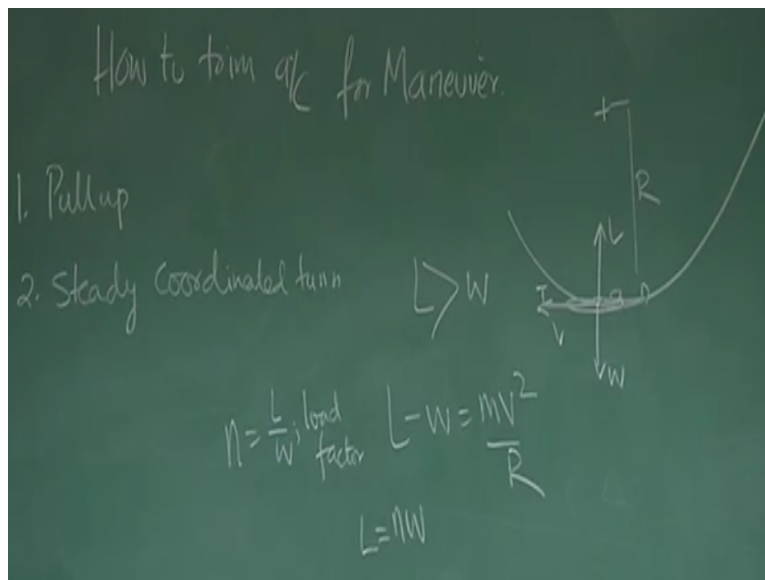


Aircraft Stability and Control
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Lecture- 17
Trim: Maneuver

So far we were discussing about cruise climb landing, now let us talk about how to trim aircraft for maneuver. We will take two cases one is pull up, another is steady coordinated turn, we will be discussing how to trim an aircraft for maneuver one is pull up.

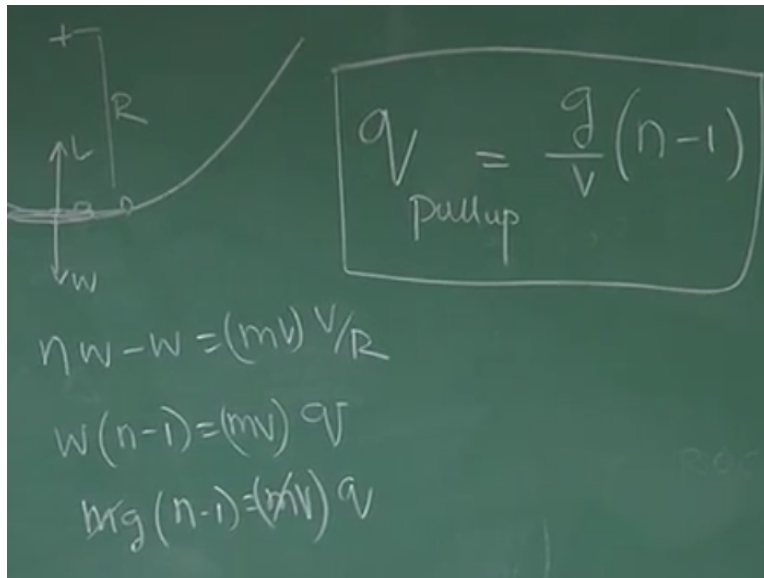
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And another is steady coordinated turn right. What is the pull up? This is the airplane this is the V you have generate lift W, this is the thrust this is the drag, and you know if I want to go for radius R then lift has to be greater than W. So $L - W = M V$ square upon R, you are all familiar with this, because this difference in lift and weight is only giving centripetal acceleration, and which is model through every square upon R.

And we know L is $= NW$ for in a sense we say N is L by W that is called load factor, the advise to read my first course for introduction to performance, if you want to know more about this thing, but I will just glance through this so that there is continuity.

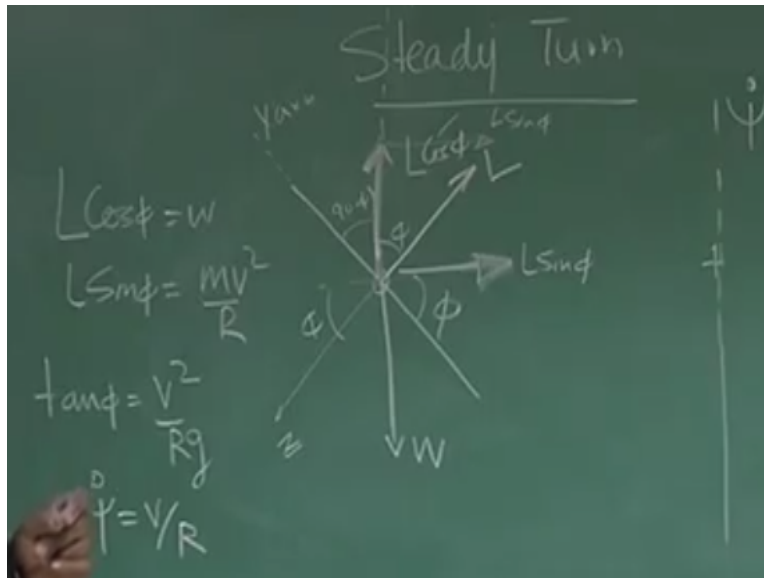
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So now from this relationship what I get $NW - W$ is $= MV$ into V by R or $WN - 1$ is $= MV$ into QV by R is what? Is the pitch rate okay, this is the V and this is the DSR, so what is the angular rate, that is Q . So this angular rate is about remember about Y axis correct? So, now I have replaced V via by Q , and I can write this $MG N - 1 = M V$ into Q MM gets canceled so, I get Q for pull up is G by $V N - 1$.

What is the meaning of this? If I plan to go for a load factor of N , at a particular speed V , then the pitch rate will Q given by this relationship right. So, if I want to go for a same load factor at lower speed, then the Q should be higher and you have to generate this Q who generates this Q ? It is through the elevator okay.

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This is one for pull up, similarly if I come for steady coordinated turn to lift this is W and this is your Y axis, this is your Z axis, YZ what are body fixed axis so, the airplane has banked by ψ so body fix axis also tilted by this for example, this is the airplane and let's say this is the Z axis this is the body fixed axis so, if the airplane banks the Z axis also rotates, that is how you are seeing this Z axis right? Similarly Y axis,

If this true it is taking a turn about with $\dot{\psi}$ about a fixed vertical axis Z , Z earth axis I say, rotating through about that axis, and that notation is $\dot{\psi}$, turn rate is $\dot{\psi}$, it is something like this it is banking like this, and turning like this okay. So, this is the definition of $\dot{\psi}$.

Now if I write the equation, I find if it is coordinated, that we should not also lose the height, so, this is $L \cos \psi = W$, ψ must have $L \cos \psi = W$. So that $L \cos \psi$ component of lift should balance the weight, and $L \sin \psi$ component was $= \frac{MV^2}{R}$ because this $L \sin \psi$ component, it has two component one this, and one this, it is the $L \sin \psi$ component, that is going to give you $L \sin \psi$, if I write it here that is responsible to give you centripetal acceleration right.

So, this is $= \frac{MV^2}{R}$ so, I can write $\tan \psi = \frac{V^2}{Rg}$ this state forward dividing this by this, and we also know $\dot{\psi} = \frac{V}{R}$ that is true let me write two expressions so, that we do not get lost unnecessarily so, $\dot{\psi} = \frac{V}{R}$ what is Q ? will there be any Q in

this see 1 thing you understand, if the airplane is like this and taking a turn like this then there should not be any Q that is local Y axis, local body Y axis but if the airplane is bang like this, then it is turning like this, then there will come a component about local Y axis which is the along the span of the airplane.

You could see that this is Y axis, and this is the SI Dot, if I draw a SI dot here parallel, there will be component along Y axis, and that is = SI Dot sign 5. you see this, any one component will come here, and that is nothing but this is 5 this angle is 90, this is again five you can find out yourself, this is 90 - 5 so, it 90 - 5, the COS component will become SIN component, because SIN COS 90 - 5 SIN 5, so Q = SIN out SIN 5 however physically also you could see.

If I am turning like this there is no motion of like this, but you have a banged like this, and my axis is this one, now if I turn I am actually turning about Y axis also, that is if you could check it, if five is 0 then, there will not be any pitch rate because of SI dot. But if there PI there will be Q because of five because of SI dot okay. We know we will substitute free expression, and try to get Q for steady coordinator in terms of load factor and velocity.

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Handwritten mathematical derivations on a chalkboard:

$$q = \dot{\psi} \sin \phi$$

$$\tan \phi = \frac{v^2}{Rg}, \quad R = \frac{v^2}{g \tan \phi}$$

$$q = \dot{\psi} \sin \phi$$

$$R = \frac{v^2}{g \tan \phi}$$

$$q = \frac{v}{R} \sin \phi$$

So you will do some manipulation, we have seen that TAN 5 = V square by RG or R = V square by G TAN 5 right. This is what we have derived. Now what is Q? Q is SI Dot, SIN PI and what

is SI Dot? SI Dot is V by R SIN PI okay. This part we have come, after that $Q = V \text{ by } R \text{ SIN } \text{PI}$ and $R = V \text{ square by } G \text{ Tan } \text{PI}$, if I put R here I should get expression.

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$$\cos \phi = \frac{1}{n}; \sin \phi = \frac{\sqrt{n^2-1}}{n}; \tan \phi = \sqrt{n^2-1}$$

$$q = \frac{P}{V} \sin \phi \tan \phi$$

sin ϕ \Rightarrow tan ϕ = ?

$$L \cos \phi = W$$

$$\boxed{\frac{L}{W} = \frac{1}{\cos \phi} = n} \quad \text{load factor}$$

If I put R here please do yourself R here, then I will get $Q = G \text{ by } V \text{ SIN } \text{PI}$, $\text{Tan } \text{PI } Q = G \text{ by } V \text{ SIN } \text{PI Tan } \text{PI}$ and $\text{SIN } \text{PI}$, is what $\text{COS Tan } \text{PI}$ is what? if I know this I can put it here, I can get this expression for Q one thing I know, that $L \text{ COS } \text{PI} = W$ or $L \text{ by } W \text{ is } = 1 \text{ by } \text{COS } \text{PI}$, this is $= N$ for the load factor, no objection you know that $L \text{ COS } \text{PI}$ was balancing the weight, so $L \text{ by } W$ will be $1 \text{ by } \text{COS } \text{PI}$ and $L \text{ by } W$ is nothing but load factor, so I can write load factor is $= 1 \text{ by } \text{COS } \text{PI}$, what is the meaning of that?

If you are turning and not losing your height or altitude, then you can easily find out what is the load factor, by just taking $1 \text{ by } \text{COS } \text{PI}$ as a computation okay. So now what our aim? Our aim is to see here, it's very straight forward $\text{COS } \text{PI}$ you know $1 \text{ by } N$, so you can show that $\text{SIN } \text{PI}$ is $=$ on the root, $N \text{ square} - 1 \text{ by } N$ and $=$ show $\text{Tan } \text{PI}$ is $=$ on the root $N \text{ square} - 1$. So if I put all these things here then.

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$$q_{\text{pullup}} = \frac{g}{v}(n-1)$$

$$q_{\text{scT}} = \frac{g}{v}\left(n - \frac{1}{n}\right)$$

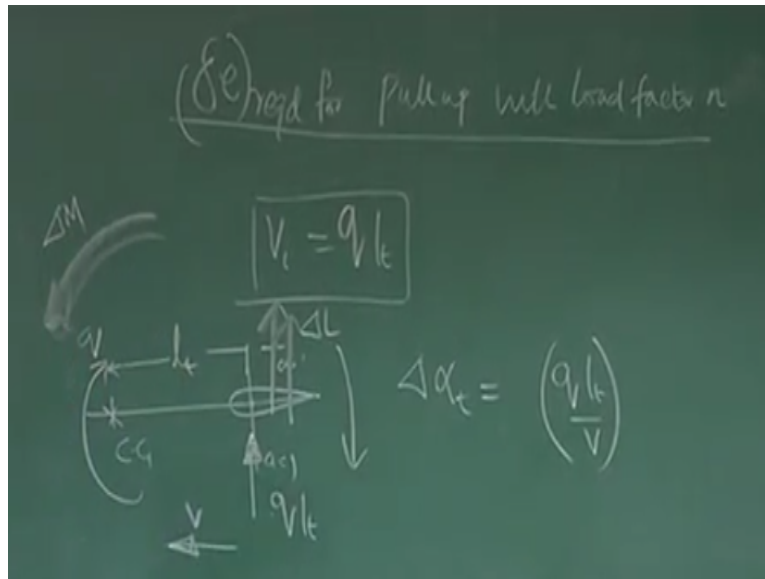
I will get Q for I write here, Q for steady coordinate turn, as G by V N - 1 by N. Let me repeat this is little bit of algebraic manipulation, and I will always suggest you do yourself, but let me repeat for your clarity, Q is SI Dot SIN PI this you have understood, Tan PI V square by RG already you have seen, so R is this so in Q PSI Dot SIN PI PSI Dot nothing but V by R, so I have put V by R here and SIN PI is here, for R we substitute this expression to get Q as G by SIN PI Tan PI.

We wanted to find SIN PI and Tan PI what are their expressions, we know L COS PI = W, so we know N = 1 by COS PI, so COS PI = 1 by N, so you can find SIN PI and Tan PI substitute there, and will get expression this. So, what is the meaning of this? Let me erase all these terms all, once again I repeat.

If you have any difficulty please read my first course, most lecture you will get all any standard textbook, you take now we are coming back to the control part of it. What sort of elevator deflection you should give? Before we decide what sort of elevator deflection you should give, let us again relook what are these, what is this Q pull up? What are these say, If I want to go for a load factor of N pull up like, this then I need to generate this much of Q, and let's say if you want to go for steady coordinator turn, for a given load factor N.

Then I need to generate Q given by this expression at particular speed, and also realize that if the speed is lower, then Q will increase, but we also know that there is a limit on V , it cannot be less than V_{stall} okay. Now what is the question?

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So, what is the ΔE required for pull up for pull up with load factor N right? So this is important please understand, what I am going to say now. We are looking for ΔE required for pull up with load factor N , what is the pull up the airplane going remember this is the fuselage, assume there is a wing and this is the tail right.

White part is the tail so, I am going in a cruise then I have to pull up means, I have to generate a Q so I do like this, the moment I am giving a Q and this Q is proportional to whatever load factor I want. So where is I give you a Q what is happening? This tail is going downward about CG, going downward so, it is it will be seeing induced air component, which are like Q / LT right. This is the induced V , relative that speed right? This is going Q up this is going down LT is the distinguish CG, and AC of the tail right, if I draw it this is the tail, this is the AC of the tail.]

And this is the CG going for a Q like this, so it is going down like this so, there is a relative air speed which is Q into LT , where LT is the tail moment arm we know that and this is Q . We could see from here, if this airplane is going for a pitch rate Q , then the tail is going downward like this, so they will relative a Q / LT that $V = \omega$ right, that will be seen at the AC of the wing,

So and this airplane is also moving forward, so $\Delta \alpha_T$ tail will be $Q L T$ by V , this much of additional angle of attack,

Will be seen by the tail whether the airplane is doing a pull up like this, that is when there is a Q and the moment $\Delta \alpha_T$, which is now more than the α_C during cruise, because he was cruising like, this then he is doing the pull up, so have to again put additional elevator so that this $\Delta \alpha_T$, which will cause a nose down moment is now taking care right.

So I have to give more ΔE , that this $\Delta \alpha_T$ will give some ΔL and that will give some ΔM . So If I want to trim at Q , I must ensure that this additional ΔM should be corrected by elevator deflection, over and above whatever ΔE was there for cruise this part is clear? I repeat again when I am cruising, I have put some ΔE elevator deflection. Now I am pulling up, the moment I try to pull up then I see additional angle coming at the tail, so that additional angle of tail will give me nose down moment.

I do not want that I have to counter it. So I have to give the elevator deflection little more up, so that I counter whatever change of $\Delta \alpha_T$ is there, so that will be my new ΔE required for a pull up. One thing we are understand I can say that in a pull up, since there is a additional moment nose down moment is coming, which is try to opposite so I say,

It is apparently increasing its stability okay. That is why I need to give additional elevator deflection, because it has become on the in the process, in that become more stable right. So now how I do I find what is the ΔE additional required if this is clear then the job is simple.

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$$\tau = \frac{\partial \alpha_t}{\partial \delta_e}$$

$$\tau \Delta \delta_e + \frac{q V l_t}{V} = 0$$

$$\Delta \delta_e = - \frac{q V l_t}{\tau V} = -1.1 \frac{q V l_t}{\tau V}$$

I know that what is TOW, TOW is $\frac{\partial \alpha_t}{\partial \delta_e}$, so I can write TOW into $\frac{\partial \alpha_t}{\partial \delta_e}$, the additional Delta E required to change Alpha T by how much amount? By $\frac{q V l_t}{V}$, If they balances, so I put = 0, so I get $\Delta \delta_e = - \frac{q V l_t}{\tau V}$. This is the additional Delta E up required for a positive Q. But generally what happens we have concentrated only tail, if it is customary you multiply by 1.1 to take care of wing fuselage which are ahead of the wing or wing fuselage combination,

It is the typical number; around 10% you can add right okay. So what is the method now? It should be very straight forward to you, what we have got.

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$$\Delta \delta_e = -1.1 \frac{q V L_t}{Z V}$$

$$\delta_e = \delta_{e0} + \frac{d\delta_e}{dq} q_{trim} - 1.1 \frac{q V L_t}{Z V}$$

The Delta Delta E, required to trim at maneuver is $-1.1 Q L_t$ by TOW V, I repeat Delta Delta E required, the additional elevator deflection required, for a pull up by Q is $-1.1 L_t Q$ TOW V right. So what was at during it was Delta E0, + D Delta E by DCL Trim into CL trim right? Now what will happen? Because of this Q I have to add this also right.

This is clear? So this is the additional thing you have to add to whatever you are in the cruise whatever elevator deflection of the cruise, the moment you want to go for a pull up, I have to calculate what is the value of Q, and with this expression I must give additional Delta E to trim for that pull up maneuver, this is the understanding okay. So if I ask you a question if I am going for a pull up then what Q I should use for pull up I should use this Q, if I am going for a steady coordinator turn which Q I should use? I should use this Q that's all as simple as that that.

That answers the question how much Delta E required to trim the airplane for the particular pull up, or particular study coordinated turn right for a given load factor N correct.

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$$\Delta \delta_e = -\frac{g l_t}{\tau V} \times 1.1$$

$$(\Delta \delta_e)_{\text{pull up}} = -1.1 \frac{l_t}{\tau V} \cdot \frac{g}{V} (n-1)$$

$$\Delta \delta_e = -1.1 \frac{g l_t}{\tau V^2} (n-1)$$

$$(\Delta \delta_e)_{\text{SCT}} = -1.1 \times \frac{g}{V} (n-1) \frac{l_t}{\tau V}$$

So, Delta, Delta E you are trying to find out what will be the Delta, Delta E for pull up. We know that Delta, Delta E is given by $-Q \text{ LT by TOW } V$, and into 1.1, we agreed for giving some consideration for fuselage and wing etc. So then I should know this $= -1.1 \text{ LT by TOW } V$, and for pull up I will pick up the expression of Q for pull up,

And that is $G \text{ by } V \text{ into } N - 1$, so effectively it becomes $-1.1, G \text{ by TOW } V \text{ square LT into } N - 1$. This is the Delta, Delta E required for pull up meaning thereby for a given velocity, if you are going for a pull up, over and above if you are cruise Delta E, I have to give this much of elevator deflection up.

So Similarly, for Delta, Delta E for a study coordinate turn, will be again -1.1 , now for Q I should take the expression for, study coordinate turn which is $N - 1 \text{ by } N$, So $-1.1 \text{ into } Q \text{ into LT divided by TOW into } V$. So what is expression then becomes, so Delta, Delta E for SCT $= -1.1, \text{ GLT by TOW } V \text{ square, into } N - 1 \text{ by } M$. What was for pull up? it was $-1.1 G \text{ by TOW } V \text{ square, LT into } N - 1$ but here it is $-1 \text{ point GLT by TOW } V \text{ square } N - 1 \text{ by } N$, that is the difference.

So let me write it neatly as the final result, because we will be using this in your next class. I repeat please all of you derive this yourself, unless you derive this yourself in pen and pencil, it will not go to your mind.

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Pullup

$$\delta e = \delta e_0 + \frac{d\delta e}{dC_L} C_L - \frac{1.1 g h}{\tau v^2} (n-1)$$

$$C_L = \frac{1.1 W}{\frac{1}{2} \rho v^2 S} \quad L = 1.1 W$$

$$(\Delta \delta e)_{\text{pullup}} = -\frac{1.1 g h}{\tau v^2} (n-1)$$

$$(\Delta \delta e)_{\text{SCT}} = -\frac{1.1 g h}{\tau v^2} (n - \frac{1}{n})$$

$$(\delta e)_{\text{SCT}} = \delta e_0 + \frac{d\delta e}{dC_L} C_L - \frac{1.1 g h}{\tau v^2} (n - \frac{1}{n})$$

So what you have to derive this Delta, Delta E or pull up is given by -1.1 , GLT by TOW V square into $N - 1$ and Delta, Delta E for steady coordinate turn, as -1.1 GLT by TOW V square into $N - 1$ by N , so what happens?

If I am trying to find out how much Delta E, you have to give for a pull up? Total one part will be because of cruise, which is already there Delta E $0 + D$ Delta E, by DCL into CL now it will come -1.1 GLT by TOW V square $N - 1$ and what is the CL here? CL is nothing but, NW by half row V square S, because $L = 1.1 W$ so CL will be NW by half row V square S, this is for pull up okay. Similarly, Delta E for steady coordinate turn, already equilibrium cruise that much Delta E is required by DCL into CL.

Where CL is again NW by half row V square S, and then -1.1 GLT by TOW V square $N - 1$ by N . This is our final finding, So, I block it here, focus here. Let us come back here, so if $N = 1$, that is just cruising Lift is = weight, how much Delta E is required? They will require Delta E 0 , D Delta by DC into CL trim. CL trim is nothing by CL cruise with $N = 1$ right.

But as N is increase, then Delta E has more negative component from here that is you have to put elevator up and up. Similarly proof here for steady coordinate turn right. The difference is coming only because, the Q R different one is G by $V N - 1$ and, G by $V N - 1$ by N right. One thing you could see loosely as a designer, if N is very large, then $N - 1$ becomes N and $N - 1$ by N also becomes N .

So that time the Q both the Q 's are same right as n increases then Q will remain same so, the ΔE is required additional will be complete will required will be always same right. That you should know that clearly. From here next lecture we will try to find out what is the CG location at which $D \Delta E / DN$ goes to 0, because CL for CL I put in here, here N is there if I find this gradient that is $D \Delta E / DN$, $D \Delta E / DN$ that is for different load factor how much, what is the gradient of ΔE in terms of load factor increase or decrease.

That derivation we will find out and will see something very interesting, we will find there is a CG location, at which $D \Delta E / DN$ becomes 0, and that will define it as stick fix maneuvering point.