Aircraft Stability and Control

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Lecture- 28
Hinge Moment and Hinge Moment Derivative

We will now try to develop the mathematical expression to model stick force. What is the stick force? It is the force applied by the pilot on the stick, to deflect the elevator in a particular angle correct. To deflect the elevator in a particular angle. Now the question is all this stick force or stick free stability these have a relevance, if I am talking about control system which are reversible.

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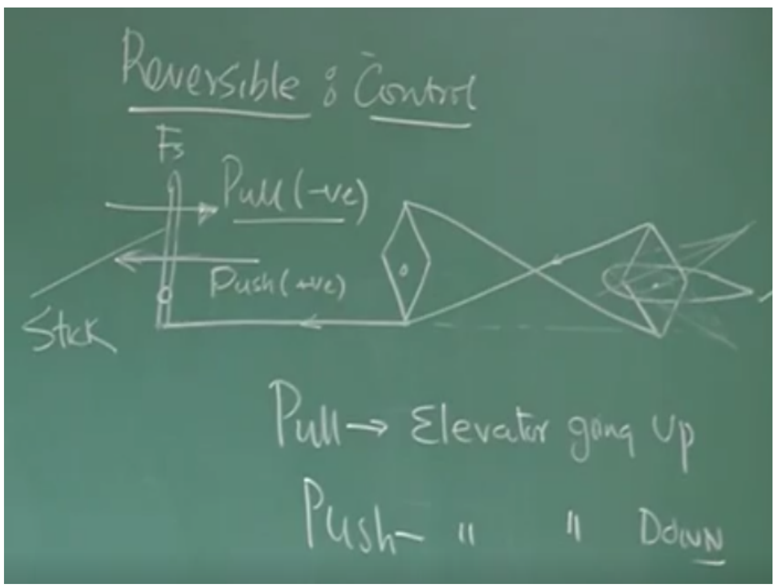

Reversible means, let us understand what is, the meaning of reversible is although I have already explained you in some time, let me draw this diagram. To appreciate the stick force, let us see this simplified diagram okay. This is the stick and through lever you could see, if I pull this stick if I pull it, then this elevator will go up like this correct. Again if you see if I push this stick push it, then if I am pushing it then this elevator will go down.

So if I pull you could see that elevator going up, and if I push elevator is going down, clear. And you will see that as a matter of convention, for our lecture here, we have taken pull force to be negative. So let us see what is the meaning of pull and push in actual flight, which I will be demonstrating you when you will be flying an aircraft, but let us say I am cruising at a particular speed, or cruising at a particular CL, now suddenly I want okay, I want to reduce the speed, but same time I will try to maintain lift = weight, so what will happen?

Reduce the speed by psychologically it is like I am pulling the stick. As I am trying to reduce the speed, as I am trying to pull the stick, as I want to try to reduce the speed to maintain lift = weight, CL has to increase. So what will happen, so as I pull CL has to increase means, the elevator has to go up, you see if I pull the elevator goes up like this, as I pull then this gets stretched like this and this stretches like this and it rotate like this, so that is why when I want to reduce the airplane.

I pull that stick, but still I want to maintain lift = weight so, My CL should increase so the elevator will go up right. In a reversible control if I have the elevator here, and if I try to pull the elevator up you will find the stick will automatically go in a pull force, will experience a pull force, it will try to move that side. That is why and if I want to put the elevator down like this, the stick also will move this way, it will generate a push force.

That is why it is called reversible, that is why it is easy to give feel to a pilot, but a irreversible control which has most aircraft are there are boosters, there are activators and there is sort of direct reversible relationship is not there, so the stick force relationship will not hold true strictly, and one may ask for what is the relevance also okay. But we will do this we will see that there are important things to learn.

So this is what we are talking about, we are talking about reversible control, and when you are developing stick force required we have this diagram or this picture in mind right. And when you are talking about stick force we also have relaxed one thing that now the elevator is no more fixed right. Elevator is allowed to float okay. So we will revise this what is this meaning of elevator float.

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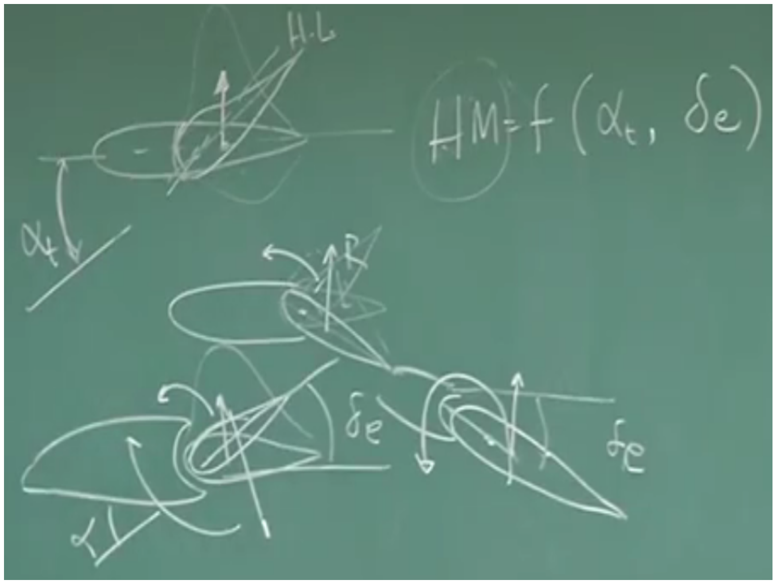

Already we have discussed something but we will further discuss, because now this background to be reinforced before you write an expression for stick force okay. If the elevator is mounted on a hinge and you know that yes it is indeed, yes it is the elevator, there is a hinge line right. And assume that is fiction less, then what will happen, see that if there is a positive Alpha tail seen by the tail then there is a pressure distribution over this.

And as you know if the resultant is behind the hinge line, then I get a moment which will try to take the elevator float like this right okay. And nose down so it is negative in nature, and we said this effect is due to whatever hinge moment is coming is due to Alpha tail.

But now see also one more thing, if I am trying to deflect the elevator down. So again there will be pressure distribution over it and there will be a resultant here, and this hinge line is ahead of it, this also will give a moment, which will try to oppose the deflection clear that is if I am trying to take it down, it will try to oppose it, if you are trying to take it up then again it will try to oppose it, so what is happening?

There is a hinge moment generated because of the Alpha T, as there is a positive Alpha T it will try to float, as it rise to float now this Delta E angle is being introduced this Delta E. Because of that will give a force downward with you try to oppose it right. So we have hinge moment coming not only because of Alpha T but also because but also because of Delta E, is this cleared.

I repeat again if this is your elevator and because of pressure distribution, and if hinge line is a head of resultant for a positive Alpha it will try to take the nose of the elevator down,

So it will have pitching moment negative but you also see that if elevator is taken up like this. Because, it generates elevator angle Delta E and then that will generate a force downward, which will try to take it for a negative Delta E it will generate a positive pitching moment, similarly for a positive Delta E you could see here, positive Delta E force will be somewhere here so about the hinge line, it will generate a nose down moment, so what is the message?

Message is with positive Alpha T, with positive Alpha T there is suppose the elevator tail is herewith positive Alpha T it tries to float like this as its float like this because of the elevator angle is change, it generates the force downward its try to discourage it right. So, there has to be an equilibrium where the net hinge movement because of Alpha T and Delta E is 0 and that is that angle we call it floating angle. Physically and fundamentally okay.

Let us try to see little bit of nature of this hinge moment generated because of Alpha T and Delta E.

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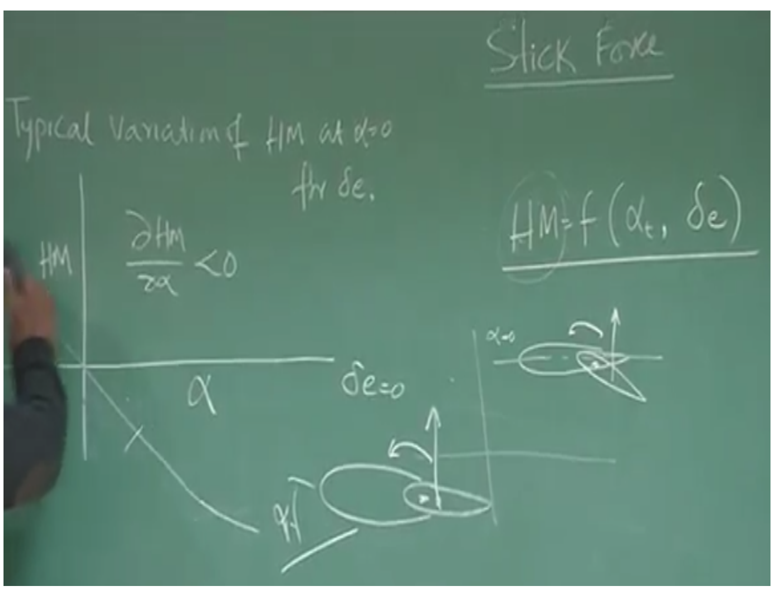

Typical variation of hinge moment at Alpha = 0 for Delta E, any way I am trying to see what is the hinge moment that is coming nature wise we will see that typically. The nature will be something like this. This is your for different Delta E of course Alpha = 0, so what we are seeing that this linear to some extend and at larger Delta E is become little bit of non linear. What is the meaning we are getting here? At Alpha = 0 right.

We are seeing that or near Alpha = 0, almost things are linear and if I write this is hinge moment and Delta E, then I know that D hinge moment by D Delta E sign is less than 0, correct. This is the nature of hinge moment generated by elevator deflection at Alpha = 0. Which you can physically see if I am seeing the hinge moment, if I draw it like this and this is your elevator.

I am keeping Alpha = 0 I am putting elevator deflection positive, the moment i put elevator deflection positive I know hinge line is here, resultant forces here which will give me a nose down moment, so indeed the slope should be negative right, similarly.

If I try to see hinge Moment versus Alpha at constant Delta E, that diagram also looks like this, this is hinge moment and this is Alpha for Delta E = 0 let us say, there also you will find to some part it is linear at higher Alpha, It tries to become non linear not that much but yes, but for our interest will find that the variation of hinge moment with Alpha, at Delta = 0 so DHM by D Alpha this is also less than 0, which physically we understand if this is the elevator.

So if there is a positive Alpha T we have seen the hinge line is here, and central pressure is here this also gives a moment nose down, for a positive Alpha or we say the hinge moment with Alpha the slope at equilibrium at around is negative, but at higher Alpha it changes it becomes little non linear but we have more focused here. The catch what is linear right. We will be exploiting that in expanding this hinge moment at Alpha T and Delta E.

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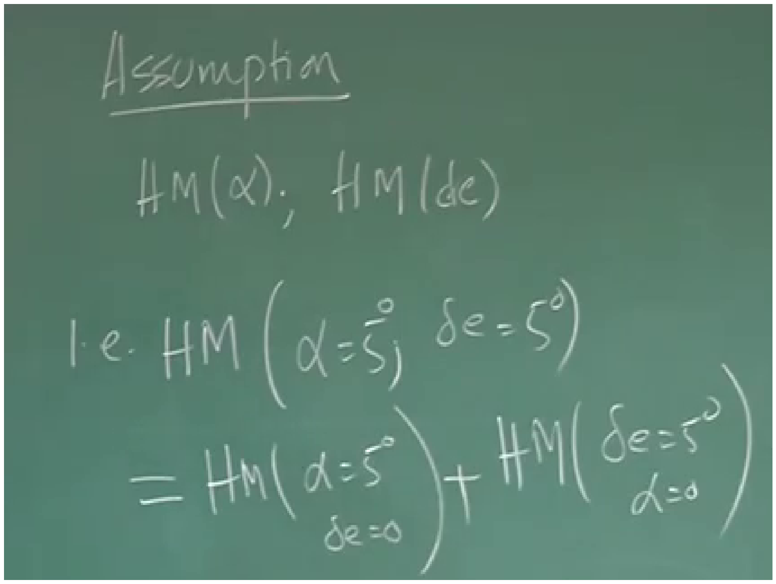

So now let us make assumption. Let's not forget we are trying to develop model for stick force, mathematical model for stick force and we have kept this matic at back ground. For we know that pull force we have taken negative, and push force we have taken it as positive okay. Let's say assumption we have seen hinge moment is function of Alpha we have also noted hinge moment is function of Delta E right.

And if we assume them to be linear the whole variation of hinge moment with Alpha E and Delta E is linear. Then I can write linear means please understand we have been doing it for long, suppose I want to find out hinge moment at Alpha = 4 degree and Delta E = 4 degree. The linearity will help mean. Expanding this problem as hinge moment calculate hinge moment, at Alpha = 4 degree Delta = 0, then add to it hinge moment at Delta E = 4 degree and Alpha = 0,

That is the beauty of assuming linearity okay, it makes life very simple if this is understood then you are already you are smart enough, you know whenever there is a assumption then things are linear.

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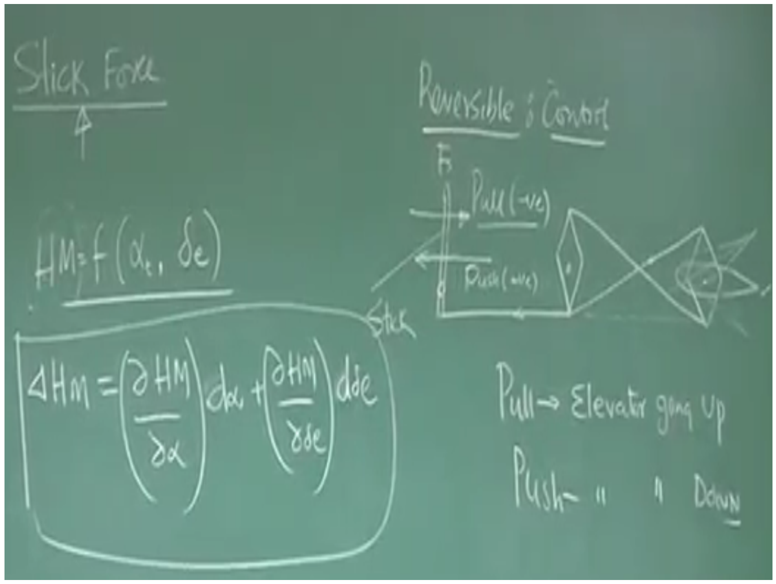

Then I can write Delta HM I can write as DHM by D Alpha into Alpha or strictly speaking D Alpha + DHM by D Delta E, into Delta, Delta E or D Delta E is right. Ward or mathematicians will not allow me to write so casually, so if that is true then I can write 1 thing clearly that all through my text when you will be doing the stick force, I will be more focusing if I know the value of this derivative and this derivative, then I will be able to find out what is the hinge moment coming at a given Alpha and Delta E through this model, okay,

This is the model we will be using it is simply like when you write CL, CL = CL Alpha into Alpha + CL Delta E into Delta E in a same linearity concept word also used. I have been repeating it here because once you see hinge moment there is every possibility that, because of this diagram you may lose the insight. So again I want to repeat you that nothing new we are doing, we are still focusing on the assumption, thus whatever operation we are doing on the linear domain.

So, you can always use this partial derivative concept okay, and because of linearity.

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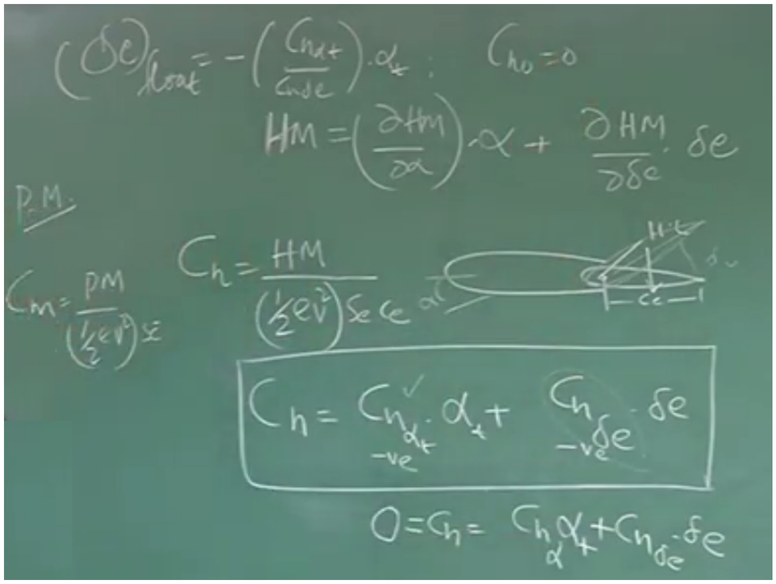

Now also we can write this as any hinge moment at any Alpha. I can write as DHM by D Alpha now I can write Alpha and DHM by D Delta E in Delta, this is also beauty of the fact. That we are all operating in a linear domain, when I write hinge moment here, I will try to take you back we are also writing pitching moment write, but we never evolved with pitching moment we evolved with a non dimensional pitching moment that is CM . So here also you will be working with non dimensional hinge moment so how do I define that.

I define CH as hinge moment divided by half row V square in to S elevator C elevator, remember C pitching moment how CM we define the pitching moment divided by half row V square S C bar right. So here we are defining hinge moment non dimensional coefficient as hinge moment divided by half row v square SC CE, And for your information, already we have been told that if this is the hinge line C is this from the hinge line and AC is the elevator area that you know.

So using this expression now I can write CH = CH Alpha tail into Alpha +, CH Delta E into Delta E, and what is the SIN of CH Alpha T? You know it is negative, CA Delta is negative. So I know that if I am having a positive Alpha here because of CH Alpha T, nose will go down so it will try to float up like this and because, now it has into those Delta E this will try to resist for a negative Delta E it will give a positive hinge moment.

That is why CH Delta E is negative so there will be a balance between this 2 moment, 1 is trying to float try to float 1 is trying to bring it back. So there will be an equilibrium and that equilibrium at which CH is = 0, is basically CH Alpha T into Alpha T + CH Delta E into Delta E and from here we have calculated Delta E float, = - CH Alpha T by CH Delta E into Alpha T of course we have assumed that CH 0 = 0.

We have already known that so this the Delta E float okay, now there is an interesting thing you should understand.

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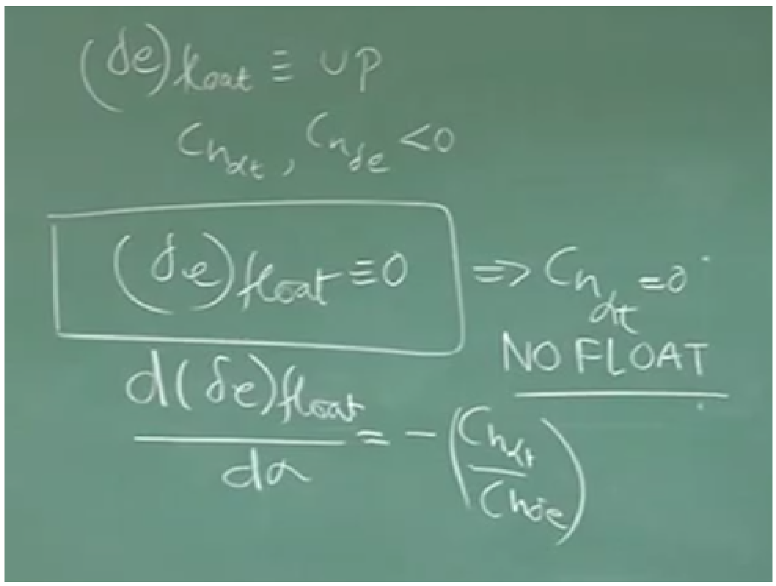

Now let us also carefully see this we could see that Delta E float, it will float up if CH Alpha T and CH Delta E less than 0. So this is it can float down depending upon the signs also. But mostly you will find the airplanes are designed such that CH Alpha T and CH Delta E are less than 0, now suppose you want Delta E float should be 0, I don't want any Delta float suppose I want Delta E float to be 0, what i have to do?

I could see that Delta E float will be 0 CH Delta E cannot be 0, because it becoming infinite this derivative only way to ensure is that CH Alpha T is made 0, that means if CH Alpha T is 0 then there is no float, this should be in your mind you will understand why I am mentioning this thing at this point.

And how can you make CH Alpha T 0? That is 1 should also have to give a thought to it, if the hinge line and the resultant to the center of pressure, of the same place same point then theoretically CH Alpha T should become 0 okay. But there are other many ways to reduce CH Alpha T because, you don't want much of a floating tendency of an elevator, we will talk about.

How it has been done okay but, before you go to that you also should understand this is also very important this gradient that is D Delta E, float by D Alpha you could see that is nothing but ratio of CH Alpha T by CH Delta E. So, this ratio will tell you at what rate the elevator will float per unit deflection, or unit Alpha tail angle, This is also a very very important parameter, design parameter when you are trying to link all these floating tendencies to a reversible control to the pilot right.

So this concept you should keep in back of your mind will be using it, as you develop all those forces. Since we are talking about how can you make CH Alpha T low so that this is less floating tendency, or CH Alpha T theoretically possible to make it to 0, what is the meaning of CH Alpha T 0 means? Whatever elevator deflection pilot is giving right, so it stays there right no floating tendency what are the popular way of doing it and this concept is used, in trimming the pilot force how much force he will apply. Is through a horn balance.

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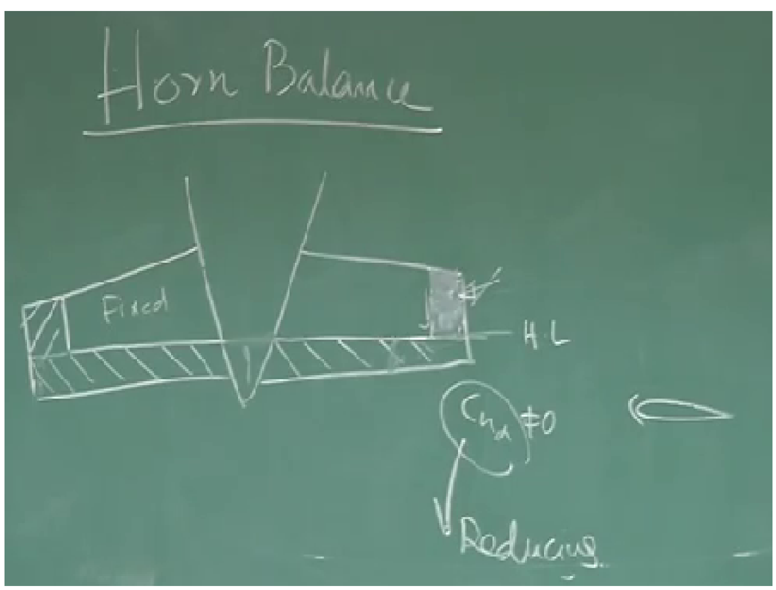

We call it horn balance; it's just a passing remark for you to get little bit of better feel, other than only getting lost into suppose this is the hinge line. Okay and this is the fixed surface this is vertical part of vertical stabilizer, and this is the elevator surface it can go up and down elevator right both sides, now naturally here you will find the CH Alpha T is non 0 its most probably negative, because of this tail , because hinge line will be ahead of the central pressure over the elevator ,if you want to see cross sectional wing it is something like this okay.

Now suppose if I do like this, I make the elevator like this, this whole is elevator do you see? now this whole portion moves up and down so as this portion sees an Alpha T, please note down because of this portion of the elevator it will try to make for a positive Alpha T it will try to go float, towards negative float up, but because of Alpha T on this surface , it will try to discourage it , it will try to take the nose upward or come to the side left, I will explain what is it there, we are discussing about floating tendency of elevator you could see this is the horns of 3 aircraft.

And these are the elevator and this is the hinge line about which the elevator rotates up or down, up or down now you see if there is an Alpha T, Then you could see easily that the center pressure on this elevator will be somewhere here, and hinge line is here so it will have a floating tendency like this okay clear or not. This is cleared now you see in normal aircraft this is only up to this portion will be elevator, but here see this extension has been put here and that is part of a horn balance, what happens you see?

As Alpha T is seen by the Tail this portion, will try to float it up okay because hinge line is a head of central pressure on this surface, however this front el surface this surface, you could see this surface, this is here, here the force will act in this direction so that will try to give nose up, so what is happening? This portion floating tendency is trying to take it nose down, Or floating up and because of this horn balance here it's trying to take it nose up or floating down, so is actually reducing the effect of CH Alpha T or reducing the magnitude of CH Alpha.

Again listen for a positive Alpha T this will try to take float it up, but because this portion is there horn balance this much portion, here the force will act upward so it will try to take a down so, effectively CH Alpha T is marginalized it is reduced, so that is 1 of advantage having this sort of a horn balance that is, elevator + some extra surface which is a head of leading edge. This surface a head of leading edge will give a moment opposite to the moment generated by surface behind leading edge, so if you want CH Alpha T to be 0.

So ensure that they cancel each other, and CH Alpha T become 0 okay, this is how we reduce the value of CH Alpha T or, that is how we reduce the floating tendency of an elevator through horn balance. Okay come to class room so once you have understood this horn balance, what is how we summarize we say because of this portion a head of a hinge line, it will try to discourage floating tendency by reducing the value of CH Alpha T.

If I reduce in this value right, there are many mechanism of handling say CH Alpha T but, at present we are now going towards some definition and we will be discussing that, so that once you develop the force expression all these primary concepts are cleaner, concepts which are physics based, concepts which are convention based.

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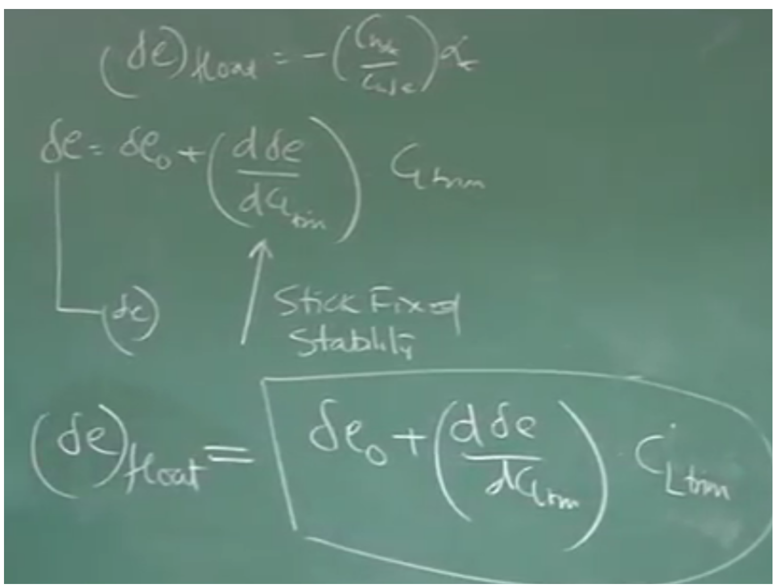

If I now again right Delta E = Delta E 0, + D Delta by DCL trim into CL trim , so how much Delta E required for a particular CL its comes from the stick fixed, stability longitudinal. We are clear about it because, D Delta E by DCL trim has - DCM by DCL so, that - DCM by DCL is stick fixed case no issues on that, now think of a situation if Delta E float is so happen that Delta

E float is = this value, for the particular CL what is right hand side ? It is the Delta E required for a particular CL trim, to trim a particular aircraft or particular CL.

So it is basically CL trim, that comes from the longitudinal fixed stability, now say the way CH Alpha CH Delta etc has been designed, that you know Delta E float = - CH Alpha tail by CH Delta E into Alpha tail and suppose, The way CH Alpha T and CH Delta E are designed that Delta E float, when I compute through this that become exactly = what is the Delta E required, by itself by floating that means the stick force applied will be 0 correct.

You don't require any stick force so that is the case where we say stick force is 0, physically that means if the airplane is designed that whenever it goes to a particular CL, automatically the float is exactly = what is the Delta E required, so it do not require to apply any stick force.

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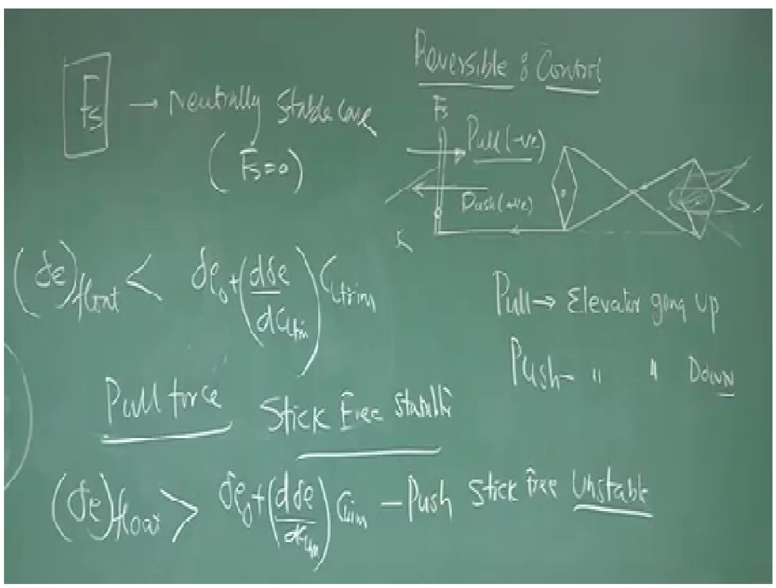

And that is the case, in our term through stick force we say that is the neutral stable case. That is FS = 0 okay, we see a again we are trying to build some convention, second case we will say it. This Delta E float is less than Delta E 0, + D Delta E by DCL trim into CL trim at a given CL that means what happens? please understand suppose Delta E required was 4 degree, and by float it has come to 3 degree that means, pilot has to increase the elevator up by 2 degrees so, how does he do that? He has to put the elevator up, so he has to pull the stick okay.

So he has to pull the stick. You could see here he has to pull the elevator up, so has to pull the stick and pull is negative so, you could see here if Delta E float is less than the Delta E required which is Delta equilibrium. Delta trim then pilot has to pull to increase further angle so that it is = this equilibrium, and Delta E required or trim Delta E required so ,we have apply a pull force in such cases where you have to apply a pull force, we will say this is stick free ‘stability as far as, Stick force is concerned through stick force is concerned, and obviously.

If Delta E float is greater than Delta E 0 + D Delta E by DCL trim, into CL trim that is whatever Delta. He has floated it has exceeded, the requirement of Delta E through this relationship, as I have given you example if 4 degree was required, now it has by floating it has become 6 degrees, so the pilot has to reduce that elevator so it he has to push it, so he has to push and this is stick free unstable, please understand these are the part of the convention and this convention has an understanding, that from handling qualities point of view if pilot wants to increase the speed.

Then he will prefer psychologically to push the stick so, the speed is increasing if he wants to reduce the speed, he will try to pull it okay, from that convention this if i translate into FS I come back to this, stick free unstable or stick free stable this understanding has to be very very clear, before you go for any development of stick force.