

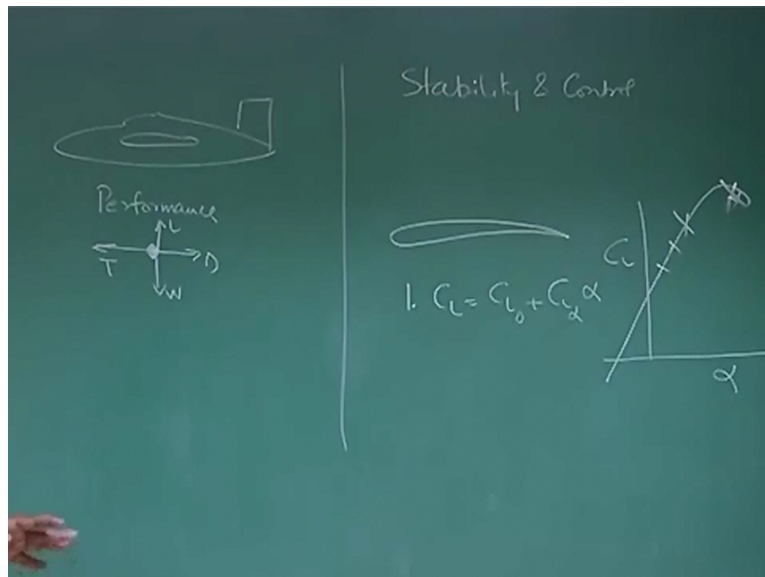
Aircraft stability and control
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Lecture- 03
Stability and Trim

Good morning, we will be discussing about stability and control of an airplane. The aspects which need to be understood in the exhausted manner, we will be putting extra effort on that, and there are some mathematical parts, which we will also mention but, it is expected that you will refer text book to understand the mathematics related to understanding of this phenomena. If I take you back to airplane performance.

Let us understand one thing when you are trying to understand airplane performance, we will have a different approach we will see an airplane in a different manner, then when you are trying to understand stability and control, we will try to understand an airplane same airplane in a different manner.

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For example, when I am talking about performance, even if there is an airplane our approach and understanding it was through point mass model that is we assume that all the masses are concentrated at the center of mass, why? Because this was okay for us because we wanted to know the response of this airplane in a rectilinear motion, or response of this airplane for the given external forces right?

No angular rotations were considered, since no angular rotation was considered so we are happy with the point mass perception, and in that the moment I talk about performance, this diagram was good enough, trust equal to drag lift equal to weight that was good enough and we took each component.

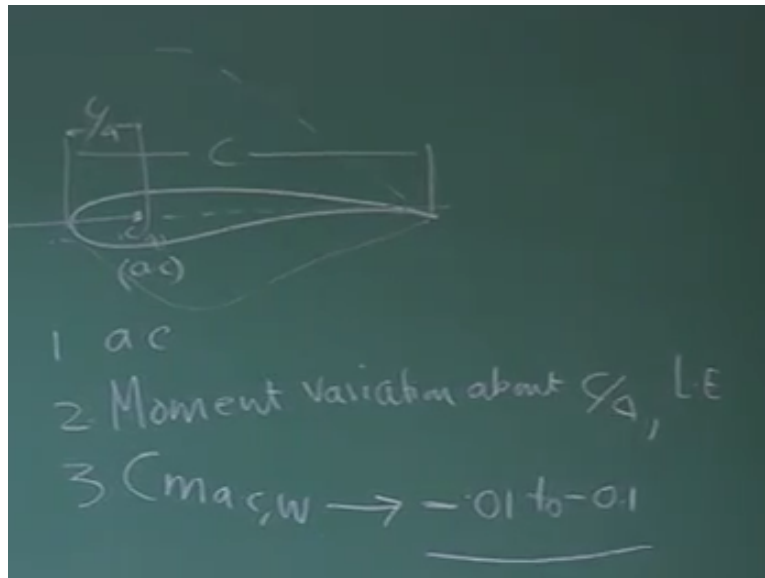
The drag lift thrust and weight and try to understand the implication of this in terms of airplane performance which are primarily take off prime, cruise, lighter, landing and manual okay? And of course yes, we also wanted to have an understanding about what is the ratio of L by D means to us for an aerodynamics and for a flight mechanics man, typically we try to understand what is the significance of having CL by CD maximum.

But nowhere explicitly, we talk about in a rotational motion. Now see when you come to stability and control, this will not work okay? How will you see this for example in performance if we are visualizing a wing we are only concerned about one, how CL is modeled $CL_0 + CL_{\alpha}$ into Alpha, that is you know we have CL versus Alpha graph, and why it was required from performance point of view.

Because I wanted to know if required to generate this much of CL, what is the angle of the attack required? I will ensure that the angle of the attack is not here, it should be somewhere here, somewhere here in the linear zone, (()) (03:44) you should not going to the stall, so that was the only thing we are looking for an aerofoil, explicitly we would like to know that whether the aerofoil is cambered or symmetric depending upon what is the value of CL_0 , and how I can see model CL? With this specific goal in performance.

I want to know whether the CL, which is required to balance lift equal to weight can be achieved with a requisite Alpha or not? And the Alpha should be such that it does not go into the stall, right? So that was the performance perception.

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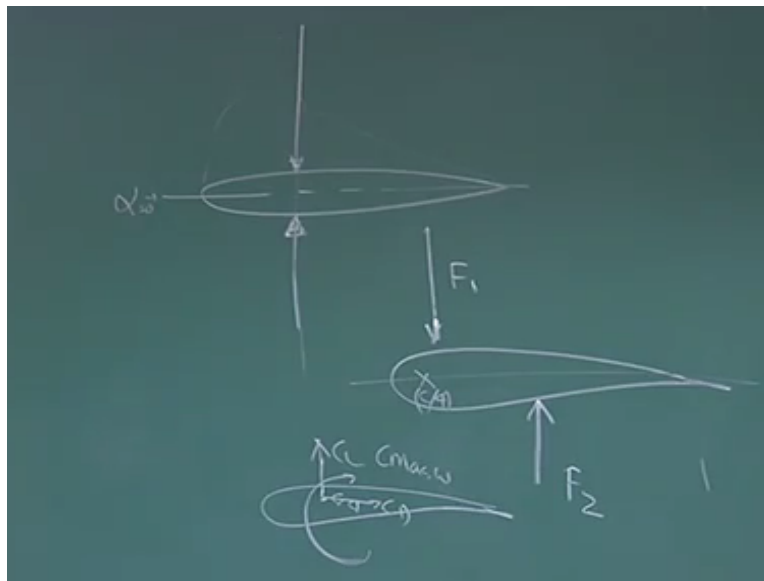
But now from stability and control point of view same wing, now our approach will be different. What we like to know first of all, where is the aerodynamic center which is which was also there here in this case. Also last case performance also, and if we recall the aerodynamic center is a fictitious point about which the pitching movement is independent of angle of a attack. And typically it is at C by four or quarter chord point.

That is if I tell this is at the chord, then this will be C by four correct? So one is yes we need to know the aerodynamic center, second thing which is extremely important, is how the moment variation about a point about? let say C by four it could be about leading edge of the of the aerofoil, with the leading edge of the aerofoil at any point if I know I can easily find out about any other point.

In stability and control we are not only interested in what is the aerodynamic center, we are also interested the moment variation, because of aerodynamic forces and about any point and as you know, this aerodynamic center is a point about which moment is independent angle of the attack, so it is a convenient way to represent the effect of all the aerodynamics or aerodynamic forces generated around the aerofoil, on the wing and they are represented conveniently at aerodynamic center C by four.

And as you know, ideally when there is a pressure distribution about the aerofoil, say typically let say something like this, and if you want to transfer the resultant of all these things about a point or to a point C by four, you need to transfer into the force, net force and a moment okay? And that is why it is extremely important, to also know what is CMAC wing. This was never considered in performance, what is CMAC wing let us have some understanding, we have already explained this and we will showing you those lectures.

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But just to have little bit of warm up for newcomers, If say this an aero file which is symmetric, and you know let's say at Alpha equal to zero, if there is a pressure distribution and further there is a net force in the top surface, I can get by integrating the pressure distribution over the area, similarly in the bottom surface we will also have in this direction right? And if it is symmetric these two points will or these two forces resultant forces will be on the same point, and they cancel each other.

That is why you say at Alpha equal to zero the net force will be zero, and net moment also will be zero, okay about any point. But if it is a cambered aerofoil, you will find on a top surface if the forces in direction, the bottom surface this is F1 this is F2 the bottom surface.

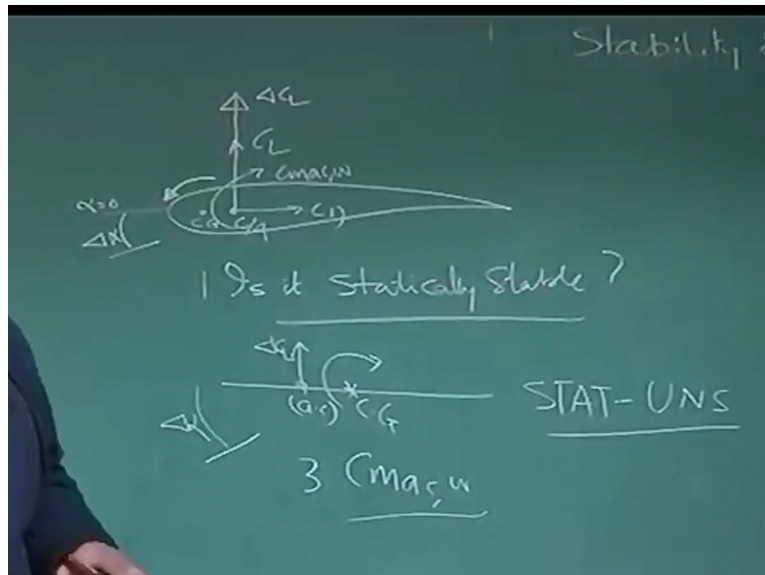
Is not align with the top surface resultant, and so net force will be definitely F1- F2, however, when I try to shift this point shift these forces to a point C by four, so now when I shift this force

from here to here, I would shifted by the force and the moment, so this also a force into moment so finally what you will find? This will have cambered aerofoil at C by four, will have CL, CD, and CMAC wing. What CMAC wing is consecutive moment that came into existence because, we are transferring all these forces to a point C by four right?

And that is an important thing what we must understand when you are studying stability and control okay? Typically, for a cambered aerofoil these values are minus let's say point zero one, to minus point one typical number. So If I see this cambered aerofoil, it has a consecutive moment about C by four which has a negative sign, and we know as our pitching moment is concerned, the convention is nose up is positive nose down is negative right?

So this a difference major difference that I see that there will be a CMAC wing are that is going to create an effect terms of the angular motion of the airplane okay?

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Now let us go little bit more into it before we go for our regular systematic build up, so let us now do an experiment, this is a cambered aerofoil and you this is CL, CD and CMAC, wing and we know CMAC wing is negative. Suppose if I throw this cambered aerofoil or a wing made up of a cambered aerofoil, if I throw it what will happen? Now, there are two cases I will discuss one assume that.

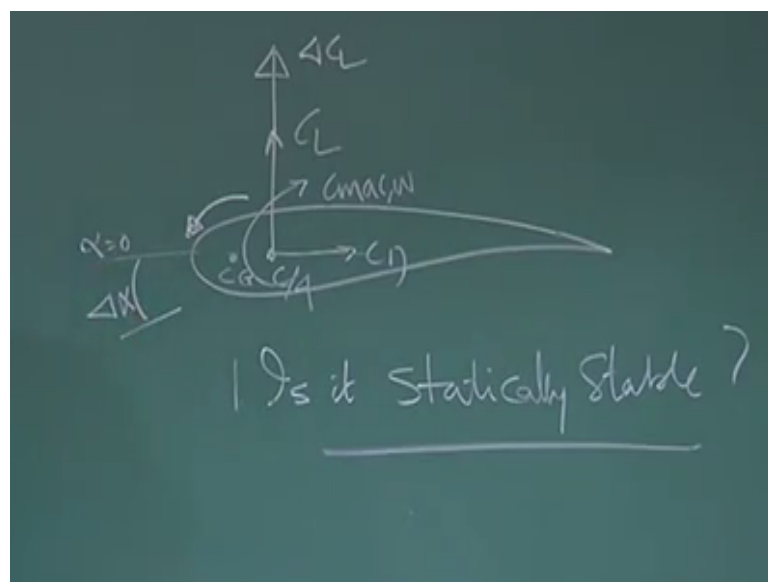
The CG is ahead of C by four, why I have brought CG now that you know in free space when I throw something or somebody is flying or some aircraft flying, this rotation will be about an axis passing through the central of gravity right? In free space rotates about an axis passing through central gravity. So now let's see when I am throwing this wing made up of cambered aerofoil.

And if the is CG ahead of it, what sort of response you expect? First of all, I will ask a question is it statically stable? Right, as I was telling let us do an experiment, this is a wing made up of a cambered aerofoil, and let say this size I am just throwing it in air. The first question which comes to my mind is, is this configuration statically stable or not?

And you remember what is the meaning of static stability? Static stability means if a body is distract from its equilibrium state, and if it has initial tendency to come back to that equilibrium we say it has static stability right. Now see here, If I am throwing it like this and let say some disturbance of Delta Alpha is here, okay and then what did Delta Alpha will do? you know that it will give additional lift Delta CL also Delta CD but I am taking Delta CL.

We are being predominant in this case, and then what happens the Delta CL will do what? It will give a nose down moment okay, correct? That means it will try to nullify this disturbance, it will try to come again back to the Alpha equal to zero which was perhaps this is the case right.

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So it will try to give a nose down moment, So it has initial tendency to counter this disturbance initial tendency to come back to Alpha equal to zero, initial tendency to come back to its equilibrium. So this is a statically stable case and we know very well, as long as aerodynamic center is behind center of gravity it will be statically stable. If it was a other situation if AC was here and CG was here then you could see slight disturbance.

Delta Alpha, there will be a Delta CL here and that will give a nose up moment and the angular disturbance will further increase and it will not have a initial tendency to come back to Alpha equal to zero or Alpha equilibrium. So it is statically unstable right. This is one thing, now let us see what is this CMAC will be doing right? CMAC wing will do. CMAC, wing negative cambered aerofoil, so if I throw it CMAC negative means it will try to always give a constant negative moment. It is not proportional to angle of a attack in constant negative moment so the airplane or the wing here will have a tendency to do like this, it will very difficult to trim, even if it is statically stable right?

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The image shows a chalkboard with the following handwritten equations and a diagram:

$$C_L = \frac{Lift}{\frac{1}{2} \rho V^2 S}$$

$$C_D = \frac{Drag}{(\frac{1}{2} \rho V^2 S)}$$

$$C_M = \frac{Pitching\ Moment}{NS\ (P.M)\ Coeff\ (\frac{1}{2} \rho V^2 S) \bar{c}}$$

To the right of the equations is a simple diagram of an airfoil with an angle of attack α indicated by a horizontal line and an arrow pointing to the right.

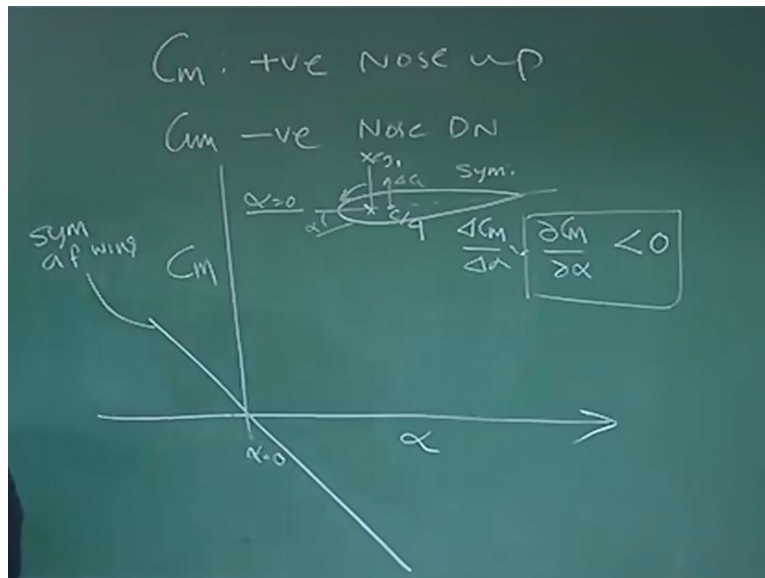
If I go back again, we define CL as lift divided by half rho V square S, we say lift non-dimensionalised by dynamic pressure and area okay. Similarly, CD we know it's a drag divided by half rho V square S again you could see here CD is nothing but which we term as drag coefficient is nothing but, total drag non-dimensionalised with dynamic pressure and area. Now what is CM? CM is the pitching moment.

What is this pitching moment? Let us understand, if this is the airplane, and let's say this is the CG and let me draw it I will explain, is there X, Y, Z axis passing through center of gravity the pitching moment is motion about Y axis. So it goes up sign is positive, goes down sign is negative, Yawing and about Z axis is a Yawing motion that is it a Z axis this is the Yawing motion, if the right wing going back, that means Yawing moment is positive, left wing going back Yawing moment is negative and about X axis is a roll so.

If right wing is going down it is a positive rolling moment and left wing going down is a negative rolling moment right. We are at present talking about pitching movement, which is essentially motion about Y axis okay correct. Nose up positive, nose down negative. So what is CM? CM is again is a pitching movement coefficient like it was lift coefficient it is a pitching movement coefficient right.

How do I define it is the total pitching moment about center of gravity divided by half rho V square S, this is a dynamic pressure and into area reference area but we have to non-dimensionalised this. So we have to put a length down so generally we put mean aerodynamic chord. Okay, this is a definition of CM correct.

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Now let us see either CM positive nose up, CM negative nose down, if I plot, CM versus Alpha, let say for this case, I take a symmetric aerofoil or to more precise wing having symmetrical

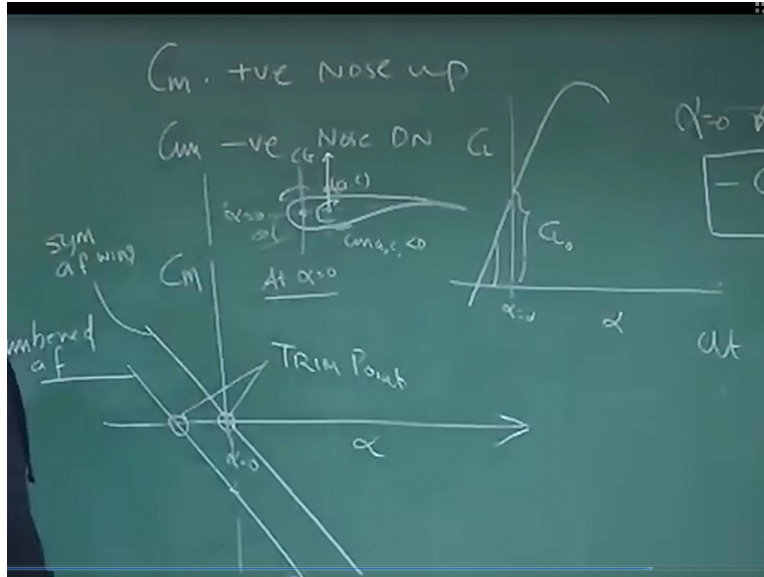
aerofoil and let say this is C by four and let say this CG expression, X_{CG} okay, X_{CG1} I put. Now, here If I want to plot qualitatively the variation of C_M and α (17:13) how do I do that?

See, first I have to assume, with my understanding that no angle of attack C_M versus α variation will be linear, right, okay. So if it is linear, if I drawing a line I need to know its intercept and slope okay. So what is the slope between the C_M and α that will first trying to see in terms of sign. From here what do you see, if I am give a some α to this wing having symmetric aerofoil immediately I know there will be a ΔC_L and that will give me a negative moment, or pitch down movement. That means for a positive α , I will have a negative movement.

Similarly, for a negative α , I will have a positive pitching movement is it clear? So this much I know that the C_M α that is ΔC_M by $\Delta \alpha$ or if I write ΔC_M by $\Delta \alpha$ these signs should be negative okay. So once I know this slope is negative I need to find out what is the intercept because we have assumed that C_M versus α variation is linear. To get an intercept means I need to know what is the value of C_M at α equal to zero since this is symmetric, aerofoil wing, I know at $\alpha = 0$ there will not be any C_L right, and there is no C_{MAC} also for a symmetric wing.

So at $\alpha = 0$, C_M will be always be zero so this is the intercept here and slope is negative so I will draw this curve C_M versus α like this, check it here, the slope is negative yes slope is negative. And intercept α is equal to zero there will not be any C_L and there is no C_{MAC} because of symmetric aerofoil wing. So that will be zero so this is the line for symmetric aerofoil wing right? Now, let us see we do one change, this is important please be careful.

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What we do is, we now take a cambered aerofoil and AC is exactly here where earlier it was there CG is also exactly where earlier it was there right only the aerofoil has changed nothing has changed. In this case if you see again the slope C_m vs α will be what positive or negative we could see here if I give a disturbance it is like this and nose down moment and I know that since the aerodynamic center is behind center of gravity it will be statically stable as we see here the aerodynamic center is behind center of gravity and you know it will statically stable.

So in our language now we say the DCM by $D \alpha C_m$ will be negative. So slope will be negative no problem but what about at $\alpha = 0$ what will happen that is the question. Now see here the moment I draw cambered aerofoil I should also understand it will have C_{MAC} from negative value which I have not drawn earlier. So at $\alpha = 0$ what will happen let's see if I see C_L vs α for a cambered aerofoil at $\alpha = 0$ it will always had positive lift coefficient C_{L0} .

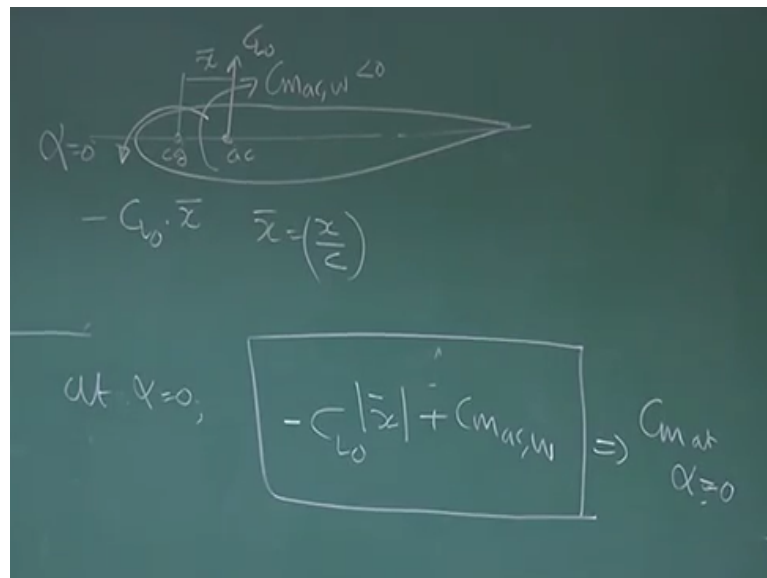
Correct? So that will also act here or let me draw a neat diagram so that you're not confused we are trying to find out at $\alpha = 0$, we know that $\alpha = 0$ there will be C_{MAC} wing which is less than a zero now at $\alpha = 0$ what happens at $\alpha = 0$ you could see from this graph for a cambered aerofoil there will be a positive lift coefficient I name it as C_{L0} so this C_{L0} will act here. For a symmetric aerofoil the C_{L0} value will be zero but for the cambered aerofoil there will be a positive lift coefficient this.

This will do what this will also give a nose down moment which is C_{L0} into let's this is be \bar{x} that is \bar{x} and I put the minus sign if I take \bar{x} positive \bar{x} is the thing but this physical distance divided by chord clear. C_{L0} into \bar{x} and if I assume this sign to be positive.

Then I need to generate a negative moment represent a negative moment because the C_{L0} will give a nose down moment is it clear? So at $\alpha = 0$ we have C_{L0} into \bar{x} I put absolute signs for that no confusion with sign as long as we put a minus sign here over and above what will be there over and above CMAC wing is here CMAC wing. So this will be basically your CM at $\alpha = 0$ are you clear with this?

I repeat this is cambered aerofoil at $\alpha = 0$ there will be a C_{L0} which I know for a cambered aerofoil CL versus α looks like this. So this C_{L0} into this physical distance which I have non-dimensionalized with chord and that will give a nose down moment so we have put this term okay if you have any confusion we can put a bar here.

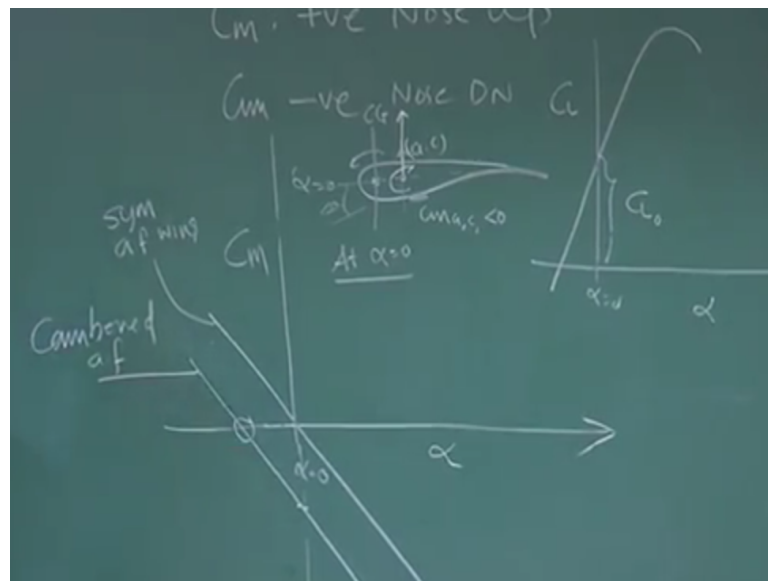
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And put this minus sign then no confusion this is the physical distance between AC and CG non dimensionalised with the cord. So that one moment will come at $\alpha = 0$ in addition do not forget because of cambered aerofoil CMAC wing already there so that also will get added so total of this see it will generate a large negative value okay to see that this part is negative this is \bar{x} is behind CG it is statically stable.

So it will give a nose up moment so negative value C_{MAC} wing is always negative for cambered aerofoil the total will be negative so if I come back here for a cambered aerofoil for this slope will be negative, which ensures statically stable but the intercept will be negative this is for cambered aerofoil wing. See it has static stability because AC of the wing is behind CG of the wing however because of cambered aerofoil at $\alpha = 0$.

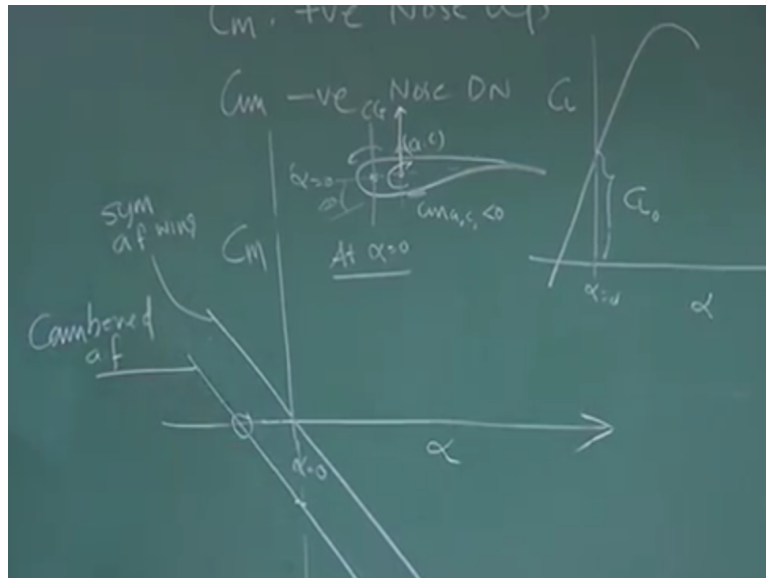
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It will have contribution of negative moment because of CL_0 and because of concentrated C_{MAC} wing so in this intercept will be negative. Now what is the repercussion of this why you want fly you want to fly at an angle of attack so that it produce enough lift to balance weight now if you're flying at this point which is negative angle of attack you may not be able to generate that much of lift even if it is cambered to produce the enough force for a restricted speed to balance the weight.

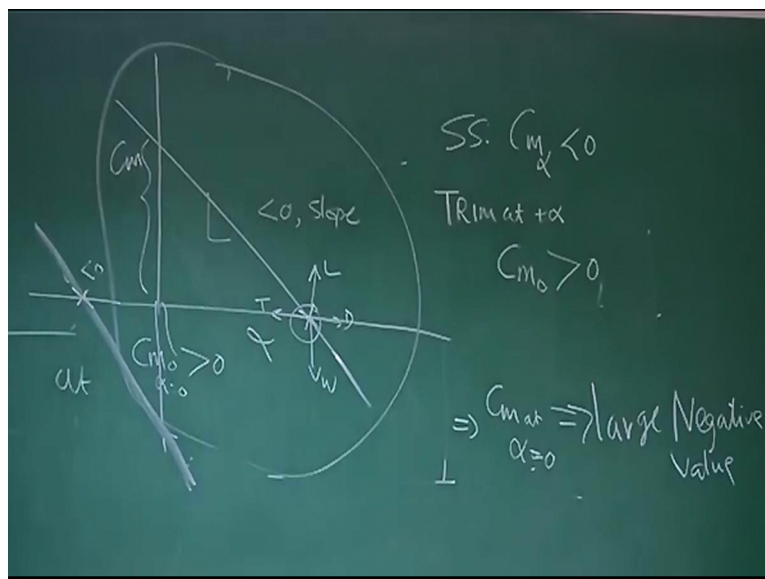
So that is why we have selected this point because these are called trim points why they are called trim points you will be knowing it explicitly why they are called trim points because these are the equilibrium point at which net moment net force is zero or in this case net moment is zero right.

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So if I want fly cruise I want the moment should be zero so my trim point for this aerofoil configuration is this for symmetric it is this for symmetric unfortunately $\alpha = 0$ so I will not be able to produce any lift so this is meaningless for me for cambered theoretically speaking some lift to generate because of some angle here but that is not a good way of flying when you have velocities so this is also not accepted for us.

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What we look for this can I configure whole airplane so that C_m and α variation look like this that is its slope is negative so statically stable slope is negative slope so statically stable and C_m at $\alpha = 0$ which you called C_{m0} is greater than zero what is the advantage here this

variation will tell you the yes statically stable but trim point is that a positive angle of attack, so I can easily generate a configuration where lift = weight and thrust = drag.

I can fly the machine at the positive angle and give enough speed so that in our (()) (27:12) dynamic pressure so that I get enough lift to produce force which balances the weight okay so why my aim is to fly at a positive angle of attack. So that is why most of the airplane will be designed in such a manner that CL vs Alpha graph (()) (27:32) is like this or in a nutshell that CM_0 is get us a zero and CM_{α} is less than zero CM_{α} less than zero ensures.

It is statically stable and CM_0 greater than zero ensures that you can trim the airplane at positive angle of attack. If CM_0 was not zero if it was somewhere here even it was statically stable the slope was negative you are not able to trim it at positive angle of attack. If the angle of torque less than zero we do not want that we want this and trim my airplane $\alpha = 0$ so what are the conditions I must have for static stability I must ensure CM_{α} less than zero and for trim positive Alpha at positive Alpha.

I must ensure CM_0 is greater than zero how much CM_{α} how much CM_0 these are matter of details and as you go through the lectures you will get this would be answered. Is it clear so up to this point this is totally (()) (28:31) warm up. You have got enough exposure what you are going to have for first 10, 15 lectures and my lectures on static stability will continue after one or two lectures of this and we suddenly enter into again with something called dynamic stability or lateral stability directional stability I do not want to mix up mind focus here try to understand the static stability concept.

And once my other force surround seven lectures are over we will be again discussing about little more on this topic in a lateral mode, directional mode and we have a nice time we will be solving lots of the examples in this course, please remember lot of numerical problem we will be solving here may be in a week if the full lectures there might be one or two numerical lectures or lectures solving in numerical okay. So please when you are watching my video take a note book calculator pen and try to solve those problems okay thank you very much.