

Aircraft Stability and Control
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Lecture – 29
Aircraft Handling Qualities

We are continuing our lecture on how to model stick force and try to understand the importance of Hinge moment coefficients on designing a system, reversible control system for aircraft, so the stick force is well within the capability of the pilot, the pilot can fly at ease, right. So, if you recall before I come to stick force modeling let me write few statements.

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The image shows three equations written on a chalkboard:

$$\bar{N}_0^{\text{Fixed}} = \bar{X}_{ac,w} - \left(\frac{dC_m}{dC_L} \right)_{fus} + \frac{C_{L_{\alpha E}}}{C_{L_{\alpha W}}} V_H \eta_E \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$\bar{N}_0^{\text{Free}} = \bar{X}_{ac,w} - \left(\frac{dC_m}{dC_L} \right)_{fus} + \frac{C'_{L_{\alpha E}}}{C_{L_{\alpha W}}} V_H \eta_E \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$C'_{L_{\alpha E}} = C_{L_{\alpha E}} \left(1 - \tau \frac{C_{H_{\alpha E}}}{C_{N_{\alpha E}}} \right)$$

We have already done N_0 stick fix XCG or this is XAC wing - DCM by DCL fuselage + CL Alpha tail by CL Alpha wing $V_H \eta_E$ into one - $D \epsilon$ by $D \alpha$, similarly this is stick fix or control fix. But we have also derived stick free, that is where we allowed the elevator to float, here it is XAC wing, which is same DCM by DCL fuselage.

What was the change we know because of floating and floating there will be equilibrium angle of the elevator and CL Alpha tail will now be CL Alpha tail with prime CL Alpha wing $V_H \eta_E$ into one - $D \epsilon$ by $D \alpha$ and we know how CL Alpha tail is related CL Alpha prime tail is nothing but CL Alpha tail into one - Tou $C_{H_{\alpha E}}$ tail by $C_{N_{\alpha E}}$ elevator so, if I know stick fix CL Alpha tail.

And if I know the stick fix CL Alpha tail and if I know this parameter I can find CL Alpha prime tail which had need to substitute here to get and N0 bar free where we use the notation N0 bar with dash.

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The image shows a chalkboard with handwritten mathematical equations. At the top, it states:
$$\bar{N}_0 - \bar{N}'_0 = \begin{pmatrix} C_{L_{\alpha t}} \\ C_{L_{\alpha w}} \end{pmatrix} V_H \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \begin{pmatrix} C_{h_{\alpha t}} \\ C_{h_{\alpha e}} \end{pmatrix}$$
 Below this, an arrow points to a more complex equation:
$$\frac{\partial M}{\partial C} = -SM - (\bar{N}_0 - \bar{X}_{CG}) \frac{\bar{N}_0 - \bar{X}_{CG} + \bar{X}_{CG} - \bar{N}'_0}{\left(\frac{dM}{dC}\right)_{fix}} + \left(\frac{dM}{dC}\right)_{free} = \begin{pmatrix} C_{h_{\alpha t}} \\ C_{h_{\alpha w}} \end{pmatrix} V_H \eta_t \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) \begin{pmatrix} C_{h_{\alpha t}} \\ C_{h_{\alpha e}} \end{pmatrix}$$

We know also we know this we have done at N0 bar - N0 bar prime is = CL Alpha T by CL Alpha wing into VH Neeta T one - D Epsilon by D Alpha Tou into CH Alpha T by CH Delta E. Let me come back to this again. This is N0 fix this is N0 prime free stick free and this is CL prime Alpha T which is related like this CL prime Alpha T CL Alpha T into one - Tou CH Alpha T by CH Delta E.

These are basically hinge moment coefficients related to elevator. Now you could to see another interesting thing from here, if I now write simply if I take this and if I write N0 bar - XCG bar and then here and + XCG bar - N0 prime, this will become what? What I have done? In the left hand side I have added XCG and subtracted the XCG. So, this remains same, but you could see carefully this is nothing but DCM by DCL fix, is it correct?

DCM by DCL fix this is N0 - XCG static margin stability margin and you know DCM by DCL is = - static margin and this is - of N0 bar - XCG bar. So, this when I write this term here if I say this is DCM by DCL fix I am wrong this is - DCM by DCL fix now here it could see this is N0 prime this XCG - N0 prime so, this is DCM by DCL free that is stick free or control free control in any book will use the term.

So, this is + this is = this term CH Alpha T by CL Alpha wing VH Neeta T one - D Epsilon by D Alpha Tou CH Alpha T by CH Delta E, what is CH Alpha T? CH Alpha T is because of Alpha tail and since hinge line is ahead of the center of pressure distribution over the elevator so, this will give a negative moment.

So, CH Alpha T is negative and CH Delta E as I put Delta E down which is positive it also generates a negative pitching moment. So, CH Delta E is negative so these are basically huge moment coefficient or derivatives right? What is important here? You could see from this what we see from this?

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The image shows a chalkboard with the following handwritten content:

- At the top right, a box labeled "Stick Force".
- The main equation: $\left(\frac{dC_m}{d\delta_e}\right)_{free} = \left(\frac{dC_m}{d\delta_e}\right)_{fixed} + \left(\frac{C_{L_{\delta_e}}}{C_{L_{\alpha}}}\right) \frac{1}{V_H} \eta_e \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) > \frac{C_{H_{\delta_e}}}{C_{H_{\alpha}}}$
- Below the first term, it is noted that < 0 .
- Below the second term, it is noted "stab is ten stable".
- Below the third term, it is noted "+ve destabilizing".
- At the bottom, another equation is written: $\left(\frac{dC_m}{d\delta_e}\right)_{free} = \left(\frac{dC_m}{d\delta_e}\right)_{fixed}$

From this you get clearly see DCM by DCL free is = DCM by DCL fix + CH Alpha T by CH by CL Alpha wing okay? VH Neeta T one - D Epsilon by D Alpha into Tou into CH Alpha T by CH Delta E I am getting from here okay? Now see the right hand side, what does it represent? First let us take the sign, this is negative, this is negative, this become positive, all are positive, negative and this is positive. So, what is happening? This is not CH Alpha T this is CL Alpha T sorry this is CL Alpha T please correct this.

Please do a correction, this term will be CL Alpha T I have written here by mistake CH Alpha T which should be CL Alpha T as able to here okay? Right. So, now let as take the sign, this is negative, this is negative so cancels this is positive this is positive so, what I have seen here what is statically stable airplane I want DCM by DCL contribution should be negative right.

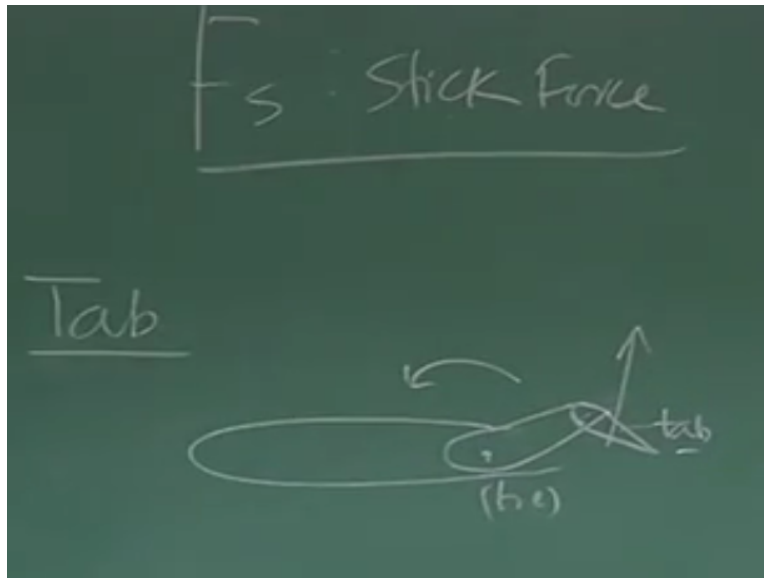
So, now it happening DCM by DCL free is whatever DCM by DCL fix stabilizing was there this is less than 0 so, this is stabilizing but this contribution is positive so, this is destabilizing right? I repeat this sign, if the sign of this term you could see this is negative than negative this become positive and everything positive here so this contribution is positive so, destabilizing that is why DCM by DCL free we say is less stable compared to DCM by DCL fix.

So, we say when allow the elevator to float up actually your airplane DCM by DCL elevator control free it comes down to a lesser stability compared to DCM by DCL fix. I repeat again here DCM by DCL free we are trying to model to relate stick force with hinge moment coefficient etc., and etc.

But as far as airplane is to be flown it has a single Delta E or Delta required which is dictated by DCM by DCL fix only right. That is $\Delta E = \Delta E_{not} - \text{DCM by DCL fix} / \text{CM}$ Delta E into CL trim that you know okay. So, this is only we are trying to find out the model FS stick force okay. This background is necessary that is why I thought I will revise it. Now we will go for the derivation okay?

Please remember one thing you must used the open forum to ask questions about the derivation but if you have any doubt? You must write and derivation does not look nice it was same when I was also a student but the more important thing would be from this derivation once they are done, how can I extract the juice in terms of the physical interpretation and I can talk to a pilot again commonly get to an aircraft that is more important right. So, do not worry about a much of derivation any doubt to put in the open forum will be giving those derivations right.

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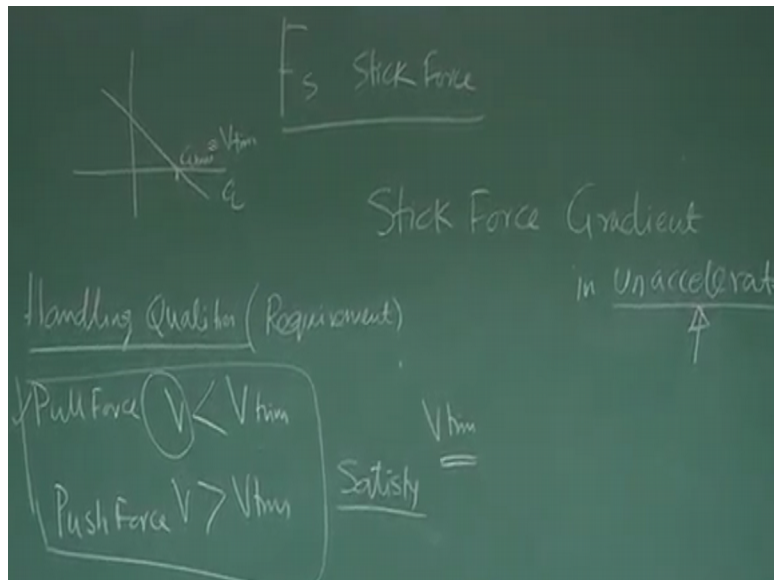


Another concept you need to revise before we go for stick force is the tab which I show you a tab and now we should see this, suppose this is your tail plane and this is the elevator I have a tab here, if I deflect this tab like this okay, then it will generate a hinge moment which will assist in flying or trimming the airplane because this will give a hinge moment like this and this is the hinge line, right.

So, once properly this is set by a trimmer, the pilot did not applying any stick force we can fly hands off at the total hinge moment on the elevator hinge I can say that stick force required 0 so, already at equilibrium hinge moment $CH = 0$ okay, but we talk about the stick force than will say is stick force is 0 because now we are trying to relate stick force to stability, stick force to a hinge moment floating tendencies okay, elevator floating tendencies.

So, this is you know that I have already done so, please remember if it is like this a tab this is the tab that will be deflected like this and it will generate a hinge moment and it can finally through the tab we can set that stick force required is 0 so, $SS = 0$ and you fly hands off. So, this is one of the use of trim tab, so this background is also required. Now with this background, we will start developing the stick force right.

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Typically the topic which I will be talking is stick force gradient in unaccelerated flight. The stick force gradient that is we may like to develop an expression $\frac{dF_s}{dV}$ that is what is that gradient $\frac{dF_s}{dV}$ that is for stick force and speed that will very important and let a lot of implication on the handling qualities of an airplane.

As far as handling qualities this is requirement because when design an airplane that many handling requirements are there, so one of the requirements is that if I am flying at V_{trim} that is suppose I am flying here, this is the CL, this is the CL trim and it is correspondingly having some V_{trim} because CL and beta related if I flying at V_{trim} and if we want to flight at a speed lower than V_{trim} , then it should apply a pull force.

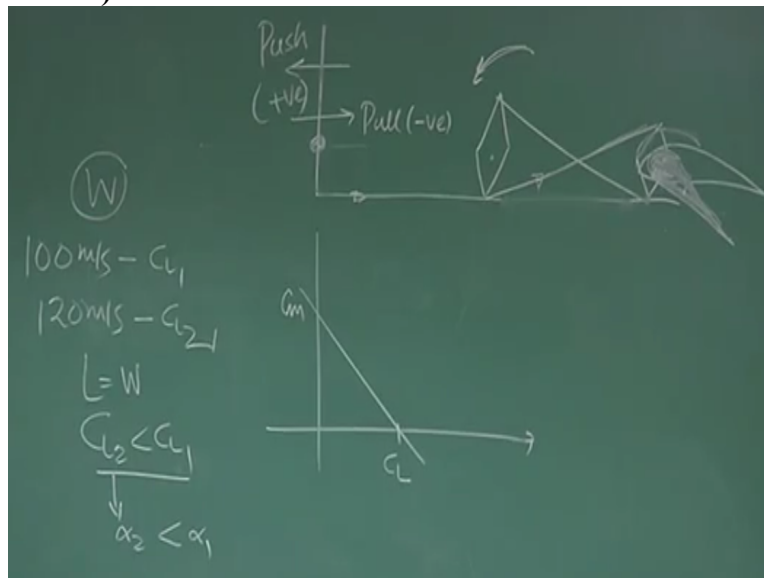
And if V is greater than V_{trim} then applied a push force. Please watch this carefully. It says that if I am flying at a V_{trim} if I want to trim the airplane at a V which is less than V_{trim} the statement is if I want to trim the airplane at a V less than V_{trim} then I have to apply a pull force that is if I am going speed say 100 meter per second now I want to trim meet at 90 meter per second keeping unaccelerated flight remember. This is the catch statement keeping unaccelerated flight I still remain lift is = wait no acceleration right.

Then I should be able to do through pulling the stick I m pulling as a pulling the aircraft reduce the speed. If you want to fly and trim at V which is greater than the present V_{trim} they say present in 100 meter per second I want to trim the airplane at 120 meter per second so I need to

push the stick, because psychologically it goes well and pushing the aircraft it is going to higher speed. So this handling become is we need to satisfy.

It is not only developing a model FS but also must take care of this requirement from handling quality of the airplane this part is clear? Right? So, we this now let as develop stick force expression it may loop it will involved lot of term will come do not get scare try to understand its term and any doubt it have got simply post it in the block.

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Time has come to revisit this diagram it is the stick and you know by now you know this is push and we will be taking a convention push will be taking as a positive and this is the pull, pull will be taken as negative right. See what happens, if I push this stick it rotates about this point okay. Rotates like this, push is this rod, push is this rod and this is the elevator is coupled here this elevator will go down like this come down like this it is clear.

If I am pushing this stick which is going like this so this there will be a forcing this direction that will trigger, this will rotate about this, so this elevator will go down and this is true. Please understand if I am flying at a particular and it is trimmed right. I want to go far another trim that is lift = weight but the speed is higher because we want to push it, I want pushing it means I want increase the trim speed.

Now if I am going at higher speed still lift will be = same weight so, CL has to reduce CL reduce means the angle of attack as to go down so, that is possible only when the elevator goes down which gives a nose down moment so angle of elevator reduced appropriately. So, this is consistent with whatever we have learned. I repeat here again so to that you do not get messed up this is CM and CL right.

If I am going to speed up it means what? We want to speed up suppose I have I want to trim at 100 meter per second. I want to trim at one twenty meter per second correct. Say weight same as altitude is same so this cause want to CL say want CL 2 as they I have increase the speed if lift is to be = the weight which as not change than speed as increased so CL to as to be less than CL one right.

The lift is equal to same weight, CL 2 less than CL one how it can do you have to increase the angle of attack if you want to make CL 2 less than CL one ensure CL 2 less than CL one that means the Alpha at this point this less than Alpha here so Alpha here is less than Alpha here so how do I reduce the Alpha by giving elevator down, which gives the nose down moment so, airplane does like this correct. So, that is the consistency which will be designing expression for stick force clear.

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The image shows a chalkboard with the following handwritten equations:

$$\frac{l_s \delta_s}{2} \cdot F_s = \frac{HM \cdot \delta_e}{2}$$

$$F_s = HM \cdot \left(\frac{\delta_e}{l_s \delta_s} \right)$$

Below the second equation, the term $\left(\frac{\delta_e}{l_s \delta_s} \right)$ is circled and labeled "Gearing Ratio".

$$F_s = G HM$$

If I say this diagram and I write something called hinge moment here and this is FS and it could see that in a very preliminary analysis okay, if I take this distance to be LS and I say during

rotation it has become Delta S so roughly the work done. On the stick is LS into Delta S let's say average divided by 2 into of course whatever stick force we have applied, and thus the work done on the top of the stick, work done on the hinge moment are on the hinge roughly will be hinge moment into Delta E, whatever deflection average by 2.

We see without writing with this expression you could see this stick force and hinge moment are related in the fraction but there is some sort of a gearing that involve this I call gearing ratio. So I can write FS equal to gearing ratio into hinge moment.

Of course moment and forces are related it could see here some force here generating here applying here some moment is there to overcome the moment are to be counter moment right? Because as the elevator goes down it will have a stick CH Alpha will try to opposite so, that's why this is the moment and this is the force and they the gearing constant okay? G.

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$$F_s = -G \cdot HM$$

$$HM = \left(\frac{1}{2} e v^2\right)_e \cdot S_e C_e C_h$$

$$F_s = -G \left(\frac{1}{2} e v^2\right)_e S_e C_e C_h$$

$$F_s = -G \frac{1}{2} e v^2 S_e C_e C_h$$

F_s is the Stick Force Required
F_s +ve Push
-ve Pull

Now the question is, question is simple question is FS required by pilot. Remember when we are talking about the stick fix case what was the question we are addressing, what is the Delta E required for a particular CL trim, right. Here we will be talking about what is the FS required so now it is very straight forward and we will see we will write, we will go on writing expression so FS required will be - G gyration hinge moment could understand the relationship between FS and the hinge moment and now we need to right what is hinge moment okay?

Hinge moment is already we know this is I can write half ROW V square at tail into S TAIL or S elevator now please understand we should not committed in mistake we are not talking about elevator I have just used the what tail but we should not forgot we all focus to elevator right? this is again I repeat here this is the tail this portion elevator is the hinge line this is from hinge line to this is C well done and the SE is the elevator area right?

We are focusing on the elevator so hinge moment like a that a half ROW V square at tail I used ever scribed elevator of course for this case repressors elevator and approximately SE into CE right? And to CH CH is the hinge moment coefficient hinge moment is given by half ROW V square SE CE into CH so FS I can write as - G into half ROW V square and elevator or tail for this case SC CE into CH so this can I write as - G Q is the dynamic pressure.

So I multiplied by NT or it is the ratio of a pressure or tail or elevator of free stream pressure than SE CE into CH let me clarify this.

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The image shows a chalkboard with the following handwritten derivation:

$$= \left(\frac{1}{2} \rho v^2 \right)_{e,t} \cdot \left(\frac{\left(\frac{1}{2} \rho v^2 \right)_{e,t}}{\left(\frac{1}{2} \rho v^2 \right)_{fs}} \right)$$

$$= q \eta_t$$

And we have to get half ROW V square at elevator or tail for this so this I can write as half ROW V square free stream into half ROW V square elevator or tail divided by half ROW V square free stream. This is = half ROW V square elevator or tail this is now written as Q is the dynamic pressure which is here and this is NT the ratio dynamic at tail is this from here how I come to this point and SC CE CH as simple so I put FS please ask yourself what is the question?

Question is FS is what? FS is the stick force required to be applied by a pilot okay? So, in write FS is the stick force required right? What FS positive FS positive is push and negative is pull I repeat here FS is - G H M this is the FS required by the pilot than HM is hinge moment is half ROW V square there a pressure at the tail or the elevator SC CE into CH and this have written as - G Q NT SC CE CH which I have explained.

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The image shows four lines of handwritten mathematical equations on a chalkboard background:

$$C_n = C_{n_0} + C_{n_{\dot{\alpha}_t}} \dot{\alpha}_t + C_{n_{\delta_e}} \delta_e + C_{n_{\dot{\delta}_t}} \dot{\delta}_t$$

$$\dot{\alpha}_t = \dot{\alpha}_w - \dot{\epsilon} - l_w + l_t \quad \epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} \alpha_w$$

$$\dot{\alpha}_t = \dot{\alpha}_w - \dot{\epsilon}_0 - \frac{\partial \epsilon}{\partial \alpha} \dot{\alpha}_w - l_w + l_t$$

$$\dot{\alpha}_t = \dot{\alpha}_w \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right) - \dot{\epsilon}_0 - l_w + l_t$$

Now let's go next term what is CH we are all expect of CH not + CH Alpha T into Alpha T + CH Delta E into Delta E if there is a tab than I write CH Delta tab into Delta T okay? Let us understand? What was CH Alpha T CH Alpha T was this the tail plane and this an your elevator okay? If there is a angle of at the tail there is a pressure distribution over the elevator and if this is a hinge line the center pressure behind than because of this Alpha T.

This elevator will generate nose down moment about the hinge here and hence this will try to float like this so we said CH Alpha T is less than 0 that is for a positive Alpha it is given nose down moment right? That was CH Alpha T what was CH Delta E you know that Delta E positive is Delta E positive is like this here also see this is the hinge line is Delta positive you meet the tail pressure distribution and here is the center of pressure that will also try to give a nose down moment it will discourage resist.

And that is $C_H \Delta E$ which is also less than 0 and what was $C_H \Delta T$ let us see this this is your tail plane and let say the elevator and somewhere here right? To more consistent diagram have already drawn we see that diagram.

Will better free they draw it like this elevator up like this and tab is here this tab is that into tab setting angle ΔT positive so $C_H \Delta T$ is also negative because I just deflect that the force here that gives a moment knows down. So, this tab is used to and show that this elevator whatever is floated finally the angle is what exactly is dictated by $\Delta E = \Delta E_0 + D \Delta$ it but CL into CL trim okay?

To this sign is also negative it could see that it can always fly an airplane by setting the ΔT this very important is understand after I ΔT is used to give appropriate deflection to the elevator by creating the moment, There so I can fly an airplane trim an airplane fly means trim. I am talking about using ΔT is it trim or trim tabs and you see this facility you will utilize to calculate some interest in parameter of an airplane right? Okay?

With this better understanding, let us go forward finally we are planning to get stick force and quickly we want to come out of all of this, we have got lot of expression in demands. So, we will now see ΔT remember we have developed this expression for ΔT in longitudinal case and it can refer that this is $= -\epsilon - I_W + I_T$ you can check when we are writing the contribution of wing and tail all stability, second week lecture, this is I can write as let me write fast $\Delta T = \Delta W - \epsilon_0 - d \epsilon$ by $d \Delta$ into $\Delta W - I_W + I_T$

Nothing I have done, I have expanded ϵ which we have done earlier as $\epsilon_0 + D \epsilon$ by $D \Delta$ into ΔW so now if I take common I get ΔW one - $D \epsilon$ by $D \Delta$ - $\epsilon_0 - I_W + I_T$ right?

Now what will I do, there is one trick I will do that and for all of your information this material is from Airplane Stability and Control or Airplane Performance by Perkins and Hage. So I will give you the exact detail by next lecture, Perkins and Hage, very popular book, old book Perkins and Hage. So I will give the exact detail of the volume and edition and all in next lecture.