Aircraft Stability and Control
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Lecture- 20
Revision

Good morning friends, we have been doing so many activities, we try to understand what is static stability, we try to understand how to trim a particular aircraft at a particular CL, or CL trim. We also understood the significance of neutral points, stick fixed neutral point and on the stability aspects we have solved one problem and as I promised that we will have a session where, we will more focusing on how to implement.

Whatever we have derived, in understanding Delta E required or how to calculate CM0, how to design an airplane to have a particular CM0. What should be the elevator deflection required? To trim an aircraft while it is climbing, while it is cruising, while it is landing, we also know what happens when the aircraft lands very close to the ground. With this background we will try to solve, one or two examples may be one example in detail today. So before I solve a problem or I discuss a problem.

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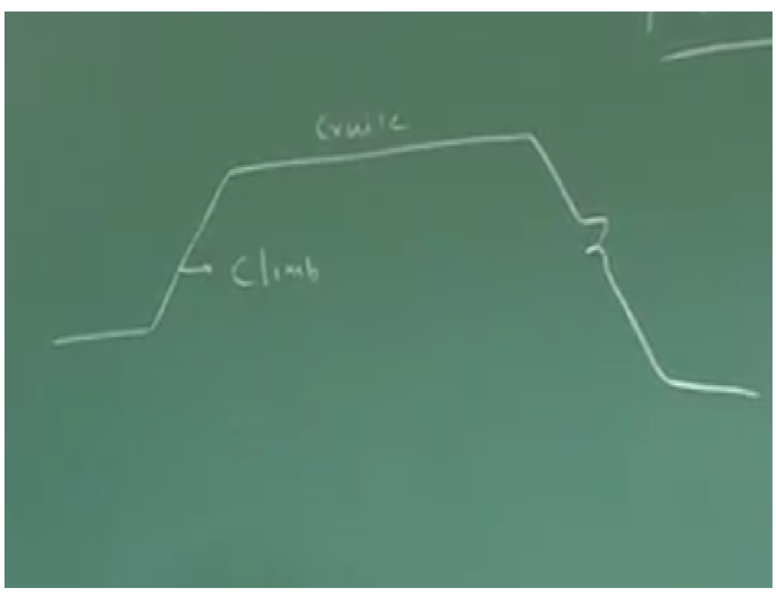

Let us again revisit few basic understanding this part is climb, this is cruise and this part is landing all you know. Now, when I am climbing, what happens?

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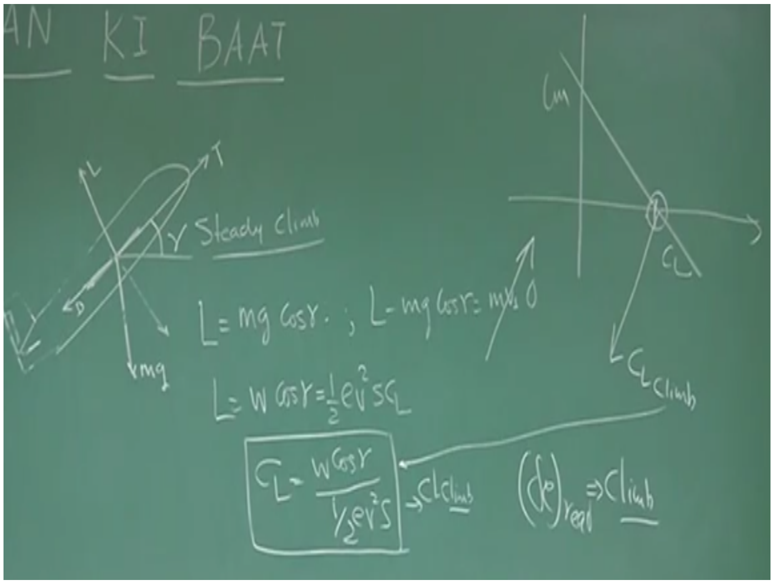

This is the wing, this is the CG, this is your MG acting here or the weight, and here it is drag, here it is thrust, here it is lift. We have assumed that, the thrust axis is along the fuselage reference line, we are also neglecting the angle of attack, or not neglecting we are saying the angle of attack is small, okay. And we are trying to develop a model for steady climb with no acceleration right.

It has going like this rectilinear motion like this, so if I use this diagram I know this is GAMMA, fight path angle and then you know, if I want to ensure that this plane is going in rectilinear direction then net force on the body across this line should be 0. So, no acceleration here no acceleration along this no acceleration perpendicular to this. We are assuming the airplane is moving in a vertical plate okay.

So if I do that and I will find L should be = MG COS GAMMA, because steady climb, steady climb means you know that L minus MG COS GAMMA = M into acceleration in the perpendicular direction, but this we are putting zero because, it is going in rectilinear motion and unaccelerated steady climb. So L = MG COS GAMMA or which we know L = W COS GAMMA and what is L?

L is nothing but half row V square SCL half row V square SCL so I get CL = W COS GAMMA by half row V square S. This is my CL climb okay. So, we are trying to model, or we are trying to ask a question, if this airplane is to remain in this rectilinear path, then what should be the CL with which I should fly? And you know if it is suppose to be in rectilinear path then the forces along this path, and forces perpendicular to this path should be balanced.

Net force along this two direction should be zero okay. And in this case, please remember what we are asking is, this is fine this diagram tells you what CL I should fly? But what is our question? Our question today we have become mature enough to ask another question is for this airplane, if I should follow rectilinear path, but it should be trimmed, trim means not only these forces should be 0, but it should have net CM 0, okay. That is what we call the airplane is at trim, okay.

That is this point CM is 0 and what is this CL? This CL is CL climb, and how do I find CL climb? I use this relationship. What is my next question is what is the Delta E required for this climb? Is this question clear? fine right. And we will just formulate a problem to solve this, and we will very well understand what I am talking about and what is the relevance of this example.

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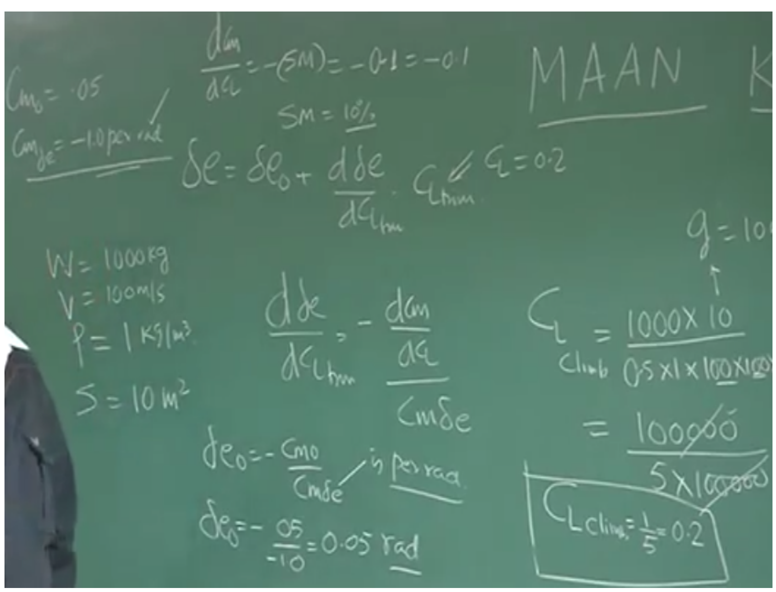

We know that Delta E = Delta E0 + D Delta E by DCL trim into CL trim okay. And what is Delta E by DCL trim is nothing but minus DCM by DCL by CM Delta E right. So let us take an example, let's say weight is thousand KG of the airplane right. Assume V = 100 meter per second is the climb speed okay. It is very high but for numerical advantage I am taking 100 meter per second, let's say row is 1 KG per meter cube, if it is at sea level will be 1.225 something like that, but I am taking 1 KG per meter cube and S I am taking as 10 meter square okay.

These are my input right? So, what is the CL required for climb we could see from here, I will erase this part. So CL climb will be how much? Will be W, W is 1,000 into 9.8 because this unit should be Newton, it is in KG, so I am taking the value of G as 10 meter per second square no problem. So put as 10, which is nothing but value of G, which is typically 9.8, 9.79 depending upon the location. But for computation of simplicity I am taking as 10 meter per second square.

So W, then half is point 5 into row is one into V is 100 square, 100 into 100 and into 10, 10 is your area. So how much it comes down to 1, 2, 3, 4, 5, point is going there, so here 5 into 2, 2, 4 and 5, 1, 2, 3, 4, 5, 2, 2, 4 and 5 and point 5 has gone here, so this is 1, 2, 3, 4, 1, 2, 3, 4 and for point 5 this has become 5, right. Here also 5, so this gets cancelled, so I am getting CL climb = 1 by 5 and that is = 0.2 correct?

So I have to fly at CL = point 2, so what is the next question? What is the Delta E required to trim the airplane at CL = 0.2 how do I find out? Now I know what is Delta E0 is minus CM0 by CM Delta E, so for this aircraft there should be a fixed value of CM0 and CM Delta E. So let's assume CM0 = point 0, 5 and CM Delta E = minus 1.0 per radian. So if that is the value I am assuming which are typical value for an airplane, so Delta E0 will become minus point 0 5 by minus 1.0 this = 0.05 radian.

Please understand this is radian because CM Delta is given per radian, in many problem we will find CM Delta will be given in per degree, so then Delta E0 will become in degrees okay. We should be very very careful about the unit, whenever these things are coming check, unit is per radian or unit is per degree okay. CM0 is this, now D Delta by DCL trim I want so further let's say DCM by DCL you remember DCM by DCL is given as minus static margin.

And typically let’s say the static margin is 10%, so this point 1, so this value is minus 0.1. So what is the static margin the airplane is having static margin is 10% okay. That means the difference between neutral and the CG is point one into C bar mean aerodynamic chord okay. So I know now you see for DCM by DCL the value is minus 0.1, and CM Delta is 1, so what is the Delta E by DCL trim?

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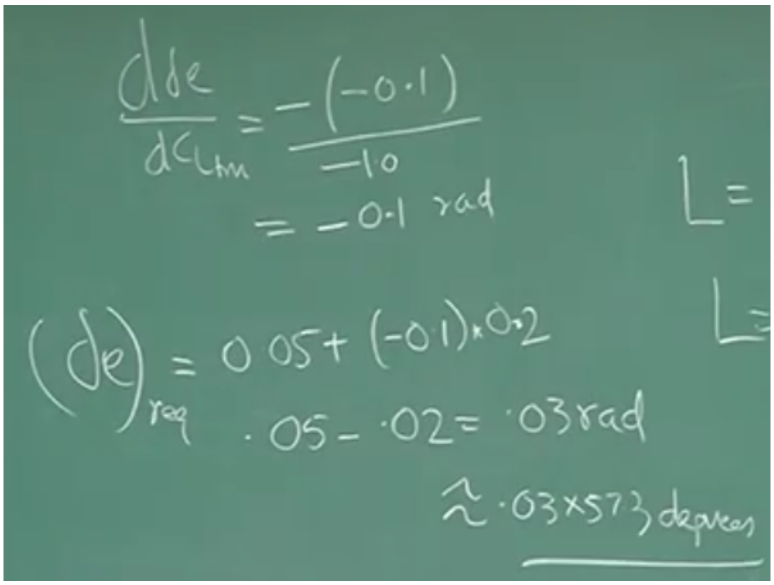

Let me write it here so D Delta E by DCL trim = minus of minus 0.1 divided by minus 1.0, so this = minus 0.1 radian. Please note that here also DCM by DCL then CM Delta is per radian so this becomes radian here also the unit is radian. So now, what is Delta E required? Delta E required will be Delta E0 that is 0.05 + D Delta E by DCL trim is minus 0.1 into CL trim is point 2 so what is Delta E required? Delta E required will be Delta E0 which is point 0 5 + D Delta by DCL trim which is minus point 1 into CL trim is 0.2 right.

So this becomes point 0 5 minus point 0 2 and is = point 0 3 radian, and which is approximately you can see point 0 3 into 57.3 this is degrees, right. So that will be around 1.8 degrees, correct. So that much of elevator deflection is required to trim the airplane during climb, which is climbing with CL = 0.2 correct. Now see if I want to solve a problem this problem little differently, now if we want to solve this problem little differently.

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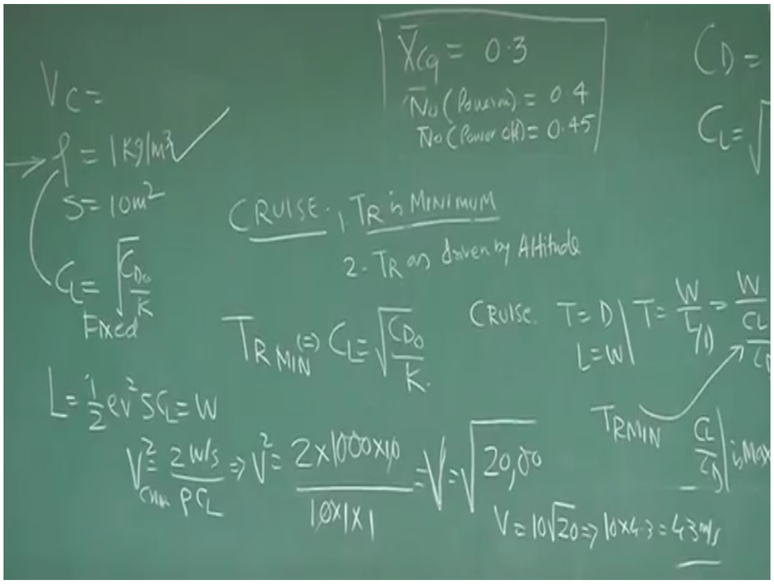

The information could be instead of given static margin 10% the information would be given like this, XCG the airplane is at point 5, or to say point 3 and neutral point power on is at point 4, and also it could be given neutral point power off = point 0 45, because you know power off is more stable compare to power on, we are assuming that engine weather it is in propeller driven engine or a jet engine they are ahead of CG, okay? So, if information is given like this, if static margin is not given directly, than you can easily find out DCM by DCL which is = minus static margin, which will be = minus of, how to find the static margin?

If the N0 minus XCG bar and the question will come, which N0 you will take N0 power on or power off? Obviously it's an airplane with engine, so when it is climbing like this power is on, so I will take, I will pick this N0, so I will write minus of point 4 minus XCG point 3 right. So this will be my static margin.

So I can solve this problem into different ways depending upon what information has been given to us, okay? So this is for the climb, now we will do the exercise, for cruise phase okay. Let's come back to the cruise, let us solve this problem for cruise case.

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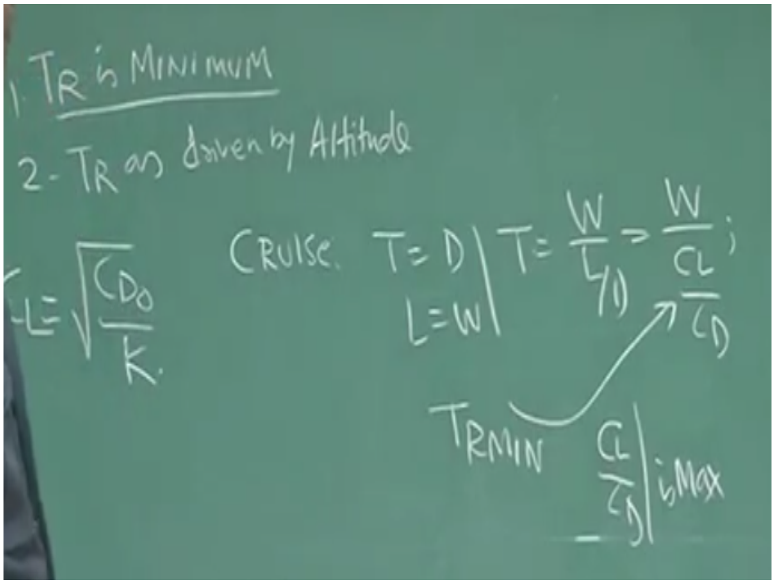

Let us see, I say that cruise speed is 100 meter per second, I say row is 1 KG per meter cube, that is at an altitude which is little higher than C level altitude, and of course same airplane is at 10 meter square right. So when I am talking about cruise, I can ask a question I want to cruise such that thrust required is minimum, okay. I can cruise such that the thrust required is minimum, one and I can also question thrust required is as driven by altitude, I will explain both the problem and we will solve both the problem.

So thrust required minimum the moment I want to solve this problem what is my question? question is what is that elevator required to ensure that the airplane when is flying, when it is flying at an altitude where row is 1 KG per meter cube and cruise speed is 100 meter per second, so what is that elevator required to trim the airplane with these condition, right?

So, now once we are talking about thrust required minimum that means you know CL = CD0 by K, I hope you remember this. Those who do not remember, for them for a cruise flight, for a cruise thrust = drag, lift = weight, so this implies thrust = W by L by D this = W by CL by CD, and that means thrust required minimum if for a given weight CL by CD is maximum and you know that.

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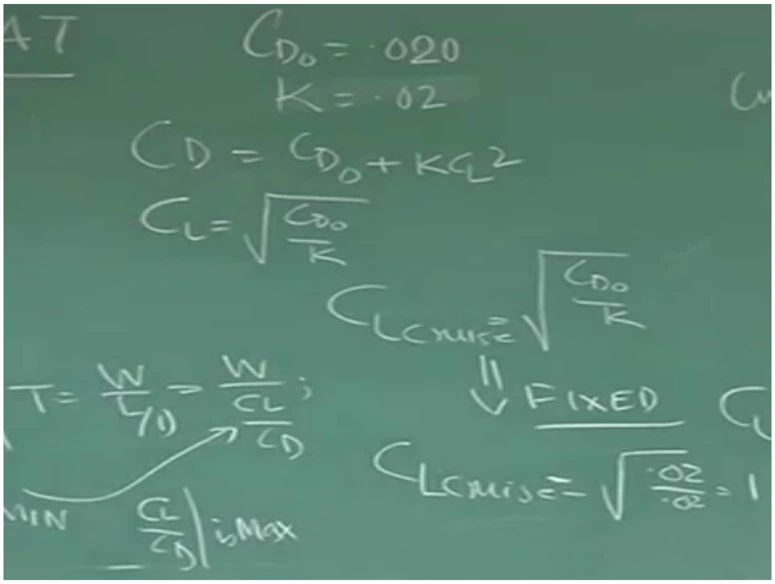

The condition which we get from drag puller that is CD = CD0 + KCL square I can show that for CL by CD to be maximum the condition is CL = under root CD0 by K, okay. So, now if I want to trim the airplane and cruise, so that thrust required is minimum it tells me the CL cruise should be even by CD0 by K. Now there will be a conflict if we prescribe what is the speed and we are telling what is the CL cruise.

Now see that's why this problem I have formulated. The moment I have to fly at thrust required minimum, the CL cruise is fixed why it is fixed? Because for an airplane CD0 and K are fixed for example CD0 let's say for this is point 0 2, or let's say point 0 2 and K we can find out, let me say is point 5. So we understand that is I want to fly such that thrust required is minimum. Then CL cruise will be given by CD0 by K, let's take for this airplane CD0 value is point 0 2, here, and K is point 0 2.

So then CL cruise we are getting as under root of point 0 2 by point 0 2 and that is one pretty high but for numerical example this is okay. The point what I am trying to tell you, if you decide altitude, right If you decide I want to fly such that thrust required is minimum, is minimum that means CL = CD0 by K.

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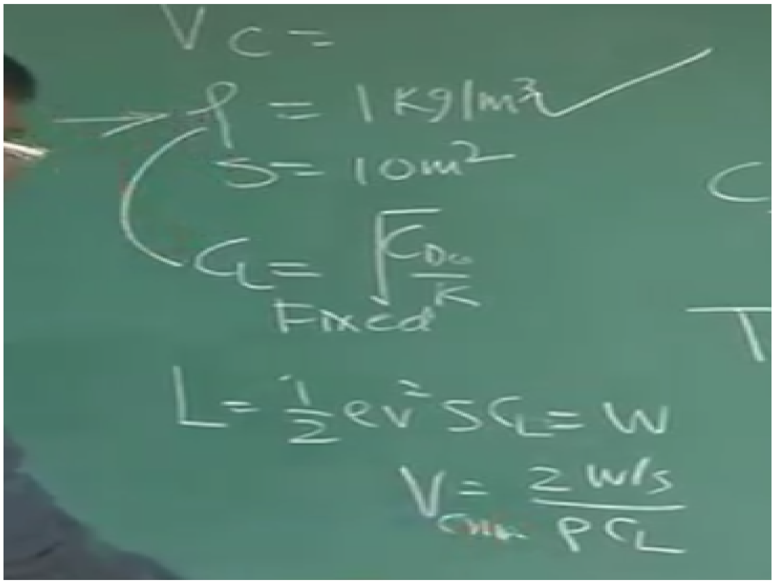

Then that means this is CL is fixed, because CL is = CD0 by K than you cannot keep cruise speed arbitrary it cannot be 100 meter per second, this has to be corrected, the cruise speed possible at that altitude to maintain thrust required minimum which gives CL fixed, VC will be or the cruise speed will get fixed by these two conditions, how because after all at cruise lift has to be = weight, right?

So lift will be = half row V square SCL that should be = weight. So V will be = 2 W by S by row CL, and what is the CL I will take? So this becomes the cruise, what is that CL I will take here for thrust required minimum? This will be this CL this is CL for thrust required minimum, that is one for this example, so I find V = 2 W is how much? W is thousand, so thousand into 10 value of G, approximately I have taken divided by S is 10, row is one and CL is one okay.

2 W by S row CL, so this gives me how much? This is of course V square = right? So V square to this, So this is, this 10 one goes here, so this is under root of so V will be = under root of 2,000. So this is typically = 10 root twenty and root twenty we could take around 4.2 or 4.5, So this will become around 10 into 4.3, so around 43 meter per second, right. If my calculations are okay, so it will be around 40 to 45 meter per second cruise speed.

What is our observation? We said that, to fly at thrust required minimum at a given altitude where row is 1 KG per meter cube, then, what we have.

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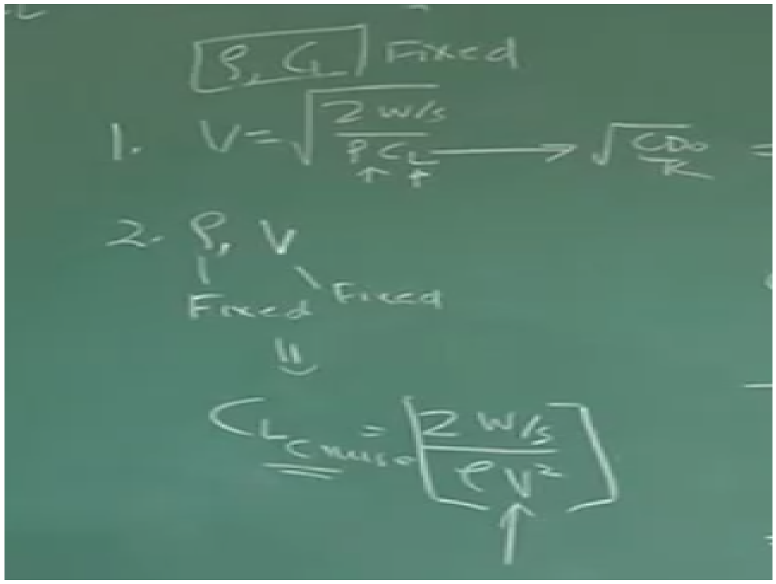

To fly will be governed by this relationship which comes from lift = weight and that is 2 W by S row CL as CL corresponds to CL = CD0 by K. So once you have fixed row and one CL is fixed from the conditions as required minimum, then the V cannot be arbitrary V will be driven by these two values for a given wing loading, right? So an incidentally for this case V is coming 40 to 45 meters per second.

Second problem is you fix a row right and you fix V, what was fixed here? Row was fixed, CL was fixed right. I repeat row was fixed because the altitude was fixed, which corresponds to density 1 KG per meter cube, CL was fixed, how CL was fixed? CL was fixed because CL has to correspond to thrust required minimum, which is given by CD0 by K under root. So first case row and CL was fixed so naturally since we are cruising lift = weight, V automatically get fixed that becomes V = 40 to 45 meter per second for this example right. But for the second example if I say I keep row fixed and V fixed clear.

I want to cruise at 100 meters per second at altitude where density is 1 KG per meter square, so this also will give me, what is the CL cruise required as 2 W by S by row V square right. If it is 100 meter per second we put 100 here, row value W by S we put here, we will get CL cruise, any altitude, but it is not necessarily CL for thrust required minimum, for CL thrust required minimum your V has to be related to row and CL because you are not changing the altitude.

Now this is what the CL concept, now what is our main aim? I want to know what is the Delta E required for let's say CL corresponding to thrust required minimum okay. And we have seen.

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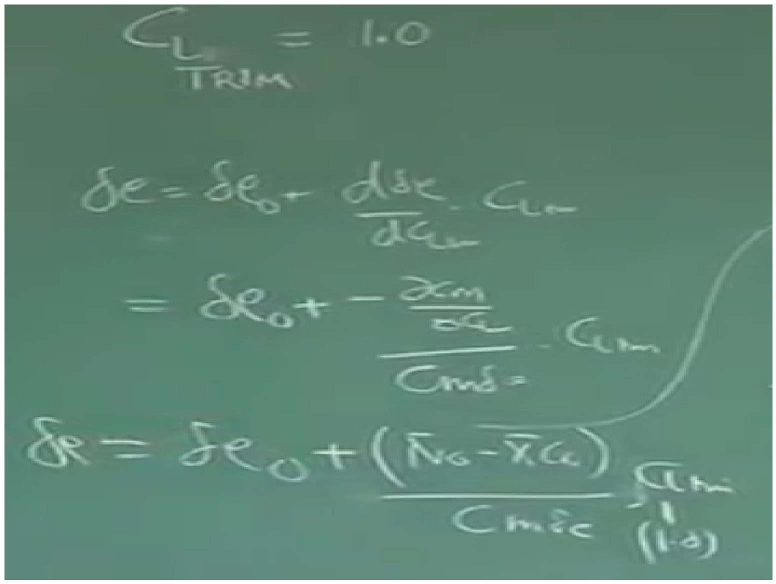

We have just now seen CL required, CL required for thrust required minimum is 1.0. I repeat this hand value, because of numerical convenience I have taken some values but typically CL cruise at a appropriate altitude for our jet driven airplane and all which flies at an around 11 kilometers, CL cruise will be point 2, point 3 not more than right. But this is an example.

So CL required is or I would say CL trim is one, so we know Delta E = Delta E0 + D Delta E by DCL trim into CL, and this = Delta E0 this value also you know and this is minus DCM. So we are trying to find out what is the Delta E required to trim the airplane at CL trim = 1.0 which corresponds to thrust required minimum during cruise. So you know Delta = Delta E0 + D Delta E by DCL trim into CL trim, which again I can write as CM Delta E into CL trim and this = Delta E0 + - DCM by DCL means this will be N0 bar minus XCG bar by CM Delta E okay.

This is cruise so N0 bar we will take from here power on point 4 and every numbers I know. So I can find out what is Delta E required, As simple as that, the trick was what is the CL I need to trim? If it is CL for any other combination, that CL has to be put for cruise, so this is into CL trim and CL trim is you know it is one in this case for this numerical that we are solving. So I am sure that you all will be able to complete this.

Now the question comes, suppose it is now coming for landing let's say, it is coming for landing and or without ground effect okay.

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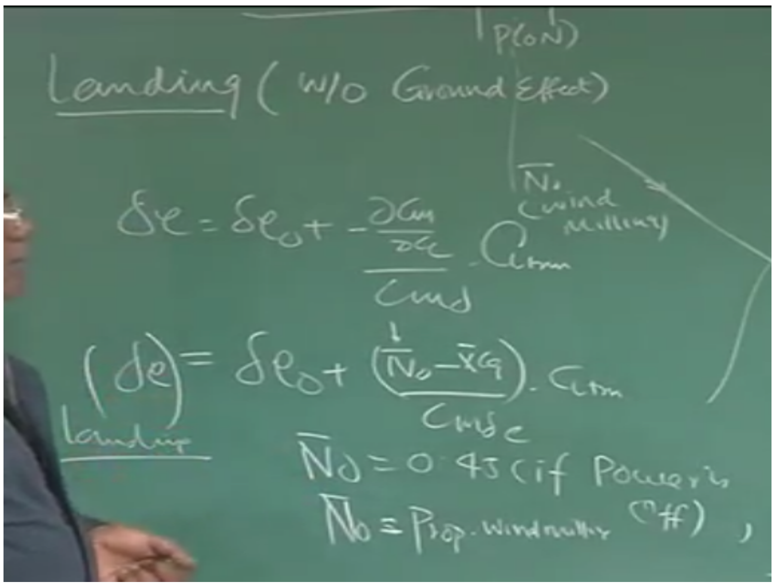

I am solving this problem without ground effect that means for a distance which is ground distance is more than 1 to 2 span of the airplane so without any ground effect because you know with ground effect I have to give additional Delta E so we not talking about that so then again that Delta E will = Delta E0 + DCM by DCL - CM Delta E into CL trim and this again = Delta E0 + N0 bar - XCG bar by CM Delta E into CL trim so what will be the CL trim required for coming for landing.

That will be again governed by same is coming like this it should follow rectilinear part it will be again the way we solved for climb you have to solve like this same equation will come so you can easily find out CL trim only point would be what N0 I should take so I will come back to this but physically see what happens when I am coming for landing I am actually trying to land at a lower speed possible right? Little more than the start speed.

So I increase the angle of attack and reduce the speed and start landing okay. So the CL that is why very high so the CL could be around point 8 1 during landing 1.2 so statically we talk about 1.2 for normal airplane it is safer to land at point 8 that is good enough by the CL but the question is when I am landing like this I have two way I can land one is theoretically speaking I put the engine off, okay and land like glide more I land is coming like a gliders this ending like a glider or I keep the engine at 10% or I have not love the propeller.

So when it is landing because of wind mill effect you know the wind is rushing at relative airspeed and the propeller will start rotating that will generate some crust okay. Typical windmill examples which life saving concept for helicopters right okay. You must do a net surfing read more about wind milling right? So, the question what I am asking is what N0 should I take for landing and when I am thinking about landing definitely thrust is not fully on it 10% 20% or wind milling.

So if you have switch off the engine typically gliding phase then you have to take N0 as N0 power off point 45 whatever value is given for the airplane. So I will take N0 bar as point 4 5 in this example if during landing power is off rest everything remains the same but if it is the pilot has initiated the wind milling that is he has not put the break on the propeller rotating because the relative wind is coming and some thrust is being generated.

Then this N0 should be propeller wind milling and that value I need to have so let's say that value will be propeller wind milling will become that value will be in between point 4 and point 5 what is it this is power on N0 this is N0 bar power on this is N0 bar power off more stable so propeller wind milling will be within this N0 wind milling okay. So between these two number it could be around point 4 2 the point is whatever the number be you need to understand N0 for propeller wind milling.

Will be between these two somewhere and when I am calculating Delta required for landing so I will be taking N0 corresponding to propeller wind milling we will not take N0 correspond to power on, is it clear? And that is what you should be very, very careful okay. Thank you very much.