

**Aircraft Stability and Control**  
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**Lecture- 11**  
**Control: Elevator**

Yes we have come out of the classroom so, if you recall last class we are discussing about elevator control power okay? See this the horizontal tail all this surface and this is the elevator which can go down and go up and we were talking about what is the elevator control power in that we have started deriving the expression for  $C_L \Delta e$  and  $C_M \Delta e$  let us understand what is  $C_L \Delta e$  what does it mean remember if elevator positive angle.

Is deflecting downward like this as if go downward you could see this text as shape like this and this almost it is like as if it has changed the cambered of this symmetric aerofoil and because of that there is additional lift and that lift will give a moment about CG that is the CM contribution, because of this change in the cambered or because of change in the elevator deflection from here to some positive value.

There is an additional lift and that additional lift per unit elevator deflection is called  $C_L \Delta e$  and because of  $C_L \Delta e$  because of this force there will be momentum about CG in non-dimensional form we call it  $C_M \Delta e$  so this is very important elevator control derivative both  $C_L \Delta e$  and more specifically  $C_M \Delta e$  we call it strong derivative depending upon the value of  $C_M \Delta e$ , your  $\Delta e$  required to trim the airplane will vary drastically for a given static margin okay.

So this is what you should know appropriately you should create a image in your mind .how and how to interpret  $C_L \Delta e$  and  $C_M \Delta e$  okay fine. I am repeating the same explanation using sinus 912 motor glider concentrate here this is the typically T tail configuration and you could see this is the horizontal tail from here to that side and this part is the elevator if I put it down what happens is initially it was symmetric completely like this.

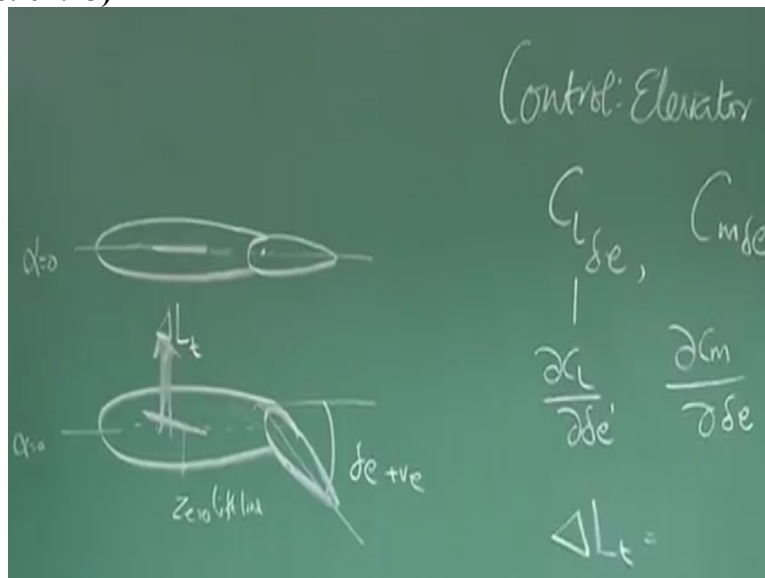
As I put it down it is like a it has zero cambered or we say the zero lift line has changed that means if its symmetric zero lift line along the chord so if there  $\alpha = 0$  there will not be any

lift but when I put elevator down I know even for  $\alpha = 0$  there will be vertical force in non-dimensional form we call it  $\Delta C_L$  and why that is happening will be happening because there is an elevator deflection and in one way to explain it that zero lift line now has changed okay. But two ask, when we talk about  $\Delta C_L$ .

What we are looking for is how much  $\Delta C_L$  is increased because of elevator deflection and that  $\Delta C_L$  how much contribute towards moment of center of gravity is what is  $\Delta C_m$  okay. So these two control derivatives are extremely important for longitudinal control and we will again go back to the class room and revisit this derivation and try to know how to calculate thing from the avenue ratio (03:46) okay? Thank you.

In the last class we have been writing so many equations so many expressions and the blackboard was full of expressions, which I personally never like the blackboard should be so dirty. So let's come back we will take out a neat part of it and try to understand it in a better way the most important thing what we talked in when we are discussing control elevator control.

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We write elevator control we talk about two derivatives  $\Delta C_L$  and  $\Delta C_m$  let us again revisit and try to physically understand what are the  $\Delta C_L$  and  $\Delta C_m$  as per the definition this means change in  $C_L$  with whole airplane because of elevator deflection keeping other thing constant and what is  $\Delta C_m$  this is changing the pitching moment of the whole airplane, per unique deflection of elevator keeping everything constant right.

Now let us see what is the mechanism, if this the horizontal tail which is symmetric and this is the elevator and one way to understand is in flight mechanism we find, we define something called zero lift line that is what is that line along which if the flow comes there will not be any lift obvious since this is symmetric, if at  $\alpha = 0$  flow comes so there will not be any lift produced right but now see what happens if I give a deflection is a positive elevator deflection.

Which we have agreed positive ways down so  $\Delta E$  positive, then one who has interpreting is as if the zero lift line has changed okay because now at  $\alpha = 0$  there will be a lift that will happens because of elevator deflection and that additional lift will be only because of the elevator deflection this one way to explain. We say zero lift line which is 0 lift line is now at an angle another way of explaining it has changed the cambered the deflection of elevator.

Has change the cambered of the aerofoil so you for cambered aero foil wing at  $\alpha = 0$  there will be lift there so there will be a CL so, we now understand 1 thing the moment I give a elevator deflection it will generate additional lift and now what is  $\Delta CL$  by  $\Delta E$  it means what is the net change in the CL of the airplane, because of  $\Delta E$  because of deflection of the elevator  $\Delta CL$  is CL of the whole airplane okay.

So, one thing we understand when I deflect the elevator then the CL of the whole airplane gets modified because there is a additional CL coming from the tail, because wing CL remains same so that additional CL coming because of  $\Delta E$  deflection let us find it out okay. Let us now see because of elevator deflection how much additional lift will come on the tail.

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Control: Elevator

$$C_{L_{\delta e}}, C_{m_{\delta e}}$$

$$\frac{\partial C_L}{\partial \delta e}, \frac{\partial C_m}{\partial \delta e}$$

$$\Delta C_L = \frac{\left(\frac{1}{2}\rho V^2\right)_t S_t C_{L_t}}{\left(\frac{1}{2}\rho V^2\right)_f S} = \eta \frac{S_t}{S} C_{L_t}$$

$$\Delta L_t = \left(\frac{1}{2}\rho V^2\right)_t S_t C_{L_t} \Rightarrow \Delta C_L = \eta \frac{S_t}{S} C_{L_t}$$

$$\Delta L \Rightarrow \Delta C_L$$

How much because of this deflection how much additional change will come on the tail in terms of lift I denote it as Delta LT. We know by now we are expert we will write half V square dynamic pressure at tail into S tail into CL tail okay.

This is the additional change in the lift because of elevator deflection okay. Now you want to find out Delta L of the whole airplane so, I have to convert this with the same reference as the reference area and dynamic pressure free stream we are using for defining for the whole airplane. So I will Delta L to Delta CL when I want to do I have to divide this term by so this implies Delta CL = half rho V square tail S tail CL tail divided by half rho V square free stream into S reference okay.

So, now this I can easily write as Neeta ST by S reference into CLT so what I am getting I got a expression Delta CL = Neeta ST by S into CLT let us again revisit here what is a Delta CL it is change in the CL of the whole airplane because of what because there is some change in the elevator deflection right. What is this S? S is the wing area which is the reference area for the airplane right. Once I know this focus this Delta CL is Neeta STS into CLT but do not forget this CLT is additional because of Delta E deflection right?

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$$\frac{\partial C_{L_t}}{\partial \alpha_t} \rightarrow C_{L_{\alpha_t}}, \quad \frac{\partial \alpha_t}{\partial \delta e} = \tau$$

$$\Delta C_{L_t} = \eta \frac{S_t}{S} \Delta \alpha_t$$

$$\Delta C_L = \eta \frac{S_t}{S} \frac{\partial C_{L_t}}{\partial \delta e} \Delta \delta e$$

$$= \eta \frac{S_t}{S} \frac{\partial C_{L_t}}{\partial \alpha_t} \frac{\partial \alpha_t}{\partial \delta e} \Delta \delta e$$

So, appropriately if I write I should write Delta CL change in the CL of the whole airplane is Neeta into ST by S and the CLT I should know that this is to be very precise, I will also use Delta CLT is here so these are both the things are same only I can put it Delta also here to stress a point that this is the change in the CL tail because of change in the elevator deflection okay? Additional thing let us cross so now what happens this I write Neeta ST by S and Delta CLT.

I write as DCLT by D Delta E into Delta E that is a Delta CL and to be more precise because it is additional so I again put Delta here is it clear? What I have done Neeta ST by S for Delta CLT because of change in elevator deflection which I am saying Delta Delta E then I am writing DCLT by D Delta E into Delta Delta E this gives what?

This tells me Neeta into ST by S this I write differently I write DCLT by D Alpha T into D Alpha tail by D Delta E into Delta Delta E what I have done I have just broken DCLT by D Delta E using DCLT by D Alpha T D Alpha T by D Delta E, what is the physical meaning of this DCLT by D Alpha T what is DCLT by D Alpha? Very simple this is CL Alpha tail right.

And now second term is D alpha T by D Delta E this is what? This is physically telling you when I am deflecting the elevator deflection how much effectively that tail angle of attack is changing isn't it? And you know that if I try to understand from zero lift line concept as I deflect it Delta E lift line also changes so, it all in the cambered changes I can see the effective angle of attack has seen whatever way you do right.

So, this is easily I can get this term let me note the notation as tow which is called flap effectiveness parameter which I have already explained and this can be easily computed by empirical chart where tow is there which I have explained you which is AC by ST where AC is the elevator area elevator area S tail is total area this is ST okay. This you can easily find out and this typical value is 0.4 0.5 I already told so now what happens?

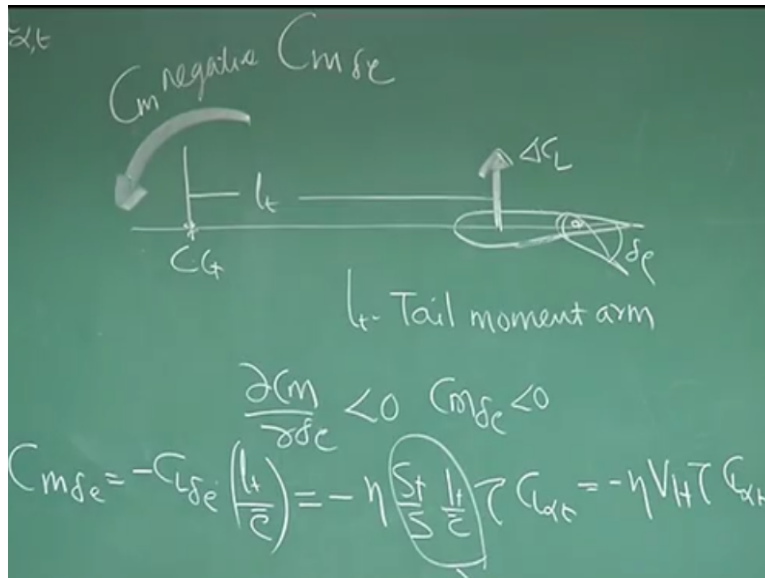
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The image shows a chalkboard with handwritten mathematical derivations. At the top left, there is a diagram of an airfoil with an arrow pointing up labeled  $\Delta C_L$  and  $C_{L_{\delta e}} > 0$ . To the right, the partial derivative  $\frac{\partial C_L}{\partial \alpha_e}$  is written with an arrow pointing right. Below these, the equation  $\Delta C_L = \eta \frac{S_t}{S} C_{L_{\alpha_t}} \tau \Delta \delta_e$  is written. At the bottom, a boxed equation shows the partial derivative  $\frac{\partial C_L}{\partial \delta_e} = \eta \frac{S_t}{S} \tau C_{L_{\alpha_t}} = C_{L_{\delta e}}$ .

From here I can write Delta CL aircraft is Neeta ST by S then CL Alpha tail into tow Delta Delta E so Delta CL Delta Delta E DCL by D Delta E which is = Neeta ST by S Tow CL Alpha tail and that is nothing but CL Delta E so neat and clean now okay so what is this CL? This CL is of the whole airplane okay.

This is very, very important and you should understand this and we know that we can easily calculate this and sign convention also you know if I put Delta E down if I put Delta E down which is positive there is additional Delta CL here so CL Delta E is sign wise is greater than 0 clear. Okay, once CL Delta is clear we will now go to CM Delta E. Now we will talk about CM Delta E.

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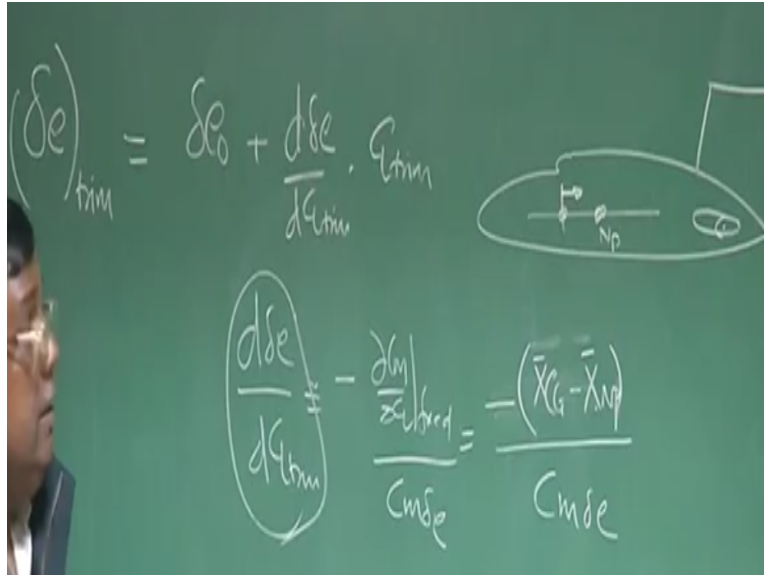
Let us see this is a line and this is your tail and this is the elevator symmetric tail, let us say CG somewhere here as I deflect it there is a Delta CL and that and this distance is LT tail moment arm LT is tail moment arm so, as there is a Delta E deflection which is positive it will give a Delta CL upward and about CG.

This will give the pitching moment downward nose down or a CM negative right? So that is why this CM is because of this Delta E so this  $\frac{\partial C_m}{\partial \delta_e} < 0$  or we write  $C_{m\delta_e}$  is less than 0 is one understanding. Now, what is the expression for  $C_{m\delta_e}$ , it is very simple  $C_{m\delta_e}$  will be nothing but  $C_L \delta_e$  into the moment arm.

Moment arm is LT but it has to be non-dimensionalized with chord LT by C and there is a - sign why - sign because we know this is sign convention given by us for positive Delta there will be a negative pitching moment, which I've already explain and this if I put the expression of  $C_L \delta_e$  I find this is  $\frac{S_t}{S} \frac{l_t}{c}$  then LT by C Tow into  $C_L \alpha_t$  and we have defined  $\frac{S_t}{S} \frac{l_t}{c}$  by S into LT by C or C bar this is called tail volume ratio so this is - Neeta into  $V_H T$  into tow into  $C_L$ .

Alpha T what is important here to see, that if I want to increase,  $C_{m\delta_e}$  I have option like for a given this aerofoil tail.

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I can increase the value of  $V_H$  because it is ST into LT by SC bar what I can do either I increase ST tail area or I increase LT or I can increase them in combination so, that is why tail volume ratio plays a important role not only in stability as you have seen but also for control.

Elevator control okay. So this is I thought I must revise this you should get lost into that expression, equation etcetera you must finally understand what you are going to do we have also done another thing in terms of control where we have derive the expression Delta E trim or required whatever you write is Delta E 0 + D Delta E by DCL trim into CL trim.

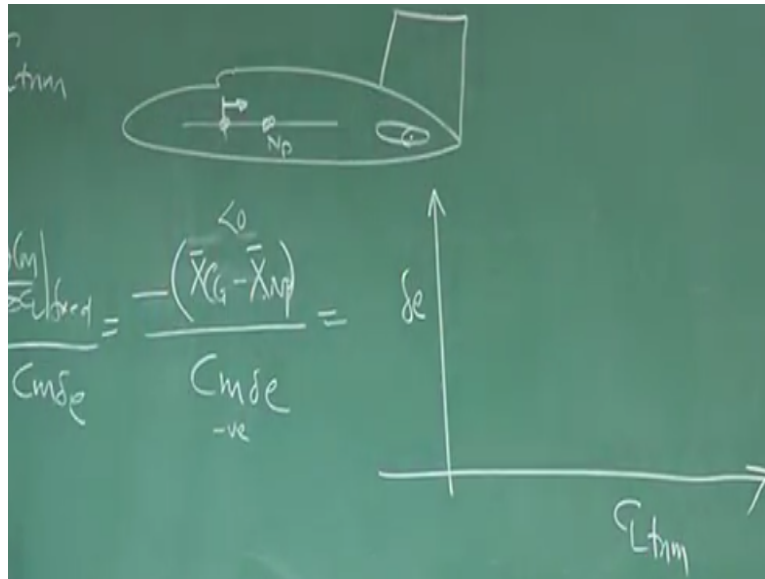
And we have also seen that in D Delta E by DCL Trim = - DCM by DCL stick fixed by CM Delta E in an approximate manner we have have neglected few terms and this is nothing but - XCG - XN or XN0 or XNP whatever notation you are using this the neutral point divided by CM Delta E right.

You can write it as, XNP please check what notation I am using this is basically neutral point by know you know neutral point is that XG location at which DCM by DCL will be 0 right. Now see here if I suppose this is the airplane okay? Suppose CG was here I let say neutral point is here and if I now start moving the CG backward let say, I ask the passenger go back side okay, If I do that or may be heavier cargo I have shifting backward what will happen?



This XCG - N0 whatever term is there this gap will reduce right. And that will give a particular value to D Delta E by DCL Trim and you could easily, see that if I go on reducing this separation right then the D Delta E by DCL trim also will go down that is if they coincide this value become 0.

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And you can see other way also if you see this DXCG minus this is the negative number right. We statically stable to negative positive here this is also negative this whole value is negative. Repeat this is negative this is negative and there is negative sign for this value is negative and what I am saying If I now plot if I plot Delta E versus CL Trim what will be the slope of this line slope is given by this and this we have seen that this sign will be negative because this is the neutral point is aft of CG because statically stable.

So this number is negative this number is negative and here it is negative sign sitting so slope will be negative we will see that and what about Delta E 0.

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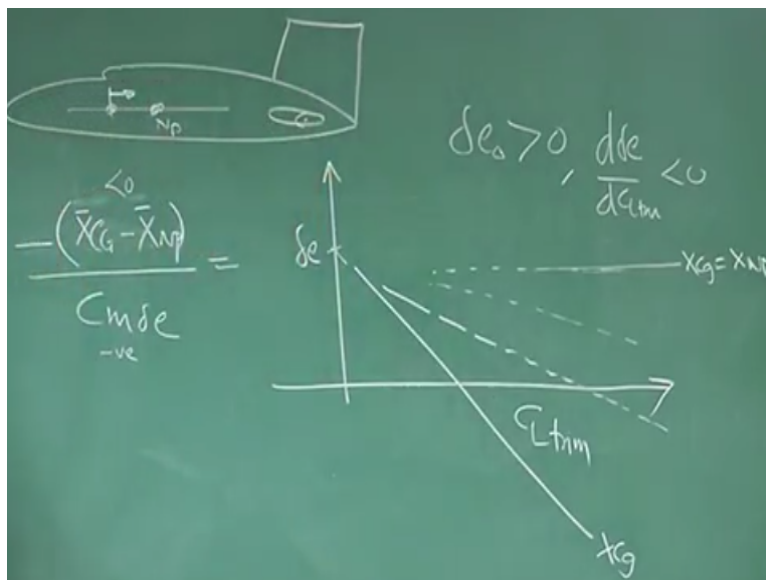
$$\frac{d\delta_e}{dC_{L_{trim}}} < 0$$

$$C_{m_0} = +ve$$

$$\delta e_0 = \frac{-C_{m_0}}{C_{m\delta_e}}$$

Let us see so I understood that D Delta E by DCL Trim sign wise this is negative what about CM 0 CM 0 is positive and Delta E 0 is CM 0 - CM 0 by CM Delta E so what happens this is positive this negative sign is here CM Delta is negative so Delta E 0 is greater than 0 correct.

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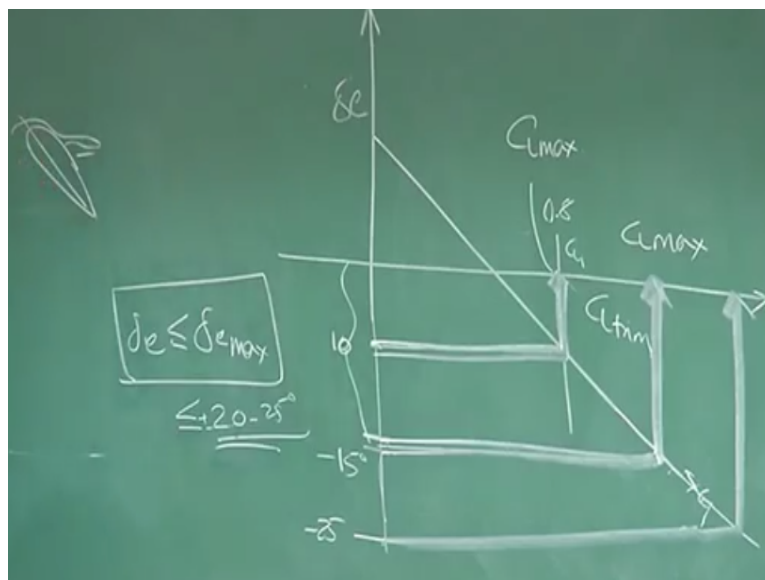


So, we have seen Delta E 0 greater than 0 Delta E 0 greater than 0 and D Delta E by DCL trim is less than 0. So, the graph will be something like this so, the slope is negative so Delta E 0 is positive Delta E 0 is somewhere here when it touch it here right. So let's say this is for a particular CG location right? Now you are moving the CG backward so what will happen as CG

goes backward as you have seen if CG coincides with neutral point this value becomes 0. This slope will become 0 as I am going towards neutral point.

This slope will go on reducing and adding some CG locations this line will be  $X_{CG} = X_{NP}$ . This is the neutral point where this line will be parallel to the X-axis here and that is the point which is the neutral point. So one message you are getting as I am moving this CG backward and backward. This slope is going to be flattened right. Now if that is the truth what is additional information we can get from it let us discuss on that let us again revisit.

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We have become next part now in Delta E and CL Trim right. Let's say this is the one graph for particular CG location who by know that this slope  $D \Delta E$  by DCL will depend upon static margin and therefore it will depend upon XCG location for a given configuration because  $N_0$  will not yet neutral point will not change because of CG right.

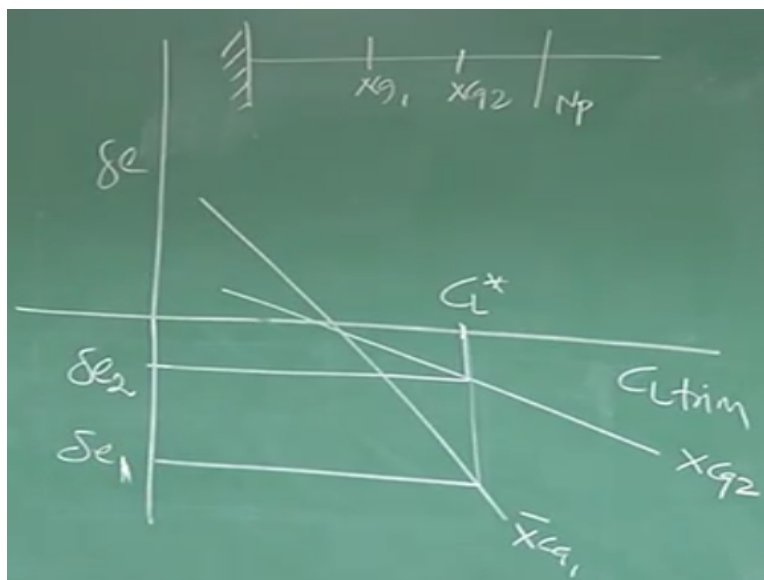
Let us see here suppose I ask a question can I trim the airplane at this CL 1 or not? Let's say this value is point 8 immediately what we will check for trimming the airplane at point 8 how much elevator is required this given by this value correct? If somebody ask can I trim it here and draw here then this is the Delta E value required how long and how far you can go along this axis who decides that. Suppose, you can really generate CL very high.

But please see that there is a limit of Delta E deflection which is governed by Delta E Max that is Delta E anytime should be less than Delta E max why? Because if you deflect the elevator too much beyond certain angle, the flow will separate here it will not effective at all and typically for normal case this will find between 20 + - 20 to 25 degrees.

So beyond, that we will not able to operate the elevator for conventional airplane in where it is 15 sometimes will find that is primarily because beyond that elevator we will lose its effectiveness. so what is the Delta E max for a elevator will also decide, how much CL practically you can trim that means I have to go to reverse direction suppose limit is - 15 degree then I go like this I come up I say only this is the CL max I can trim if Delta E Max is 10 degree let say 10 degree. This is then I say this is a CL Max I can trim because now my elevator is not effective beyond 10 degrees okay.

If somebody says your elevator will be effective after 25 degrees less then - 25 degrees then I go this graph struck here so I can trim it much higher CL so this is the relationship between your Delta E required and CL for a given static margin we will also try to have another understanding from Delta E versus CL Trim graph Delta E versus CL Trim remember.

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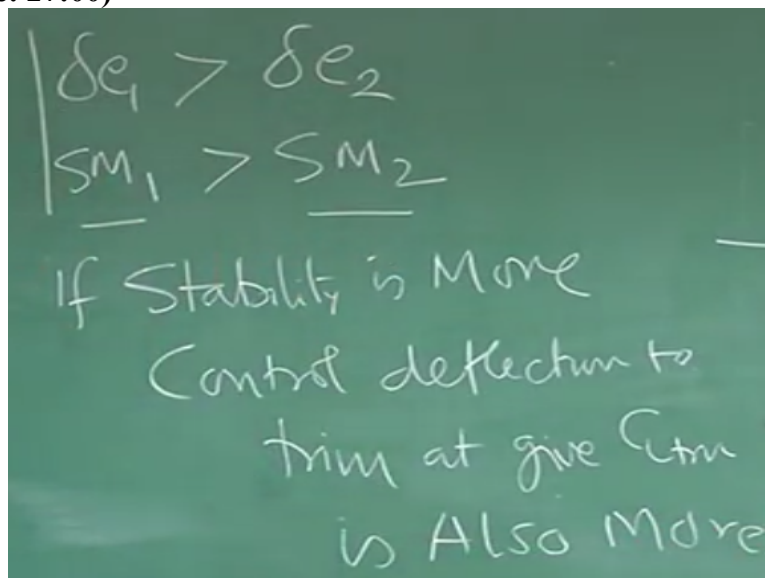


This CL Trim and you the know the slope is negative let's say this is for XCG 1 configuration and let me also. Write that this is XCG 1 and aft another is XCG 2 and let's say neutral point stick fixed it here and this is the reference line now for same airplane if I move the CG 2 XCG 2

and let's see that the graph looks like this one thing you understand because XCG 2 is closer to neutral point the static margin is less it is less statically stable, the slope will flat now right?

Now I want to check if I want to trim the airplane at CL star I will see relatively. What will happen if I try to trim the aircraft at CL star using XCG 1 configuration or XCG 2 configuration, where do I need more elevator so if I missing XCG 2 you see, I need this much of elevator I said Delta E 2 if I am trying to trim for CL star for XCG 1 configuration then I have Delta E 1 which one is more.

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Definitely Delta E 1 is greater than Delta E 2 as far as absolute deflection is concerned, and also you know that static margin in case of one is greater than static margin, for case 2 because the slope here is more for more statically stable then XCG 2 configuration which obvious, because XCG 2 is closer to neutral point then XCG 1 and this distance we define as static margin this also talks about degree of static stability right.

So what is the message you're getting here is telling if stability, static stability is more which is like SM 1 greater than SM 2 then control deflection to trim at a given CL trim is also more. So we say pilot find to difficult to trim if that stability margin because excessively high he may not be able to trim it or find it the difficulty in trimming it unless you carefully design it okay. So, this also establishes relationship between control and stability.

We know that if the airplane is when airplane is highly statically stable then it is difficult to control that means to change from one trim to another trim we have to give very large deflection elevator deflection in that since we say, more stability means more control effort okay. And to really increase the maneuverability that's why we reduce this gap to where minimum.

So that efforts are less in fact, most of the fighter airplane will find this static margin is negative in the sense it is marginally unstable statically unstable so this is the correction between stability and control which you must keep back of your mind. I hope I have made it clear? And you are say flight stability and control percent should revisit this again and again and get that feel into your mind okay. Thank you.