is, if I want to create the atmospheric description is in terms of the thermodynamic variable, I need to identify whether we are looking in a region, which is governed by the gradient or by the isotherm. And then I have applied this relationship and in practices will find, there will be standard atmosphere stables available, they are generated using this as a relationship.

And of course, the standard value for a pressure, temperature and density as C level either fix number and we all take that as in the standard condition; that is at h equal to 0; that is P standard at is 1.01325 into 10 to the power 5 Newton per meter square. Then, density standard is 1.225 k g per meter cube and temperature standard is 288.16 degree K.

So, this initial condition the standard condition at is equal to 0 along with this relationship in gradient region and isotherms are sufficient enough to generate the whole standard atmosphere stable. Why you are generate all these things, you understand that should be clear, this standard atmosphere is require primarily to relate the performance two aircraft. So, that I can check there perform is in a common reference, for all this, we are doing to address a question, which aircraft has got better performance.