

Aircraft Stability and Control
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Lecture- 27
Stick Free Stability Continued

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$C_{l_0} = 0$

$$C_{l_t} = C_{l_{\alpha_t}} \alpha_t + C_{l_{\delta_e}} \delta_e$$

Hands off

$$C_{l_t} = C_{l_{\alpha_t}} \alpha_t + C_{l_{\delta_e}} (\delta_e)_{float}$$

Let us also go back we have till this C_{l_t} will be what? C_{l_t} Alpha tail into Alpha tail + C_{l_t} Delta E into Delta E, we are assuming that C_{l_0} is 0 assuming things to be symmetric or polysymmetric. Now, we are assuming C_{l_0} , first let me write this is equal to 0. Now see at trim when hands off or I don't write at trim level, let's say in a hands off condition, what will be the C_{l_t} ?

C_{l_t} tail will be C_{l_t} Alpha T into Alpha T + C_{l_t} Delta E into Delta E float, right? That is the final C_{l_t} you required and that is why I will writing a trim that is the C_{l_t} trim, at that trim your C_{l_t} will be C_{l_t} Alpha tail into Alpha tail + C_{l_t} Delta E into whatever Delta E has floated.

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$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \left(-\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \right) \alpha_t$$

(delta_e) float

Now how much Delta you will float? that I know $C_{L_{tail}} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e$, into for Delta E float I write $C_{h_{\alpha_t}}$ by $C_{h_{\delta_e}}$ into Alpha T, let me repeat let me repeat again, at Alpha T Delta E also will float so, whatever lift or lift coefficient the tail is seeing will be $C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e$ float okay.

And that I am writing here as $C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e$, instead of Delta E float we are putting this term expression, right. So finally what is happening? Finally it is interesting to see what is happening then.

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$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e$$

$$C_{L_t} = C_{L_{\alpha_t}} \left\{ 1 - \left(\frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}} \right) \left(\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \right) \right\} \alpha_t$$

Hands off

$$\frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}} = \frac{\partial C_{L_t}}{\partial \delta_e} = \dots$$

$C_{L_t} = C_{L_{\alpha_t}} \alpha_t + C_{L_{\delta_e}} \delta_e$

I can write $C_{L_{tail}} =$, let me write this $C_{L_{\alpha T}}$, One - $C_{L_{\Delta E}}$ by $C_{L_{\alpha T}}$ into $C_{H_{\alpha T}}$ by $C_{H_{\Delta E}}$ into α_T , this is clear? From here I have taken α_T common $C_{L_{\alpha T}}$ here, and I have taken $C_{L_{\alpha T}}$ also common so, that one $C_{L_{\alpha T}}$ will come downward and rearrange this term, I could write it like this and what is $C_{L_{\Delta E}}$ by $C_{L_{\alpha T}}$? And what is $C_{H_{\alpha T}}$ by $C_{H_{\Delta E}}$? We should not forget that this are related to hinge moment properties, and these are with the elevator tail combination that property okay.

So now you know that $C_{L_{\Delta E}}$ by $C_{L_{\alpha T}}$ is nothing, but $D_{\alpha T}$ by $D_{\Delta E}$, and which is in notation wise we say Tow , what is the physical meaning of this ratio? its comes mathematically $D_{\alpha T}$ by $D_{\Delta E}$, that means per unit change in elevator deflection, how much tail angle of a attack effectively changing. This is very important performance parameter in design in terms of tail performance parameter, and that is denoted as tow ,

And you will see that when you solve an example, this tow can be estimated through some empirical relation, which goes as I will give you a right figure this it varies with a SC by ST , that is elevator area to tail area, and typical value for most of the air plane into vibrant point 4 to point 6 that is good enough for value, okay This exact numbers will be using it to the odd numericals, Now let us come back to this if this is the understanding, where we are approaching to.

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The image shows handwritten mathematical derivations on a chalkboard. The main equation is:

$$C_{L_t} = C_{L_{\alpha_t}} \left\{ 1 - \tau \frac{C_{H_{\alpha_t}}}{C_{H_{\Delta E}}} \right\} \alpha_t$$

Below this, there are several other expressions:

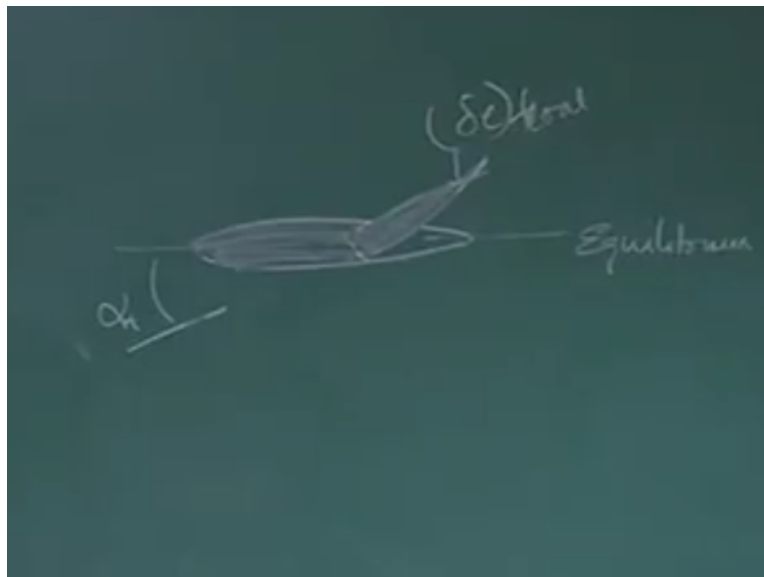
- $C_{L_t} = f C_{L_{\alpha_t}} \alpha_t$ with $f < 1$
- $C_{L_t} = C'_{L_{\alpha_t}} \alpha_t$ (circled and boxed)
- $C'_{L_{\alpha_t}} > C_{L_{\alpha_t}}$
- $C'_{L_{\alpha_t}} = f C_{L_{\alpha_t}}$

So I can write CL tail as CL Alpha T into one - Tow, CH Alpha T by CH Delta E into Alpha T, when I denote this as F so, I can write $CLT = F$ into CL Alpha tail into Alpha T or further I can write, with the float it is CL Alpha T prime into Alpha T, Note down this, If it was not floating, It was not floating then, it is CLT CL Alpha T into Alpha T because of the floating I have put prime, so that make distinction okay and what is F?

You could see F will be less than one because, this is the negative this is negative this is positive so F, will be always less than one so what we are getting is from this relationship, CL Alpha T is greater than CL Alpha T prime, because this part is nothing but, CL Alpha CL prime Alpha T is nothing but CL prime Alpha is nothing, but F into CL Alpha T and F is less than one.

So, I can always see that the CL Alpha T is greater than CL Alpha T prime, when it has been permitted to float, the elevator to float what is the physical meaning of this? Let us understand this okay, it is very simple and we will appreciate once you understand the mathematical part of it, let us see what it does mean?

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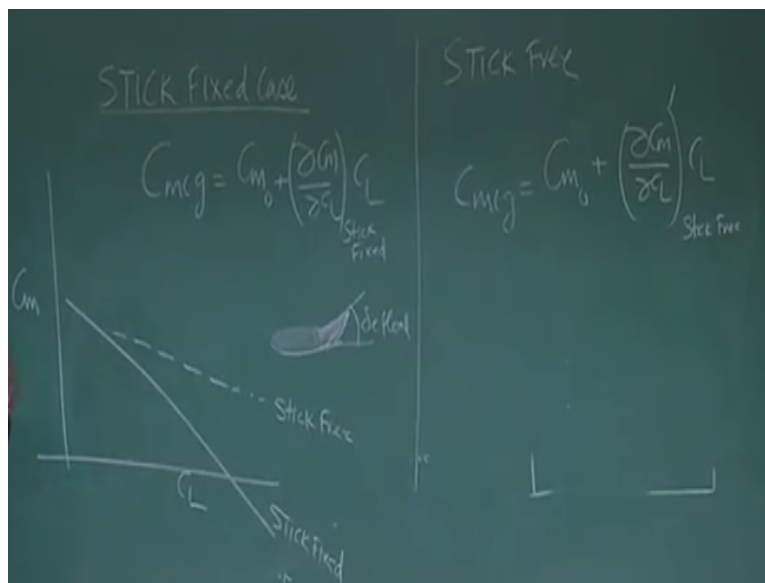
Let us say this is your elevator okay, and this is my equilibrium right, I am writing everything that is stick fixed. I am not allowing any Alpha T, I am not allowing this to float but actually, if you put hands off it will actually float and come to equilibrium. So the new equilibrium will be

something like this, which is Delta E float so now actually if I try to find out, what will happen if I try to move the airplane from this configuration to another configuration,

So I take this as an equilibrium and this equilibrium, you could see that since it has floated up its CL Alpha has to reduce, compare to the first case because you know if it is a cambered, it will further increase so that is why since it has floated up like this, its CL Alpha has to be less compared to CL Alpha, when the stick was fixed it was not allowed to float. So what is the message?

Whatever you have developed especially, you have developed for its stick fixed case, you can use those expressions but for change the CL Alpha tail, by CL Alpha prime tail, that's all, is it clear? I repeat again whatever expression you have developed, that remains same except where ever CL Alpha tail is coming, you change it by CL Alpha T prime you will get expression for its stick free case okay,

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So let us do that, let's see for this stick fixed case, C_{mCG} was $C_{m_0} + DCM$ by DCL into CL , stick fixed for stick free what will happen? For stick free it will be C_{mCG} will be $= C_{m_0} + DCM$ by DCL prime into CL . I will for time being I will not bother you with C_{m_0} , focus on DCM by DCL prime, what is the meaning here? It is stick fixed and it is stick free okay,

How they are different? If CM VS CL let say this was for stick fixed, now for stick free what has happened? For stick free the CL Alpha tail has reduced right, for the stick free now the aero foil is the elevator has moved up like this in the equilibrium right, it has floated at a given Alpha T given Delta E float so, its configuration is like this. So this will reduce CL Alpha T and as CL Alpha T reduces the static margin also reduce right, so it will be for stick free case if I am modeling through stick free case, it will be something like this and that is why I did not join them as CM 0.

Because CM0 prime I have removed, it will unnecessarily create confusion, it does not make much of the difference but yes, there will be slight difference but, at this point please understand we are not telling that pilot will be flying stick free or stick fixed this is for our modeling tools stick free we're doing to ensure that I can model stick force to the speed or CL as for as pilot is concerned there is fixed Delta E required.

Which is given by $\Delta E_0 + D \Delta E$ by DCL into CL trim Okay. That is the Delta E required I repeat in one case you take to the 5 degree or 4 degree and fix it lock it stick fixed another now we are doing one more thing because we want to develop a stick force relationship with deflections or speeds, so we are telling let us also see that since the elevator will float.

How much effort pilot has to through his force to see that it again will go back to 5 degree okay? But if it is 5 degree it is 5 degree correct? I again repeat somebody ask me the question the pilot flying at stick free or a stick fixed or a stick fixed absolutely irrelevant question pilot needs to give a fixed elevator deflection to trim the aircraft at particular CL trim as simple as that okay?

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STICK Fixed Case

$$C_{m\dot{\alpha}} = C_{m_0} + \left(\frac{\partial C_m}{\partial \alpha} \right)_{\text{Stick Fixed}} C_L$$

$$C_{m_0} = C_{m_{0w}} + C_{m_{fs}} + C_{L\alpha} V_H \eta_t \left(\frac{l_w - l_t + \epsilon_0}{l_w} \right)$$

$$\frac{\partial C_m}{\partial \alpha} = \bar{X}_{CG} - \bar{X}_{AC,w} + \left(\frac{\partial C_m}{\partial \alpha} \right)_{fs} - \frac{C_{L\alpha} V_H \eta_t}{C_{Lw}}$$

So this is one thing you understand that DCM by DCL is stick fixed will be more than DCM by DCL stick free in a sense, it will be statically more stable compared to that as per the mathematical term is concerned because we have change the equilibrium now to allow to float it, okay.

CM0 is given as CM0 wing + CM0 fuselage + CL Alpha tail VH Neeta T into IW all these things you by now + epsilon0 so CM 0 for stick free case will be as you see now will be CM0 wing which has nothing to do with elevator floating CM0 fuselage nothing to do with this + CL Alpha tail now it will become prime and rest remains same you remember IW is wing setting angle IT is the tail setting angle.

And a epsilon 0 is the down wash that Alpha = 0 typically it is for cambered wing now whatever DCM by DCL, DCM by DCL for a stick fixed case it was XCG bar - XAC wing a wall developed this + DCM by DCL fuselage FS I means fuselage - CL Alpha tail by CL Alpha wing to VH into Neeta T into one - d epsilon by D Alpha this was for stick fixed case so from this expression you can easily write DCM by DCL free will be nothing but XCG that is not going to change.

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STICK FREE

$$C_{m\alpha} = C_{m\alpha} + \left(\frac{\partial C_m}{\partial \delta_e} \right)_{stick\ free} (\delta_e)_{reqd} = \delta e_s + \frac{d\delta e}{d\alpha} C_{trim}$$

$$C_{m0} = C_{m0W} + C_{m0F} + \frac{C_{L\alpha} V_H \eta_t}{W} \left(1 - \frac{1}{W} + \epsilon_0 \right)$$

$$\left(1 - \frac{\partial \delta e}{\partial \alpha} \right) \frac{\partial C_m}{\partial \alpha} \Big|_{free} = \bar{X}_{cg} - \bar{X}_{ac,w} - \left(\frac{\partial C_m}{\partial \alpha} \right)_{fs} - \frac{C_{L\alpha} V_H \eta_t}{C_{L\alpha W}} \left(1 - \frac{\partial \delta e}{\partial \alpha} \right)$$

Because of something is slotted XAC wing - DCM by DCL fuselage and then - now CL Alpha T prime will come here CL Alpha wing VH Neeta T one - D epsilon by D Alpha so nothing new we are doing we have agreed. That because you are allowing the floating tendency the stick is not fixed so the elevator will go to a particular Delta E and will set, Delta e float and set will the curvature will change it will be going up like this so CL Alpha tail will change.

Once we allow elevator float so this is this and then okay? This is stick free we want to know the neutral point for neutral point it let me erase this write this expression and you go home happily that you have now know how to handle stick free case the most of the books are not a giving a explicit thing because this concept is out of market as you understand most of the controls are T reversible now right?

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The image shows two mathematical expressions for the neutral point location, \bar{X}_{NP} , written on a chalkboard. The left side is labeled 'STICK fixed case' and the right side is labeled 'STICK free'.

For the STICK fixed case, the equation is:

$$\bar{X}_{NP} = \bar{X}_{NP} - \left(\frac{\partial \bar{X}_{NP}}{\partial \alpha} \right) + \frac{C_{L\alpha}'}{C_{L\alpha}} \eta_H \left(1 - \frac{\partial \alpha}{\partial \alpha} \right)$$

For the STICK free case, the equation is:

$$\bar{X}_{NP}' = \bar{X}_{NP} - \left(\frac{\partial \bar{X}_{NP}}{\partial \alpha} \right) + \frac{C_{L\alpha}'}{C_{L\alpha}} \eta_H \left(1 - \frac{\partial \alpha}{\partial \alpha} \right)$$

So, far neutral point again I take the stick fixed case that is XAC wing - DCM by DCL fuselage + CL Alpha tail by CL Alpha wing VH Neeta T into one - d epsilon by D Alpha so what will happen? For Stick free neutral point XNO prime, which is stick free neutral point this will be again XAC wing - DCM by DCL fuselage + now CL prime Alpha T by CL Alpha wing VH Neeta T one - D epsilon by D Alpha is it clear?

So what I told you? As now we as understand because we have allowed the floating of the elevator the CL Alpha tail is changing if I note the expression for stick fixed case I can find out the expression for stick free case correct? Just to conclude this part you may ask me a question that how did you find stick free neutral point and how did you defined stick free neutral point so far that again.

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$$\left. \frac{\partial m}{\partial \alpha} \right|_{\text{free}} = \bar{X}_{N0} - \bar{X}_{N0} \left(\frac{\partial m}{\partial \alpha} \right)_{\text{fs}} - \frac{C_{L\alpha}}{C_{L0}} V_H \eta_e \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

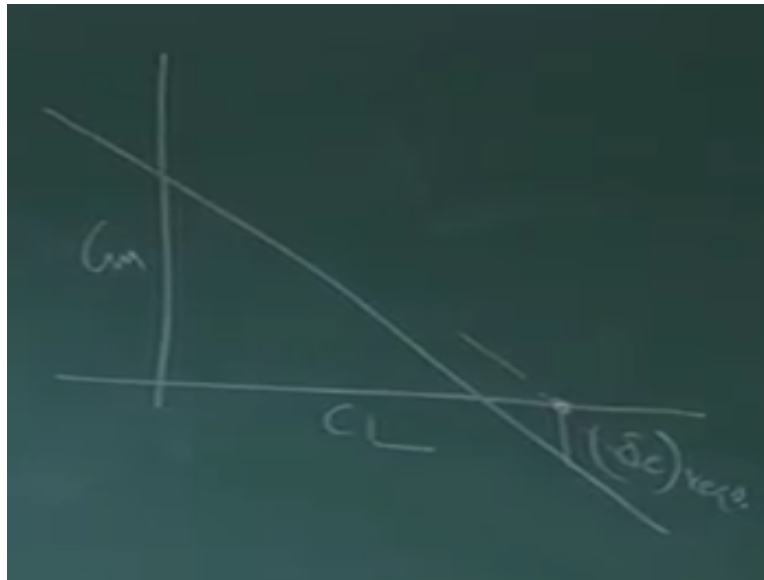
\bar{X}_{N0} For stick free case
 ? CG location at which $\left. \frac{\partial m}{\partial \alpha} \right|_{\text{free}} = 0$

I am going back to stick fixed case in a stick fixed case how did you defined XNO or neutral point stick fixed What was the definition you know DCM by DCL is given by this expression now we ask a question what is that CG location what is the CG location at which DCM by DCL is 0 that's the aircraft becomes neutrally stable and that is call stick fixed neutral point and there what you do you put is = 0 and then find out this XCG so all these terms will go to left hand side that is how we got the expression for X0 stick fixed.

Now similarly for the stick free case for the stick free case what will be your expression DCM by DCL what will be the expression for stick free you are now expert it will be all same except here CL Alpha T prime and you know the CL Alpha D prime is nothing but CL Alpha tail into F and F also you know what is how to expand it right? So, now if this is DCM by DCL free given by this expression for stick free case what is the condition what is that CG location.

At which DCM by DCL free is = 0 simple as that so now we put 0 here and then this is your then it becomes X 0 prime and you get this expression as you do it earlier clear? But they (()) (19:16) of question I repeat again if you want to trim an airplane for a particular CL.

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This is governed by simply by this that if I trim it here then I need some Delta E required and that angle if it is 5 degree it has to 5 degree only it as nothing to do is stick fixed or stick free this stick fixed means I take that angle to 5 degree and lock it.

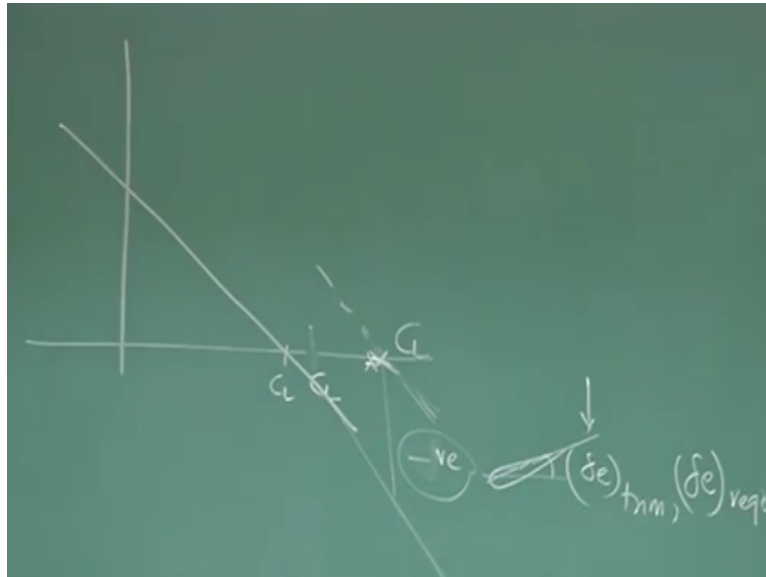
Because I know if I leave it like that then the elevator will float second thing is when the stick free case what we are trying to do we are trying to do trying to model develop background the model is stick force how much stick force is required and the stick fixed case what we answering is how much Delta E required so this is the certain difference in the modeling. In the stick fixed case our focus is on how much Delta E is required and that is the final that much Delta.

Will is necessary to be required to fly at a particular CL governed by that $\Delta E = \Delta E_0 + D \Delta E$ by DCL into CL trim. In a stick free case why we are modeling this we understand it is the reversible control then can I relate stick force to the elevator required and I can I fly hands off that is I put the hands off and after certain point the elevator float itself will take here and pair has to put that much of force to give additional thing we are totally automatically after floating it will go to 5 degree.

And I can fly hands off that is the concept to give concept to develop mathematical model to relate stick force with the flying condition CL or speed right? So please give this back of your mind and this concept are stick free reversible point is only true for reversible control but

unfortunately most of the airplane now have gone to irreversible control just for completion I will be taking perhaps one more lecture on this. So let us again revisit when we are talking about floating tendency or a stick free case what is the necessity to define a stick free case.

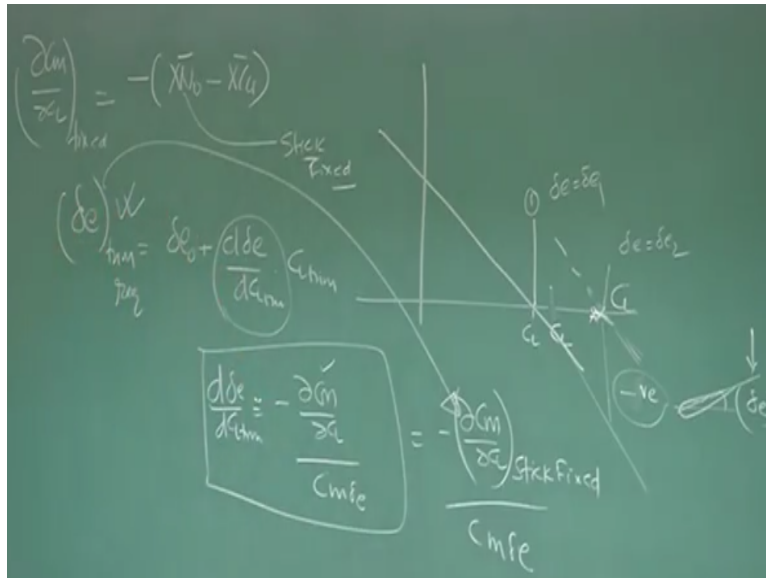
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Please recall we know one thing that if an airplane is flying at this CL so this is the CL are you want to fly at this another CL then you know that so much of negative pitching moment the aircraft will generate by itself. If I want to really fly at this point I must ensure that this negative moment is neutralized by giving the elevator up from the reference elevator angle and I need to know how much this Delta E which I call Delta trim or Delta E required.

If I know that then I can trim the airplane which was earlier trimmed at this CL from this CL to this CL I can do that by putting this much of Delta E trim who decides how to calculate the Delta trim.

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Please understand this this Delta E trim or Delta E required whatever we say will be given by Delta E 0 + D Delta E by DCL trim into CL trim and if I carefully see D Delta E by DCL trim it is nothing but - DCM by DCL by CM Delta E approximately we have seen that the DCL trim when I want to use - DCM by DCL which is actually - DCM by DCL the stick fixed divided by CM Delta E.

Stick fixed means control locked we are not allowing into float that is important so what is Delta E required is decided by DCM by DCL fixed and the DCM by DCL fixed or DCM by DCL stick fixed is nothing but you know that DCM by DCL fixed is a - static margin so - XN_0 by XCG I could bar I know you know bar means non dimensionalized with mean aerodynamic chord so the important is here the N_0 will be again stick fixed okay?

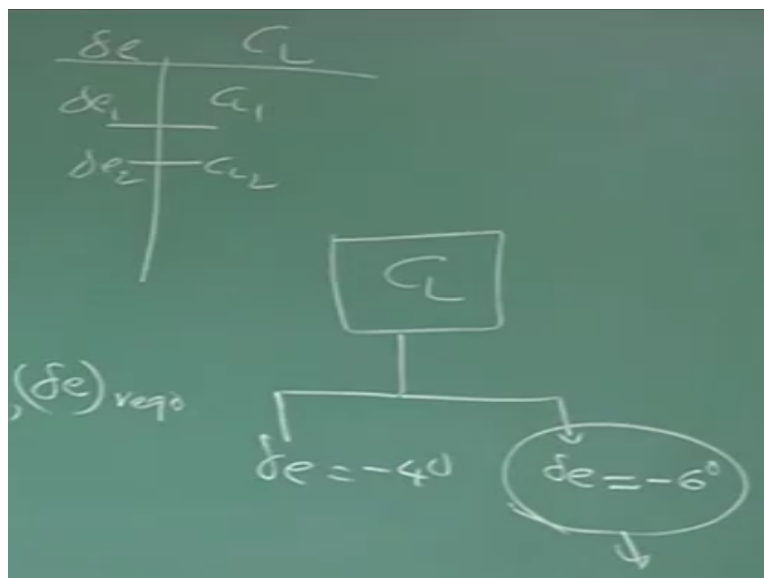
This is very important when I go to trim how much Delta E required is decided by this DCM by DCL is stick fixed as simple as that that you should understand and it should not a start thinking of when the DCM by DCL free or not Delta E required is governed by this DCM by DCL fixed by this through this expression. We need to also understand that what is the physical meaning of D Delta E by DCL trim? Many times I write it as D Delta E by DCL.

But you understand that is DCL D Delta E by DCL trim meaning there by suppose at this point it has the Delta E = Delta E one to trim this here and at this it is Delta E = Delta E 2 so D Delta E

by DCL trim is that gradient that is from Delta 2 - Delta one by CL 2 - CL one CL 2 is trim CL one is again a trim right? At different condition that is the meaning of D Delta E by DCL trim okay? This is very important because many time students get confused please understand when

We talk about floating tendency we are telling that we are developing this concept to give a feel for the pilot through stick force. which you will see how we build it up because if I am talking about stick fixed like this which is a which is that very important to decide how much Delta E required and how the pilot will develop the field he will say okay?

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For a given CL this is the Delta E required this CL one Delta E 2 for CL 2 so he will be seeing that how much Delta E required for the particular CL or particular speed from that he develops the field for example for same CL if CL remaining same if one aircraft required Delta E = - 4 degree and another aircraft or same CL for same condition which required Delta E = - 6 degrees and the pilot understands that the aircraft number 2 is statically more stable.

Because it requires larger control effort okay? So that way you can give a feel which doesn't have a direct one to one contact it is visual this much Delta E and this much CL or speed but if you want to really a give a feel to the pilot okay? Then you have to one way is to do is that okay? what is the effort pilot takes to change the elevator from one angle to another angle what is the

amount of stick force he is applying so that is also another way of giving a feel to the pilot to develop an expression.

So that we can quantify you can mathematical model to develop a feel for a pilot there you are using the concept of stick free stability okay? I repeat again only to give mathematical model to be a feel for a pilot we are talking about stick free stability building the concept of floating of a elevator so this is very, very important and you should not get messed up if you ask a question yourself finally.

If I am going to fly at a particular CL how much Delta E I required you have only use this where you have very clear this is nothing but DCM by DCL stick fixed or controls locked or elevator is not allowed to float clear? Right this should be very, very clear in your mind thank you!