

**Aircraft Stability and Control**  
**Prof. A. K. Ghosh**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology-Kanpur**

**Lecture- 16**  
**Trim-Cruise, Climb and Landing**

Yes, Good morning friends, we have been so far discussing about how to control an airplane in a longitudinal mode that is, how do I deflect the elevator to trim the airplane at different, different CL right, and in this regard we developed one relationship that is Delta E.

**(Refer Slide Time: 00:36)**

The image shows handwritten equations on a chalkboard. The top equation is:
$$(\delta e)_{trim} = \delta e_0 + \left( \frac{d\delta e}{d\delta c} \right) \delta c_{trim}$$
An arrow points from the derivative term in the first equation to the second equation:
$$\frac{d\delta e}{d\delta c_{cm}} = - \frac{\frac{\partial C_m}{\partial \alpha_L} (\bar{N}_0 - \bar{X}_{CG})}{C_{m\delta e}} ; \frac{\partial C_m}{\partial \alpha_L} = - (\bar{N}_0 - \bar{X}_{CG})$$

That is Delta E trim or Delta E required is Delta E0, + D Delta E by DCL trim into CL trim. Further we know D Delta E by DCL trim which is this one, this is equal to - DCM by DCL by CM Delta E what is CM Delta E? CM Delta E is elevator control power its sign is negative, and DCM by DCL is - static margin. So I can write this as  $\bar{N}_0 - \bar{X}_{CG}$  by CM Delta E, this is clear? Because we know DCM by DCL is equal to - static margin, - of static margin is neutral point - XCG location, so if I put it here this - sign goes so I have got this.

**(Refer Slide Time: 01:56)**

$$(\delta_e)_{reqd} = \delta_{e0} + \frac{(\bar{N}_0 - \bar{X}_{cg})}{C_{m\delta_e}} \cdot C_{L_{trim}}$$

$$(\delta_e)_{trim} = \delta_e$$

A/c. Low speed

$$\delta_{e0} = -\frac{C_{m0}}{C_{m\delta_e}}$$

$(\bar{N}_0)$  = Stick Fixed case  
 Power on  
 " off  
 windmilling

Cruise:  $L=W \Rightarrow C_L = \frac{2W/s}{\rho V^2}$

So in a simpler form I can write Delta E trim or required, whatever you use is equal to Delta E0 + N0 bar - XCG bar divided by CM Delta E. And you all know when you say bar means it is non-dimensionalized with mean aerodynamic chord, this is not complete I have to write here CL trim, so what is the message from this relationship we got? If I want to trim an airplane at tail particular CL once I know this, this, this and this I know how much elevator deflection has to be given.

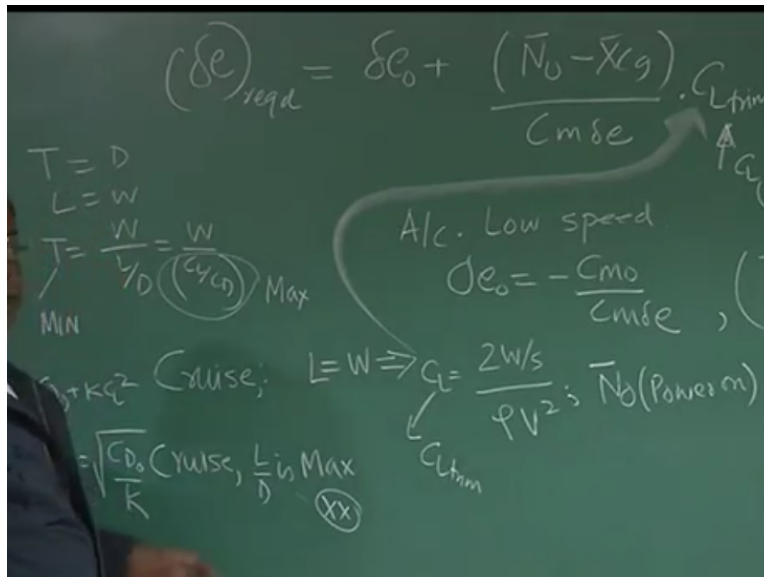
Let us say for an aircraft for an aircraft low speed aircraft, low speed means why I am saying low speed, I am assuming that at low speed this derivative will not change with Mac number. So they are even fairly constant and this statement is fairly correct, if Mac number is up to point six with all this derivatives remain fairly constant,

So I am considering at low speed, so that means I know that Delta E0 which is - CM0 by CM Delta E that is also constant, also we are assuming that if there is slight change in the CG okay, we do not attribute that might have change the change CM0 we are assuming CM0 is constant. But one can always find out exact value. Similarly, here N0 bar which is for a stick fixed case is also fixed right, but we have noticed that this N0 bar may vary depending upon for that power ON, Power OFF, or propeller wind milling right, okay.

So now, think if I have taken a particular aircraft, and if I want to find out what is the Delta E required for cruise I am cursing at the particular altitude where density is low, then what is the CL required that I find through lift equal to weight, this implies CL equal to 2 W by S by row V square. Let us say I want to fly at a particular altitude were density is row, and in particular speed V then what is the CL required? I know from here CL will be 2 W by S row V square and W by S is the wing loading, you all know that.

So what is this relation is telling me now? Then this becomes my CL trim for that cruise at altitude where density row, I will be simply using this CL value here right? As we have assume that CM Delta E Delta E0 I am not changing, what is the next thing I should check I know that N0 I have to be very careful, I have to see N0 for power ON Power OFF wind milling case which case it is? While it is cruising at cruise which N0 will be relevant for us, it is not power OFF it is not wind milling it is power on case.

**(Refer Slide Time: 05:42)**



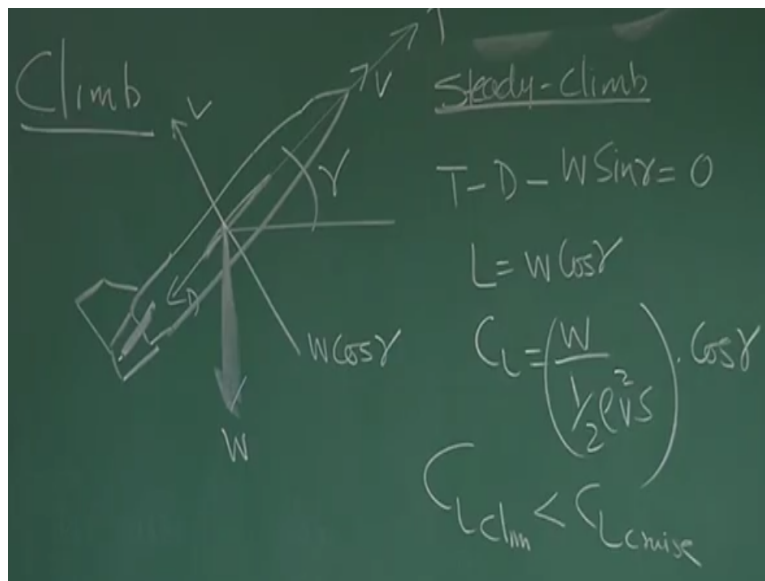
So I should put this CL trim here and along with I will pick up the value of N0 power ON and find out what is the Delta E required okay. This will go there and N0 power on will be here come here for cruise, but that's not the end of the story. Suppose you want to cruise such that L by D is maximum possible, and you know for that typically this is a condition when we say the thrust required is minimum, if we revise you remember for a cruise thrust equal to drag, lift equal to weight.

So I can write thrust equal to  $W$  by  $L$  by  $D$  or equal to  $W$  by  $CL$  by  $CD$ , so from here I can make an observation if I want thrust required minimum, that means this gentleman should be maximum for a given weight right. So for  $CL$  by  $CD$  2 maximum we know that  $CD$  I can write as,  $CD_0 + K CL^2$  and using this relation, I can find out the condition for  $CL$  by  $CD$  maximum which we have  $d1$  in your first course to be  $CL$  equal to under root  $CD_0$  by  $K$ .

Now notice this  $CL$  and this  $CL$  are not same now in general this is the  $CL$  when I am going thrust required minimum, or this is the  $CL$  when  $L$  by  $D$  is maximum right. So now how do I get what is the elevator deflection? what I will I will again take this  $CL$ , this  $CL$  value I will plug in here, whatever this  $CL$  coming for thrust required minimum. This  $CL$  for  $L$  by  $D$  maximum or thrust required minimum I will plug in here right and then next I will check what the  $N_0$ ?

Because it is also cruise although it is in a thrust required minimum configuration or  $CL$  by  $CD$ , maximum configuration but it is power on so  $N_0$ , again I will be using power on correct. Next case we see, let's say climb.

**(Refer Slide Time: 08:04)**



How did you model the climb? if we recall this is the airplane this is vertical tail, this is the horizontal tail, this is the wing and said  $T$  this is  $D$  and this is Lift, and the  $V$ .  $V$  in the same direction, this is  $W$  so this is  $W \cos$  of flight path angle, which is  $\gamma$  right so how did you

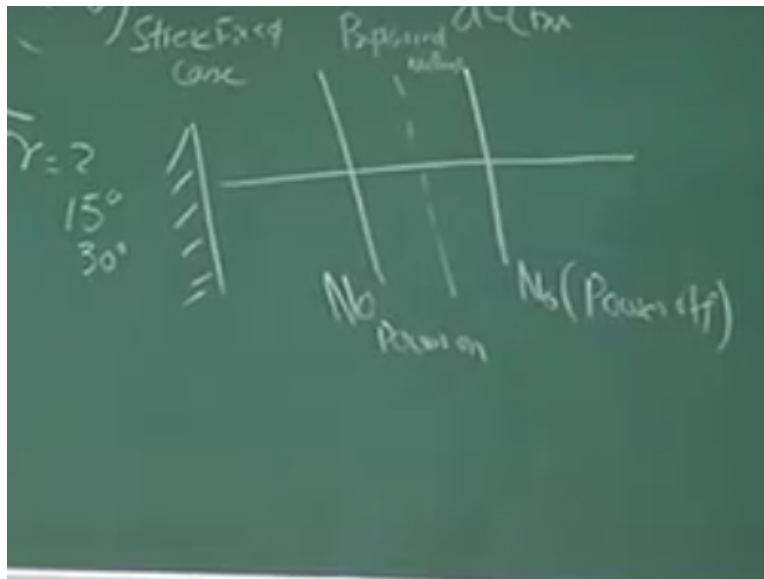
form the equation? You see  $T - D - W \sin \gamma = 0$ , because we are talking about steady climb. We have done all this thing in your first course you are advised to go through that I will be going skipped, I will just glancing through it because it is relevant and then here  $L$  equal to  $W \cos \gamma$ .

So message is If I want make an rectilinear flight along this direction, I should ensure that there is a force perpendicular to this velocity vector, if there a net force perpendicular to the velocity direction then you will take curve path. So we must ensure that, if we are flying like this steady climb rectilinear path then I must ensure this lift force should be balanced, by the component of it  $W \cos \gamma$ , so from here  $CL$ .

I find as  $W$  by half row  $V^2 S$  into  $\cos \gamma$ . And you could see that if I am climbing the  $CL$  climb is less than  $CL$  cruise right. That is exactly why we saying, induced drag during, cruise is more than induced drag during climb. What is the  $\Delta E$  required? Again I ask this question, so I will now put I will estimate this,  $CL$  value depending upon what is the value of the  $\gamma$ ? am I climbing at 15 degree or thirty degrees depending upon that I know the  $CL$  value, and this  $CL$  value.

I now put to  $CL$  trim and next question of  $N_0$  okay, this is climbing if the power is on, so I should take  $N_0$  power on this case correct, so what is the change? Change is only I need to only find out the  $CL$  for the climb, which is equal to  $W$  by half row  $V^2$  is into  $\cos \gamma$  depending upon what is the  $\gamma$ , what is flight path angle this  $CL$  value also will change  $CL$  for climb will change right. And that value I will put here, I check what is  $N_0$  it is climbing that means thrust is on the power is on, so I will be using  $N_0$  corresponding to power on okay.

**(Refer Slide Time: 10:48)**



At this point you should remember that we have already proofed that, if this is N0 power off more stable case, this is N0 on and this is power on and this is propeller wind milling correct. Here imaginary from this reference, and this graph, this variation is typically correct for a nose mounted propeller driven engine, or a engine nose mounted that means that engine is the thrust line or thrust is ahead of CG.

The engine ahead of CG okay, now the story again doesn't end here, when you are in the operation you see what are the things you do, what the pilot would like to do?

**(Refer Slide Time: 11:43)**

- Climb case

$$T - D - W \sin \gamma = 0$$

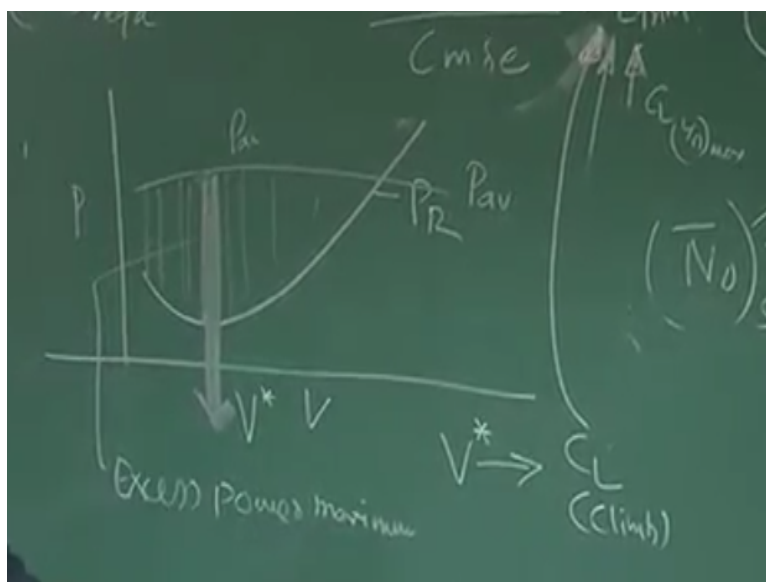
Power required  $\frac{T - D}{W} = \sin \gamma$

Power available  $\frac{TV - DV}{W} = V \sin \gamma$

If I take the climb case we have seen  $T - D - W \sin \gamma = 0$ , so I can write  $T - D$  by  $W$  equal to  $\sin \gamma$ , or  $TV - DV$  by  $W$  is equal to  $V \sin \gamma$ . We have already done all these so what is  $V \sin \gamma$ ? If this is  $V$ ,  $V \sin \gamma$  exist component, so this is rate of climb okay this is rate of climb.

And what is  $TV$  if the power available, power available and this is the power required and this is a weight so, I know how rate of climb changes with excess power that is different between power relevant, power required for a steady climb this is for a steady climb no acceleration is there.

**(Refer Slide Time: 12:47)**



Now from pilot point of view, if you see if I write power and V you know that power required graph will follow something like this, and let's say power available for particular type of engine, propeller driven, let say remain all must constant with speed where ever this excess power is maximum, this is the excess power maximum you really know this is power required is power available.

So this is the excess power, and there it starts increasing goes to maximum, and then start reducing so there is a corresponding V let say V star which corresponds to weight of climb maximum because I know excess power and this rate of climb they are related okay,  $V \sin \gamma$  is rate of climb so, they are related.

**(Refer Slide Time: 13:48)**

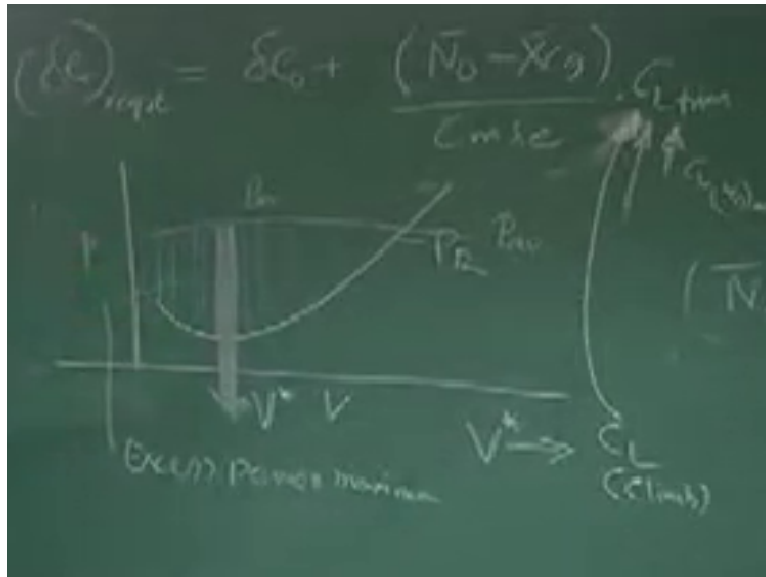
Handwritten equations on a chalkboard:

- Climb Case
- $T - D - W \sin \gamma = 0$
- $\frac{T - D}{W} = \sin \gamma$  (labeled  $P_{Req}$ )
- $\frac{TV - DV}{W} = V \sin \gamma = ROC$  (labeled  $P_{avail}$ )

So wherever excess power is maximum rate of climb also be maximum. Now suppose the pilot wants to fly at a particular speed, where rate of climb is maximum, how it is going to trim the airplane? How much Delta he is going to deflect, so you know this is V from this chart you know what is the V the pilot should fly, from that V pilot should find out CL using the climb equation, and that CL, so this is V star he find CL from climb, and that CL he as to use it here okay. That will give him the Delta E required to fly at a speed for which rate of climb is maximum.

**(Refer Slide Time: 14:32)**

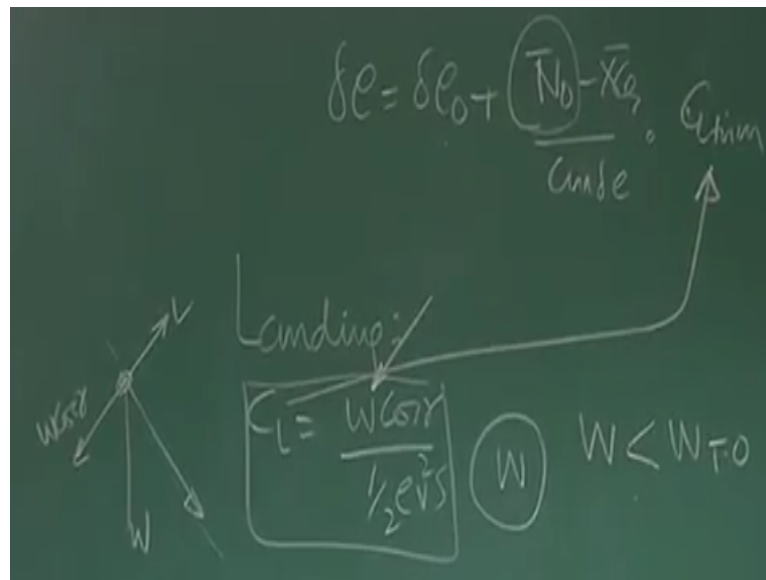




Again he will be using  $N_0$  bar, which is for power on, these are pilot will not compute, all these things, the designer you have to see that, these solutions are available pilot will go on handling elevator, and thrust extra together, but if you do not have a solution in the design, then pilot won't be able to get those thing through iteration right okay. So, these are very important for the understanding for the designer so what you are doing, what are the solutions available, what sort of bandwidth you have given for the airplane right.

We have been discussing on the cruise, climb and now landing also same story, same you have to find out what is the  $C_L$ , and then trim the airplane.

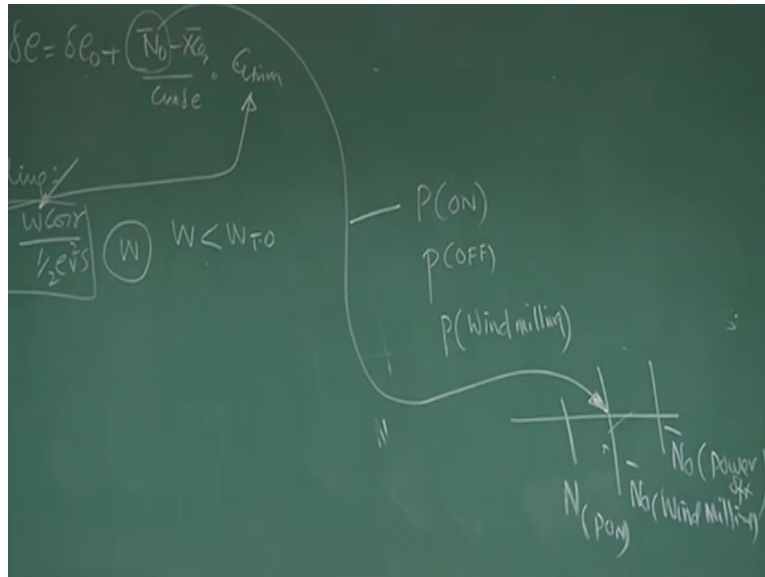
**(Refer Slide Time: 15:19)**



Based on Delta E equal to Delta E0 + N0 bar - XCG bar, by CM Delta E into CL trim, see the landing. During landing, what is happening again the velocity vector is like this, and you want to follow this path. This is the weight, this is the lift, this is the W Cos Gamma, CL will be again W Cos gamma by half row V square S, this is fine.

So what you will do, you can find out the CL by taking the weight, what weight should I take? When I am landing this weight is not equal to the take-off weight, isn't it because some amount of fuel is consumed this weight will be different, so this weight will be less than W take off, whatever when you are computing CL right, and you have particular speed so, you know what is the CL value, that CL again will be putting here for CL trim but then as we have made it a habit, we should check what N0 should I put in this case.

**(Refer Slide Time: 16:45)**

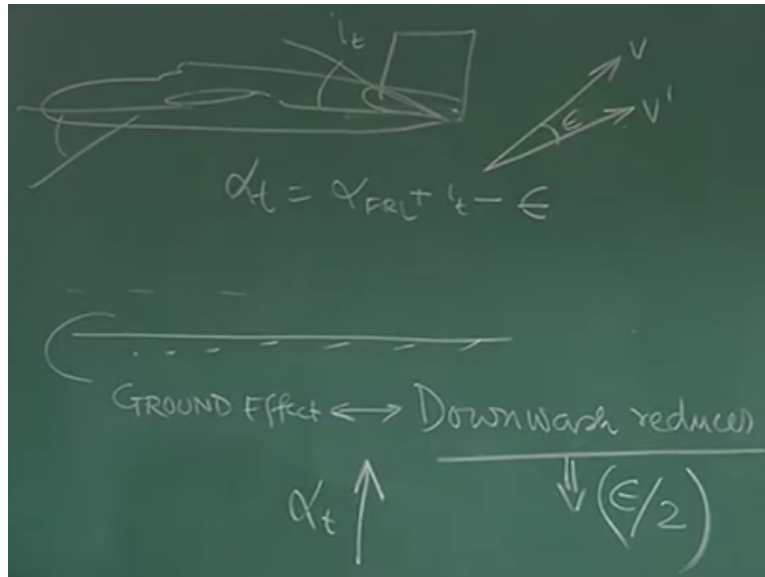


When I am landing then we have three cases power on, power off, and power that is wind milling. Which way should I pick? Generally when am landing like this, ideally power should be 0 because, why do you want unnecessary power. But that is not the case for all safety arrangements. So, the propellers are generally put in a wind milling condition that is when it's moving it is also rotating because of wind milling effect, and sometimes it is some pilot to set 10% of the power, 10%, 20% percent power.

But we are thinking of a case where wind milling, and we know that,  $N_0$  for a wind milling is in between this is  $N_0$  power off, this is  $N_0$  wind milling, and this is of course  $N_0$  power on, this is typically for nose mountain engine. So, now when I am landing this  $N_0$  should be  $N_0$  corresponding to  $N_0$  wind milling. Because it is not full power on, nor it is power off okay.

Then you will find what is Delta E required correct? it is as simple as that, but see there is a catch, this is fine, but when you are landing what is happening, let's see in flight in cruise, you are flying in air and you have air and air all through, but when you are coming for landing.

**(Refer Slide Time: 18:26)**



What is happening you see, when you are coming for landing at some distance, see there is something called ground effect that we have talked about in our first course, what happens if this is the, this is the wing of the airplane when I am landing like this there is a high pressure bottom, low pressure at the top.

So there will be vertices they will come out of this, they will be hitting the ground and there is a reflection right, and that is typically characterized through ground defect modeling. But for us what happens? Because of ground effect primarily one of the important thing, that happened is downwash reduces, so because of the ground effect the downwash reduces, and what is effect of that let us see. Remember when your are modeling in free air or free space, where far away from the ground we said Alpha tail is whatever the Alpha FRL fuselage reference line + IT.

If I put IT some setting angle, you can check earlier note IT, + IT but - epsilon because of wing, vertices there will be a downwash component, then we have seen. If this is the freestyle  $V$  and the local  $V$ , at the tail becomes,  $V$  prime tilted downward and this is the downwash angle. that means what is the message from the flight mechanics point of view.

The effective angle of attack at tail, because of downwash is reduced from free stream angle of attack. That the only message, if the down-wash was 0, whatever the angle of attack is seen by the wing same angle of attack is seen by tail, if there is no tail setting angles right. Now, what is

happening it is you have put the elevator to counter the movement, which is coming because of Alpha at the tail, Alpha tail will give moment like this down so, you have put the elevator of if you are balancing it. But what happens as we come near to the ground, maybe within the one span length of the aircraft, this downwash generally reduces approximately it becomes half so that means what the airplane which was trimmed earlier.

As it comes close the ground, suddenly the tail angle of attack, Alpha tail increases because downwash has reduced, if the tail angle of attack suddenly increases what will happen, it will a nose down movement like this. So, if you want really land safely then you have to ensure that this sudden increase in the nose down movement is also connected by putting the elevator up otherwise it will hit the ground.

So this where while you are landing, you need to have additional care and keeping the Delta E or elevator deflection or additional deflection reserve okay only to be used during landing, extra amount right. How do I do that? How do I solve that?

**(Refer Slide Time: 21:45)**

The image shows handwritten mathematical derivations on a chalkboard. At the top right, the equation  $\epsilon = \frac{2C_{LW}}{\pi AR}$  is written. Below it,  $\Delta\alpha_t = \frac{\epsilon}{2} = \frac{C_{LW}}{\pi AR}$  is circled. In the middle,  $\tau = \frac{d\alpha_t}{d\delta_e}$  is written. Below that,  $\tau \Delta\delta_e + \frac{C_{LW}}{\pi AR} = 0$  is written. At the bottom,  $\Delta\delta_e = -\frac{C_{LW}}{\tau \pi AR}$  is boxed. On the left side,  $(\delta_e)_{MAX} = \pm 15^\circ$  is written, with  $3^\circ$  and  $12^\circ$  circled below it. The word "reduced" is written vertically on the left, and a circled "2" is at the bottom left.

We know because of downwash reduced, by half changing the tail angle of attack is, it will be CL wing by PI atake for an examplespect ratio, because you know epsilon is 2 CL wing, by PI aspect ratio.

We are taking  $E$  to be one Oswald efficiency to be one, so this is this much change in the angle of attack, that means angle of attack, of the tail will change by this much amount right? Earlier it was twice of this, which was subtracted from tail of angle of attack. Now, this value itself as become half, so now effectively tail angle of attack has increased, which will give a nose down movement right. So, how do I find out  $\Delta E$ ? We know that definition of  $Tow$  which is  $D \Delta E$ , that means per unit change in elevator deflection, how much  $\alpha_T$  gets change.

So, I can write  $tow$  into  $\Delta E + CLW$  by  $\pi$  aspect ratio should be equal to 0 that means whatever additional increase in angle of attack is there in the tail, should be compensated by changing the  $\alpha_T$  through  $\Delta E$  deflection using  $tow$ . So from here I get  $\Delta E$  equal to  $-CLW$  by  $tow$ ,  $\pi$  aspect ratio that means, this much of elevator deflection should be kept in reserve, meaning there why, suppose  $\Delta E_{max}$  is  $\pm 15$  degree what we have to do you find out what is this  $\Delta E$  is required for landing if it is 3 degrees, suppose it is 3 degree this is, suppose then that means you to operate in the flight will be maximum of 12 degree

Because anyway this 3 degree you have to keep reserve clear. Let's see an example there you will get better idea.

**(Refer Slide Time: 24:05)**

$Alc: \quad \bar{C} = 0.5$   
 $C_{L_{max}} = 1.5$   
 $AR = 5$   
 $(\delta e)_{max} = \pm 25^\circ$   
 $-5 - 11 = -14^\circ$   
 $\Delta \delta e = \frac{-1.5}{0.5 \times 3.14 \times 5} = \text{rad}$   
 $\approx (10.9^\circ)$

Let's take an example, this for typical airplane, let's say  $\bar{C}$  is point 5,  $C_L$  max let's say is 1.5 and aspect ratio is 5, so essentially I am talking about low aspect ratio wing. you could see that if aspect ratio is large, this value will be less right. So this effect will not be Delta E required will not be very large, but if aspect ratio is small, for the high speed airplane this Delta, Delta E will be pretty high, I repeat you could see here, this Delta Delta E, is inversely proportional to the aspect ratio.

Aspect ratio is more than Delta, Delta E required will be less, you have taken a aspect ratio 5 because, typically this phenomenon will be for an high speed airplane, what called is recognizable right. So, if I now put Delta, Delta E this is equal to  $-C_L$  that is  $-1.5$  by pie, let's say  $\bar{C}$  is point 5, pie is 3.14 and aspect ratio is 5. So typically these value if you calculate, it will come 10.9 degree, because this will be radian and then multiply 57.3, you will get in degree, 10.9 degrees. What is the message?

Message is if you have Delta E max limit, is let's say  $\pm 25$  degree, then please remember that  $25 - 10.9$ , let's say 14 that is 14 degree is only available for you to trim in free air, while landing because, you know as I come to land right, I need suddenly additional of an 11 degree up, so this is - sign, 11 degree up, so if that time if I have trimmed this whole airplane at 25 degree at for the landing, and trim the airplane at 25 degree, the moment I come closer to the ground it will demand another 11 degree to be taken up.

But, the movement I put 11 degree over 25 degree, the elevator will not work because it will stall. So what is the message you find out what is the Delta, Delta E required. So, whenever you are trimming before you come down to landing, make sure from your maximum elevator deflection allowable, this much angle is left reserved and you land there only okay. That is how you plan your maneuver, and this is strictly true for very significant for lower aspect ratio aircraft that is high speed airplane okay.