

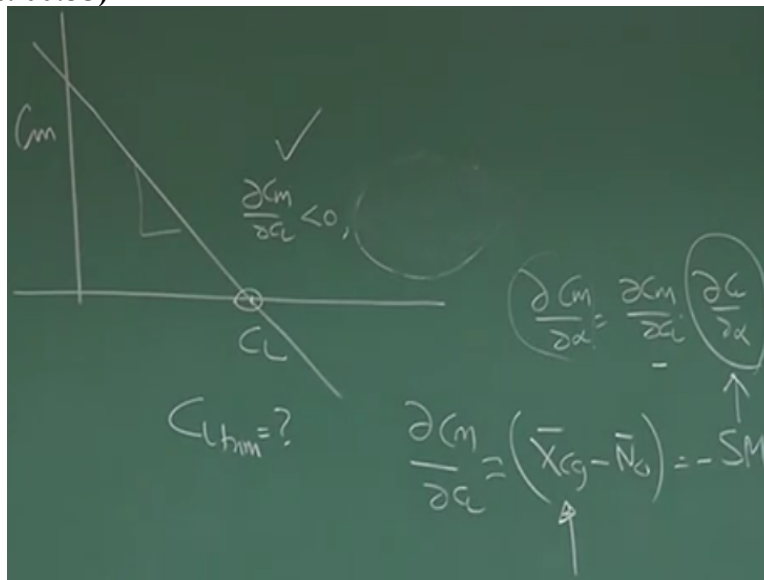
**Aircraft stability and Control**  
**Prof. A. K. Ghosh**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology Kanpur**

**Lecture- 12**

**CLtrim vs. DeltaEtrim**

Yes welcome, back to this session where we will be again solving one numerical to clear or crystallize our concept of elevator, angle required to trim an aircraft for a particular CL we call it CL trim and for various CG locations right. Why this is important?

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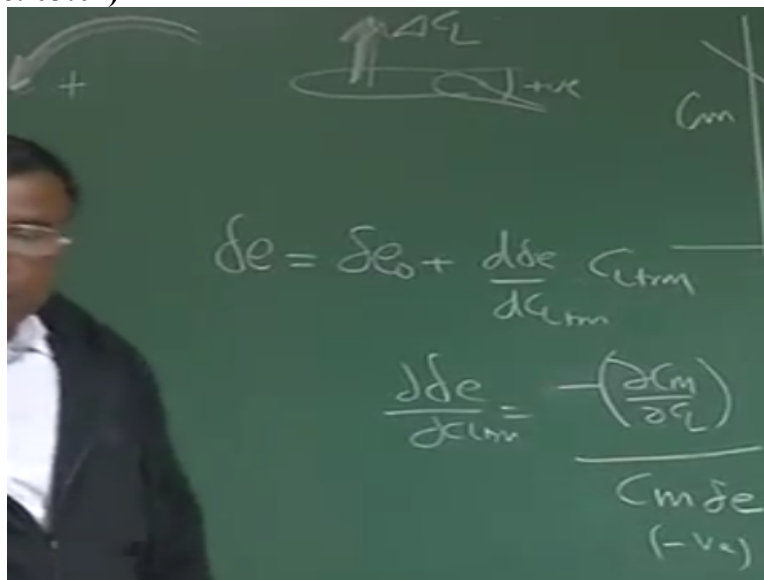
You know that, if I draw CM versus CL and this is the variation of CM versus CL at equilibrium or at trim it has the negative slope that is DCM by DCL is less than 0 or DCM by D Alpha is less than 0. You know that both are equivalent because for DCM by D Alpha I can write it as DCM by DCL into DCL by D Alpha and this is CL Alpha this is always positive so If DCM by DCL is less than 0 and DCM by D Alpha is also less than zero and vice versa so

We will be writing whenever we are writing CM versus CL variation, we talk about DCM by DCL is less than 0 for a static stability and when you are writing CM and Alpha here, then we write DCM by D Alpha less than 0. So, both are equivalent statement right. So now if I see this variations CM versus CL what is this point? This point we call CL trim, why it is CL trim because this is at the CL the aircraft is in equilibrium right maybe one of the cruise conditions it could be

And also you know this slope of this line which is DCM by DCL is loosely given as  $X_{CG} - N_0$  bar, we have already derived at which is nothing but minus static or stability margin, we have derived this. So you could see that this slope will depend upon  $X_{CG}$  location right because  $N_0$  doesn't depend upon the CG,  $N_0$  depends upon the configuration right. It is the neutral point, a stick fixed neutral point and how did you find the neutral point.

Neutral point is thus CG location at which the DCM by DCL = 0, or the neutral point is thus CG location at aircraft becomes neutrally stable or further, if even if I say if I put the CG beyond neutral point the aircraft will become statically unstable right.

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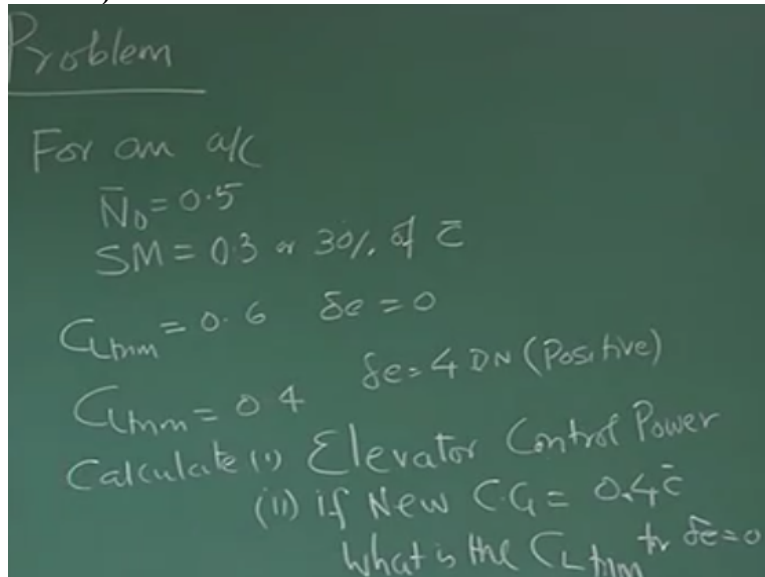


This is the concept beyond what the problem will be doing, also another concept will check we know  $\Delta E$  trim or  $\Delta E$  required is given by  $\Delta E_0 + D \Delta E$  by DCL trim into CL trim and for approximate values we have seen that,  $D \Delta E$  by DCL trim I can write and to be more precise, for yourself minus DCM by DCL divided by CM  $\Delta E$  what is CM  $\Delta E$ ? CM  $\Delta E$  was elevator control power what was the physics behind CM  $\Delta E$ .

Let us also revise here. This is the tail, this the elevator, elevator positive is down, if I deflect the elevator down there will be additional CL  $\Delta E$  and aircraft CG is here, this will give me a nose down movement, nose down pitching movement which is negative. That is why DCM by  $\Delta E$  that is pitching movement generated because of positive  $\Delta E$  is having negative sign that is positive  $\Delta E$  will generate a negative pitching movement.

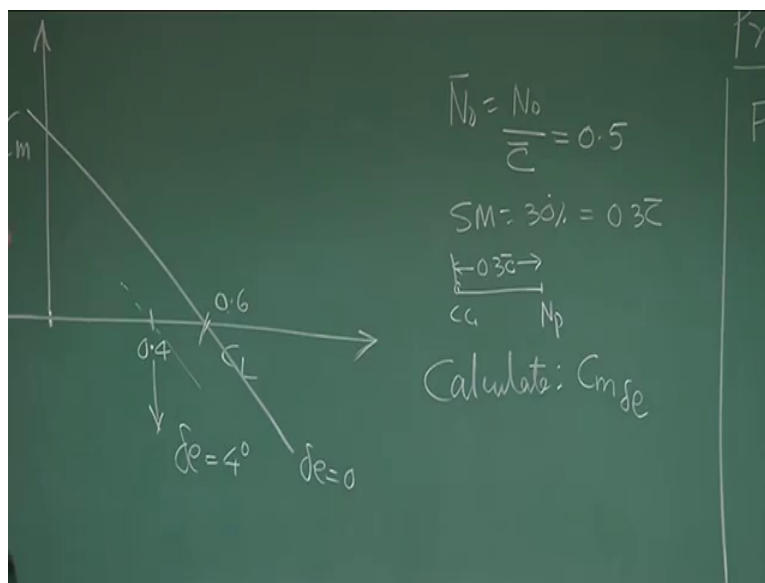
So the sign of CM Delta is negative and this is also called a control power, more precisely elevator control power okay. Now with this understanding we will try to solve this problem let us read this problem carefully.

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It says for an aircraft neutral point stick fixed is point five, neutral point bar that means what? N 0 bar means this neutral point location non-dimensionalised with mean dynamic chord it is 0.5.

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That means from the reference point where from you are measuring, the  $N_0$  is located as 50% of the chord. Mean dynamic chord right. Then SM is point 3 or 30 percent, static margin is 30% it

means it is = point 3 of mean aerodynamic chord, the absolute value okay The absolute value the separation between neutral point and the CG, this distance is point 3 C in absolute phase right So that is the starting margin. Then CL trim is 0.6 so if I draw it like this, try to draw this problem this is CM and this is CL.

What does it say it says that CL trim is given as point 0.6 so I mark here this is 0.6 and what further it says, it says when I have made the CL trim. When I have trim the airplane plane at that CL, I did not require any elevator deflection that means whole of this CM 0 was generated, otherwise from tail setting or from aerodynamics center ahead of wing, so aerodynamics center of wing, ahead of CG of the airplane.

So point this elevator deflection was not given, it was not required but the aircraft could easily trim at point 6, so this is Delta E, elevator deflection is 0, then what further says. Here when this he was trying to trim at CL = point 4, then you required a elevator deflection of 4 degree this is degree, 4 degree downward and you know as per the convention elevator deflection down is positive okay. So, this is Delta E.

Delta E is for CL 4.4, when it's trimming it requires a Delta E = 4 degree or 4 degree down okay this question clear. First is when the aircraft was trimmed at CL = 0.6, we didn't require Delta E 0 but for the same configuration nothing has changed, no CG has changed nothing as changed. Same aircraft when you are trying to trim at point 4, then Delta E required was 4 degree down okay.

So that is the understanding okay. Then what is the question? Question is calculate elevator control power that is calculate elevator control power by now you're smart there is nothing but CM Delta E okay. And second is, if new CG is 0.4 C bar then what is the CL trim the aircraft can be trimmed with Delta E = 0. Second question is if the CG of the airplane changes now changed it into point 4 C, then what is the CL trim that I can trim the airplane but I will still not give any Delta E 0, second question will come in elaborate way, first let me complete elevator control power okay.

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$\delta E = -\frac{C_{m0}}{C_m} \delta E$

$\delta E = \delta E_0 + \frac{d\delta E}{dC_{trim}} C_{trim}$

$\frac{d\delta E}{dC_{trim}} = -\frac{C_{m0}}{C_m}$

$C_{trim} = 0.4, \delta E = 4^\circ$

$C_{trim} = 0.6; \delta E = 0^\circ$

$\frac{d\delta E}{dC_{trim}} = -20^\circ$

$\frac{d\delta E}{dC_{trim}} = \frac{4 - 0}{0.4 - 0.6} = \frac{4}{-0.2} = -20 \text{ degree}$

So you know CL trim = 0.4 that time Delta E = 4 degree, then CL trim = 0.6 that time Delta E is = 0 degree, and what is the relation I know the Delta E trim = Delta E 0 + D Delta E by DCL trim into CL trim okay.

What is our aim? Our aim is to find CM Delta E don't forget this, where is CM Delta E lying in this expression. Let us look into this is there any CM Delta E in Delta E 0. Let us check yes, Delta E 0 is nothing but - CM 0 by CM Delta E. Do you think CM Delta is here. Let us check D Delta E by DCL trim is also - DCM by DCL by CM Delta E so indeed it is also here, so what are the information I've got?

You could see that CL trim 0.4 CL trim 0.6 and corresponding Delta E are given, that means I have been supplied with D Delta E by DCL trim as how much D Delta E by DCL trim is 4 - 0 by 0.4 - 0.6. D Delta by DCL trim is nothing but what is the D Delta E? Delta E is changed from 4 degree to 0 of 4 - 0 here, what is the change in CL trim? It is when it was 4 it was 0.4 towards CL trim, when it was 0 degree 0.6 was CL trim. So if I find it out I find this is = 4 by - 0.2 and this value is nothing.

But minus 20 degree let me right like this 20 degree right. 4 by point 2 is 40 by 2 - 20. So this is the information I got D Delta E by DCL trim is = - 20 degree, why 20 degree you could see this is 4 degree, 0 degree that is why and this is non dimensional CL has no dimension so this is

actually 20 degree unit is degree - 20 degree very, very important you must not lose sight of this what you call their units.

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The image shows handwritten mathematical derivations on a chalkboard. The equations are as follows:

$$C_m = C_{m0} + \frac{dC_m}{d\delta_e} \cdot C_{Ltrim}$$

$$0.4, \delta_e = 4^\circ$$

$$C_{m0} = 0.6; \delta_e = 0^\circ$$

$$\frac{dC_m}{d\delta_e} = \frac{-\Delta C_m}{\Delta \delta_e} = -20$$

$$\frac{-(-0.3)}{C_{m0e}} = -20$$

$$C_{m0e} = \frac{-0.3}{20} = -0.015 \text{ per deg}$$

$$\frac{\partial C_m}{\partial \delta_e} = -SM = -(N_0 - X_{CG})$$

$$\frac{\partial C_m}{\partial \delta_e} = -0.3$$

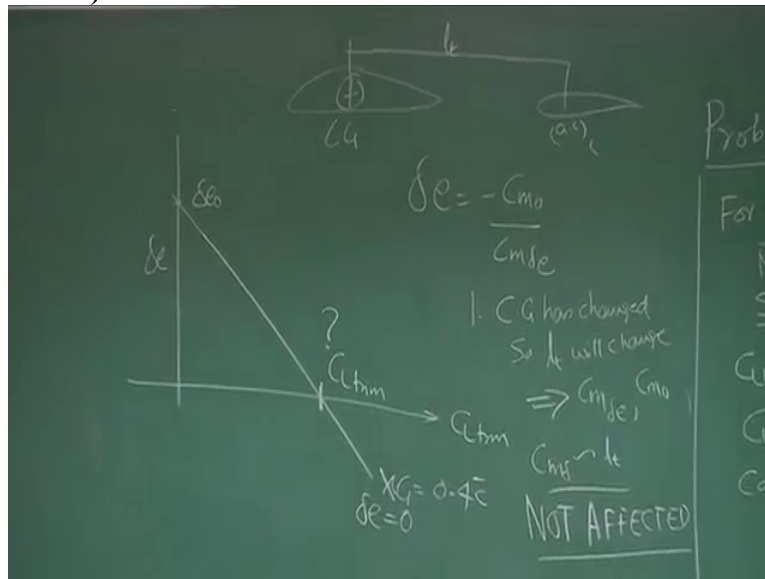
Okay once you know this what is our aim, our aim is to find CM Delta E, that is the aim okay, so let us go for the next step I know D Delta E by DCL trim = - DCM by DCL by CM Delta E this I know now which is - 20, because D Delta E by DCL trim already we have calculate to be - 20. So now check here what is our aim? Our aim is to find CL Delta E, now the question is do I know the value of DCM by DCL see the data here, what does it say N 0 SM starting margin. What is starting margin? You know DCM by DCL we have derived that equal to minus starting margin which is basically - N 0 - XCG okay.

I put the bar I know that now from the data, DCM by DCL = -0.3 as simple as that, so the problem is solved what is the solution now, it is straight forward let me erase this. So I have got - 20 and here DCM by DCL is - 0.3, so I put this no here - 0.3 by CM Delta E is = - 20, so CM Delta E is = point 3 by 20 - sign so this is - 0.015 you should do it per degree and writing this per per degree because, this 20 was in degrees okay and this is 0.3 clear.

So you could see that if I have got this minimal information, I can easily find out what is the elevator control power okay. Now the second part we will attack, the second question let us check here if new CG is 0.4 C that is new CG location is 40 percent of the mean aerodynamic

chord, what will be the CL trim for Delta E is = 0 right. That is the question we are going to ask that is now new CG location means that DCM by DCL.

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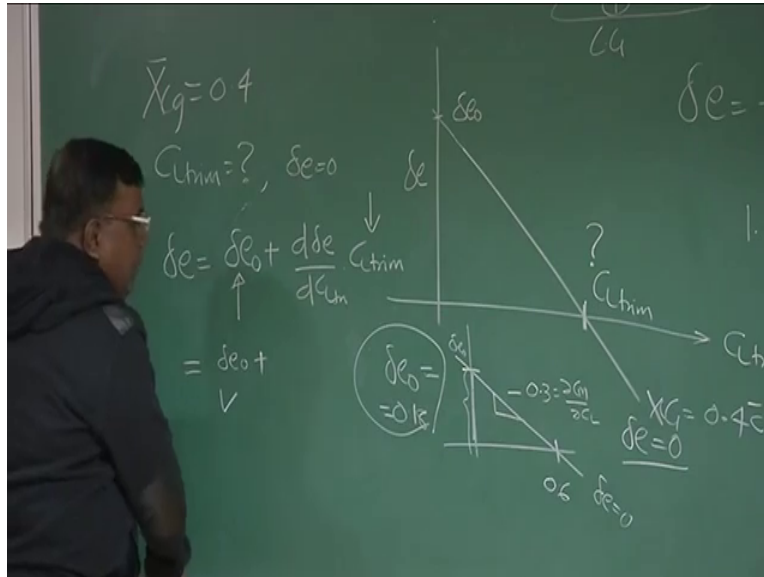
Whatever was early given is no more valid because now CG has changed, okay a new CG so CG as changed means static margin will change. So now the question is it is given now  $X_{CG}$  is = 0.4 C bar. This point I want to find out what is that CL trim given is Delta E = 0 again so now the question is what is this is CL trim value right okay. So this is CL trim this Delta E if I increase this is Delta E 0. We will make what assumption here ideally if you see what Delta E 0 is Delta E 0 is - CM 0 by CM Delta E.

So if the CG is changed then C from here, this is the wing this is the tail this is the CG of the airplane, from here to the AC of the tail is the LT, and all those movement CM Delta E will depends upon LT now you have changed the CG so CM Delta will ideally will change but we are assuming that this change in CG is not affecting the value of CM Delta E or CM 0. Please understand this assumption practically it is true also right.

The variation will not be large but the problem we are constraining that although that the CG has changed, that will change LT we are assuming that CM Delta E CM 0 are same, what are the assumption we understand that CG has changed so LT will change and that will affect the value CM Delta E and CM 0 to some extent because CM Delta E.

We know is proportional to the tail movement R, but in this problem we are assuming we are not affected these two derivatives are not affected this is the assumption which is not very unrealistic okay. So if I do that then now I come back here I try to find out what is that CL trim I can trim the airplane for a new CG location which for 0.4 C but I want to keep that Delta E 0. So what is our aim now, our aim is to find that CL trim for a new CG location.

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But we want to keep Delta is = 0 so let us solve this problem, if I write it down XCG is 0.4 okay. CL trim I want to find out what but I know Delta E I have to keep it 0. That is Delta E trim I have to keep 0, okay so now what is Delta E? Delta E = Delta E 0 + D Delta E by DCL trim into CL trim okay.

So what is Delta E 0 value? so this is since we have assumed that the change in LT DCM Delta if CM not affected, so I can easily find Delta E 0 from the first graph where this was 0.6 Delta E was = 0 so this is basically Delta E 0 and Delta E 0 by geometric you could find that, and since this slope is D Delta E by DCL trim is - 0.3 that was given from first case so, I can easily find out that this Delta E 0 value is 0.18 that is 0.6 into 0.3.

That is it will be the this Delta E 0, from triangle you can find out. This I am able to get from the first condition first problem here, because I am assuming that CM Delta E and CM are not changing okay.



So I know Delta E 0 so I put here Delta E 0 value which is known, whatever DDL trim by DCL trim we have to find out this, now D Delta E by DCL trim let us see what is happening to that D Delta E by DCL trim is = - DCM by DCL by CM Delta E, but we know that in the second case the CG has changed so, DCM by DCL is no more I can use the earlier DCM by DCL clear.

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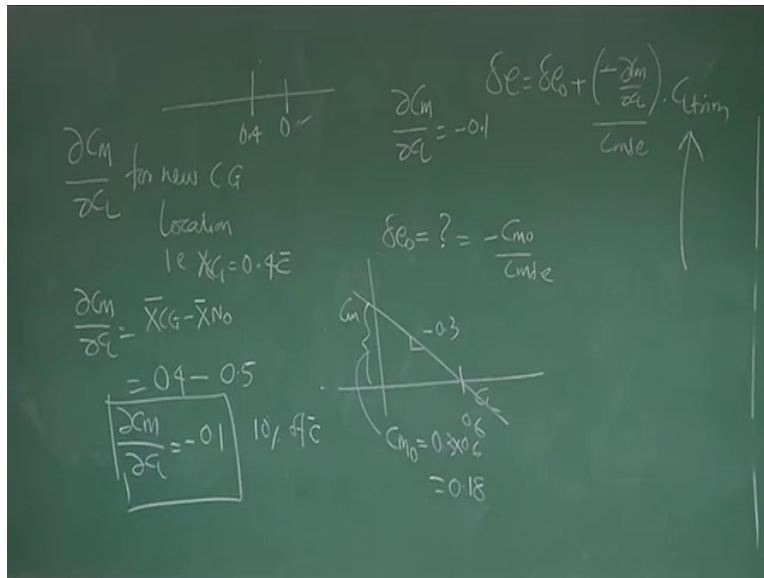
Handwritten notes on a chalkboard:

- $\bar{X}_g = 0.4$
- $C_{trim} = ?$ ,  $\delta e = 0$
- $\delta e = \delta e_0 + \frac{d\delta e}{dC_{trim}} C_{trim}$
- $= \delta e_0 +$
- $\frac{d\delta e}{dC_{trim}} = -\frac{\partial C_m}{\partial C_L}$
- $\frac{\partial C_m}{\partial C_L} = -(\bar{X}_{N_0} - X_{CG})$

Because we have only assume that because of change in CG, CM Delta E and CM 0 are not changed, with the change in CG DCM by DCL will definitely change the drastic manner, because we know DCM by DCL is nothing but - of X 0 that is neutral point - XCG okay. That is we say if XCG is concise with neutral point.

Then you say this is neutrally stable, that is DCM by DCL will become zero right. So we have to take this into account that as CG has changed DCM by DCL will also change, but the question is do you know the value of neutral point stick fixed x 0 do you have any idea? What is the value of that what is the neutral point let's ask ourselves.

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Now we are trying to find out what is DCM by DCL for new CG location, that is what that is  $X_{CG} = 0.4$  as in the second case there that CG  $0.4 \bar{c}$ , we need to find what is DCM by DCL for new CG configuration and you know DCM by DCL is basically  $X_{CG} - X_{neutral\ point}$  and you could see here neutral point is given as  $0.5$  and you know that neutral point is not going to change because it depends upon the configuration wing configuration.

Tail configuration, engine configuration the speed everything depending the same neutral point will not also change whether you've changed here and there so I will keep this as  $-0.5$  as first case, and this case is  $0.4$  and  $\bar{c}$  so put it  $\bar{c}$  because they are non-dimensional, so now I could see DCM by DCL is coming to be  $-0.1$ , or  $10\%$  of mean aerodynamic chord, that is separation the slope is  $-0.1$  right.

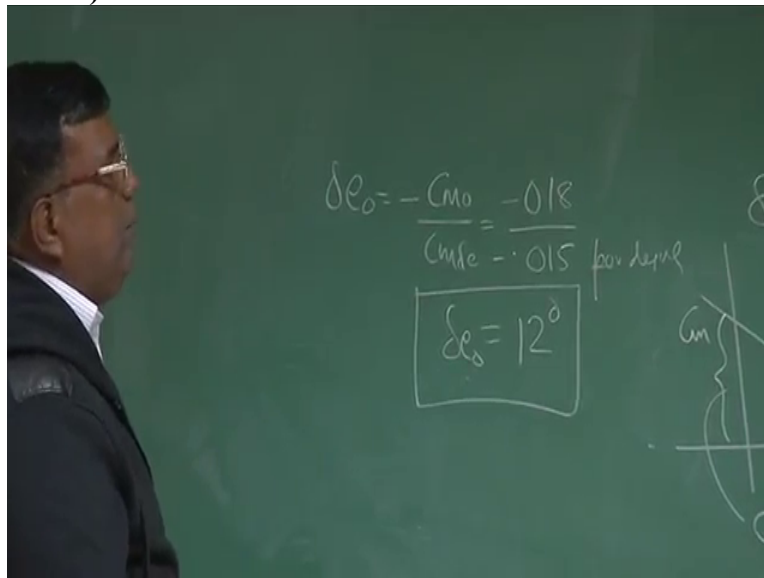
So DCM by DCL is negative so it is statically stable that means whatever new CG is there, you should be ahead of neutral point and you could see that neutral point is at  $0.5$ , and new CG is at  $0.4$  so, yes it is ahead of neutral point so DCM by DCL is negative and indeed it is coming like that okay. So now I know DCM by DCL and what is the value of DCM by DCL okay so DCM by DCL is  $-0.1$  and we know that, we are working with the relationship.

$\Delta E = \Delta E_0 + - DCM\ by\ DCL\ by\ C_m\ \Delta E\ into\ C_L\ trim$  we want to find out what is the CL trim so that  $\Delta E$  is  $0$  and we also know  $\Delta E_0 =$  what?  $\Delta E_0 =$  we know  $\Delta E_0 = -C_{m_0} / C_{L_{trim}}$  and if we recall from the first case, this is  $C_m$  versus  $C_L$  where this

is 0.6 given, and the slope is  $-0.3$  as it is given in the first case so this  $CM_0$  value from the geometry, it is  $0.3$  into  $0.6$  which is  $= 0.18$  right.

From the first case I could get the value of  $CM_0$  and since I have been telling repeatedly because, of change in CG there is no change in  $CM_0$  and  $\Delta E$  as I have assumed, so I will use this  $CM_0$  value okay. So we will use this, what is our aim, our aim is to find out  $\Delta E_0$ . If I know the value of  $\Delta E_0$  then I can easily use this relationship because this I know I have to find this, if I know this then I can easily find this so  $\Delta E_0$  will be what.

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$\Delta E_0$  is  $-CM_0$  by  $CM_{\Delta E}$  and you know the value will be  $-0.18$  by  $-0.015$ . So this will give me around  $12$  degree so,  $\Delta E_0$  please note down this is again  $12$  degree because  $CM_{\Delta E}$  is per degree, okay. We have already find it out from the first problem, right from here they have already found it out what is the elevator control power and that happened to be  $-0.015$  per degree. So  $\Delta E_0$  is  $12$  degree now what is the problem everything we know now, so what I have to do.

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$$\delta e = \delta e^0 + \left( \frac{-\partial C_m}{\partial \alpha} \right) C_{Ltrim}$$

$$0 = 12$$

$$C_{Ltrim} = 1.8$$

$$\frac{\partial C_m}{\partial \alpha} = \bar{X}_{cg} - \bar{X}_{n0} = 0.4 - 0.5 = -0.1$$

$$C_{mde} = 0.15$$

I have to simply use this relationship and find out what is the CL trim required, so Delta E = Delta E 0 + - DCM by DCL by CM Delta E, into CL trim Delta E 0 is 12 this is 0 now everything we know we have just use this relationship and in this you Delta E 0 is 12, what is DCM by DCL DCM by DCL is nothing but XCG - N 0 or XN 0 and this is 0.4 – 0.5 so, this is - 0.1 this value I can put it here CM Delta I know point 0.015 - this is 12 degree.

So I can easily calculate CL trim by inverting it so, CL trim I will get around 1.8, so see through this relationship how we have calculated different requirement of Delta E for different CG location right. So I hope if you solve this problem yourself you will get the hang of it and how to use this relationship for your practical problem that's exactly why am repeatedly supplying you so many problems so that you can crystallize all the concepts okay. Thank you very much.