

Aircraft Stability and Control
Prof. A. K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture- 30
Aircraft Handling qualities Continued

(Refer Slide Time: 00:16)

F_s required by pilot

$$C_l = C_{l_0} + C_{l_{\alpha}} \alpha_t + C_{l_{\beta}} \delta_e + C_{l_{\delta_t}} \delta_t$$

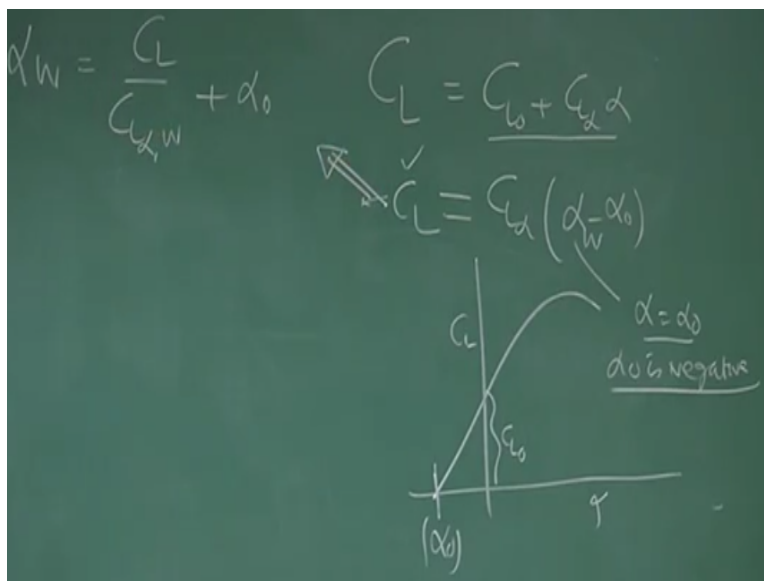
$$\alpha_t = \alpha_w - \epsilon - i_w + i_t \quad \epsilon = \epsilon_0 + \frac{\partial \epsilon}{\partial \alpha} \alpha_w$$

$$\alpha_t = \alpha_w - \epsilon_0 - \frac{\partial \epsilon}{\partial \alpha} \alpha_w - i_w + i_t$$

$$\alpha_t = \alpha_w \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) - \epsilon_0 - i_w + i_t$$

Let us come back here. So Alpha T is this.

(Refer Slide Time: 00:20)



Now you know that Alpha W I can write as CL by CL Alpha wing + Alpha 0, how? Please give a thought to this. It should not be so difficult, yes, or no? Well let's see. CL is equal CL0 + CL

Alpha into Alpha, or I can write this as CL Alpha into Alpha - Alpha 0, that is if you see. If this is the CL and this is the Alpha, so this is CL0, one way to write is this, another this is Alpha 0, what is Alpha 0? If there is physically negative angle at Alpha0 CL is 0.

So I can write CL also in this fashion, you could see at Alpha = Alpha 0 CL is 0, Alpha = Alpha 0 CL is 0 and you could see from here that CL at any Alpha I can find out okay. If at Alpha = Alpha 0, you could see CL will become 0. And typically Alpha 0 values negative. That is very important if you understand Alpha 0 is negative. So I can use this expression here and write here. Using this expression Alpha W, here I put Alpha W = CL by CL Alpha wing + Alpha 0, is this clear?

(Refer Slide Time: 02:02)

$$d_0 \frac{d\epsilon}{d\alpha} = \epsilon_0$$

$$\alpha_t = \left(\alpha_0 + \frac{C_L}{C_{L\alpha, W}} \right) \left(1 - \frac{d\epsilon}{d\alpha} \right) - I_w + I_t - \epsilon_0 C_h$$

$$\alpha_t = \alpha_0 + \frac{C_L}{C_{L\alpha, W}} \left(1 - \frac{d\epsilon}{d\alpha} \right) - I_w + I_t$$

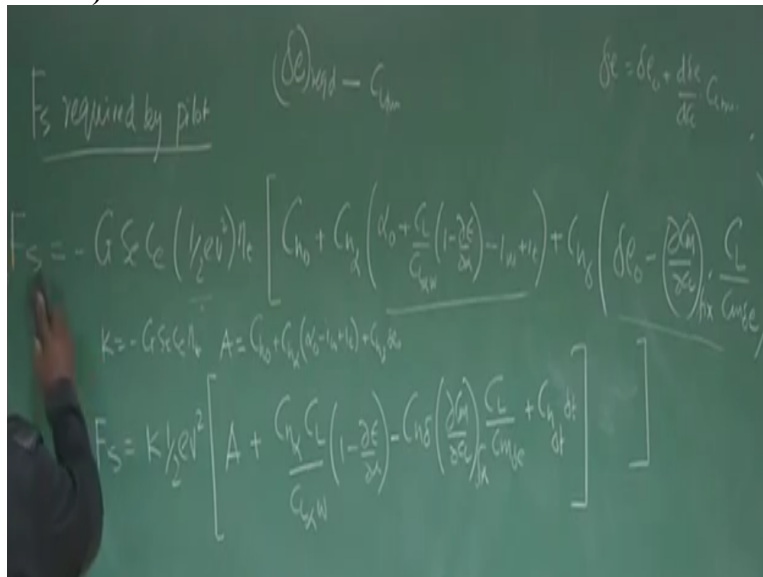
Once I do that now I substitute that expression here okay. So let me write it here, then I get Alpha T = Alpha 0 + CL by CL Alpha wing to 1 - D epsilon, by D Alpha - I W + I T, what I have done, Alpha 0 Alpha T = Alpha W, and Alpha W is CL by CL Alpha wing + Alpha 0, so I have written it like this, because the expression was Alpha W into 1 - D epsilon by D Alpha, 1 - D epsilon by D Alpha is here for Alpha wing I have put this expression, CL by CL Alpha wing + Alpha 0, then I have got ideal and then also I should not forget about - epsilon 0, now see whether we have taken care of everything or not.

This part we have taken care, this part - and + this have been take care, and epsilon 0 we forgot to take, if it is here right. Now if I expand you could see here, this Alpha 0 into D epsilon by D

Alpha is nothing but epsilon 0, but Since Alpha 0 is negative, so this and this will this time will go after multiplication, so I get Alpha T = Alpha 0 + CL by CL Alpha wing 1 - D epsilon by D Alpha right, - I W + I T. This is the expression okay, this is Alpha 0 into - D epsilon by D Alpha, I see Alpha 0 is negative and you know this into this is nothing but epsilon 0.

So, this and this will get canceled. So, this term will disappear, so you will get a neat expression of Alpha tail in this fashion okay, let we erase this because lot of expression are waiting, I repeat again don't get disturbed by this expression, anywhere if you have a doubt put in the blog in the open forum, we are there to help you out right. We have already straight forward things, looks clumsy and I also personally feel very uncomfortable, if I tell you honestly speaking. Let me erase everything now here.

(Refer Slide Time: 04:24)



So we are now here $F_b = -G \rho V h$, this is nothing but Q into CH , so for CH will write $CH_0 + CH \text{ Alpha into Alpha tail}$. Already Alpha T expression you have derived, so let me write here that is $\text{Alpha 0} + \text{CL by CL Alpha wing, to } 1 - D \text{ epsilon by D Alpha} - I W + I T$ this is this then you have CH , Delta and what is the Delta required. You know Delta required will be always dictated by $\Delta E = \Delta E_0 + D \Delta E \text{ by DCL in the CL trim and } D \Delta E \text{ by DCM will come from DCM by DCL fix}$.

So will write it as $\Delta E_0 - DCM \text{ by DCL fixed into CL trim divided by CM Delta E}$, this expression you know that, you are $\Delta E = \Delta E_0 + D \Delta E \text{ by DCL trim into CL trim}$

will see, that anything I have put here and now we have to put other term that is CH Delta T into Delta T because, of attack this completes the huge expression which was waiting to come on the black board, it doesn't look that ugly if you could see that. This is already we have derived Alpha T just now, this we have been using for long and this tab we have also discussed today.

Now what? Make it simplified, so we write it as FS = K, wait a minute I will explain you what is K, A also coming here, CH Alpha CL by CL Alpha wing into 1 - D epsilon by D Alpha - CH Delta into DCM by DCL fix, CL by CM Delta E + CH Delta T into Delta T this is the equation. Then question here what is K and what is A? This K is - G AC CE Neeta T and A is another big expression.

This is all CH0 + CH Alpha into Alpha 0 - IW + I T + CH Delta into Delta E0, where from these things are coming, we could see here that, CH0 into Delta E0 that has been taken out okay, Similarly, CH Alpha into Alpha 0 - IW + IT will be taken. This if you see to the pen and pencil, you saw paper you should be able to do this nothing, These are all big, big expression, let us try to feel out of this expression right, Now I leave an exercise to you, I am sure you will be able to do it, I erase all these things now.

(Refer Slide Time: 08:20)

$$\left(\frac{\partial m}{\partial c_u}\right)_{\text{free}} = \left(\frac{\partial m}{\partial c_u}\right)_{\text{fix}} + \frac{C_{L\alpha} + V_H \eta}{C_{L\alpha} W} \left(1 - \frac{\partial \epsilon}{\partial x}\right) \left(\frac{C_{L\alpha}}{C_{D\alpha}}\right)$$

$$\left(\frac{\partial m}{\partial c_u}\right)_{\text{control free}}$$

$$C_{m\delta e} = -V_H \eta C_{L\alpha}$$

We have already learned today, DCM by DCL free = DCM by DCL fix right, + CL Alpha T by CL Alpha wing VH Neeta T, D1 - D epsilon by D Alpha for Tow into CH Alpha T by CH Delta E

okay, this we have already you D1 it and we realize that this is destabilizing and many book it is will be saying DCM by DCL okay.

Control free, which is destabilizing for configuration where hinge line is ahead of the central pressure over the elevator. This also you need to use $VH = CM \Delta E = -VH \text{ Neeta } CL \text{ Alpha } T$ into tow. This also you know.

(Refer Slide Time: 09:35)

The image shows a chalkboard with handwritten mathematical derivations. At the top left, there is a small diagram of an aircraft with a hinge line and a central pressure point. The main derivation consists of several equations:

- Top equation: $F_s = K \frac{1}{2} \rho V^2 C_{h\delta} \delta t$ (with $F_s \rightarrow +ve \Rightarrow \text{Push}$)
- Second equation: $F_s = K \left(\frac{1}{2} \rho V^2 \right) \left[A + C_{h\delta} \delta t - \left(\frac{\partial C_M}{\partial \alpha} \right)_{free} \frac{C_{h\delta}}{C_{M\delta}} \cdot CL \right]$
- Third equation (boxed): $F_s = K \left(\frac{1}{2} \rho V^2 \right) \left(A + C_{h\delta} \delta t \right) - K \frac{W}{S} \frac{C_{h\delta}}{C_{M\delta}} \left(\frac{\partial C_M}{\partial \alpha} \right)_{free}$
- Fourth equation (boxed): $\frac{dF_s}{dV} = K \rho V \left(A + C_{h\delta} \delta t \right)$
- Bottom right note: $F_c(\text{constant})$, $(-ve) \Rightarrow \text{Pull}$

Use this into this to show that, now I can write $F_s = K \frac{1}{2} \rho V^2$ this is like this, then $A + C_{h\delta} \delta t$ into ΔT , - DCM by DCL free, now it is coming free, into $C_{h\delta} \delta t$ by $CM \Delta T$ into CL , I am sure you are smart enough to do this. Anyway we will be giving this expressions or derivations in our open forum, but I am sure it doesn't take much time to use this and get that expression right, you are all engineering students. Let us forget about all these things now, we have done enough with this expression.

And we are using the physics out of it. So let us erase this, so you can write this F_s in Neeta form, write this same expression here, this $K \frac{1}{2} \rho V^2$ right and then this $A + C_{h\delta} \delta t$ into ΔT tab - DCM by DCL free $C_{h\delta} \delta t$ by $CM \Delta T$ into CL . This further I can write as $F_s = K \frac{1}{2} \rho V^2 A + C_{h\delta} \delta t - K \frac{W}{S} \frac{C_{h\delta}}{C_{M\delta}} \left(\frac{\partial C_M}{\partial \alpha} \right)_{free}$ into DCM by DCL free, what has happened? What is the difference between these two expressions? You see $A + A \Delta T$ into $K \frac{1}{2} \rho V^2$ fine.

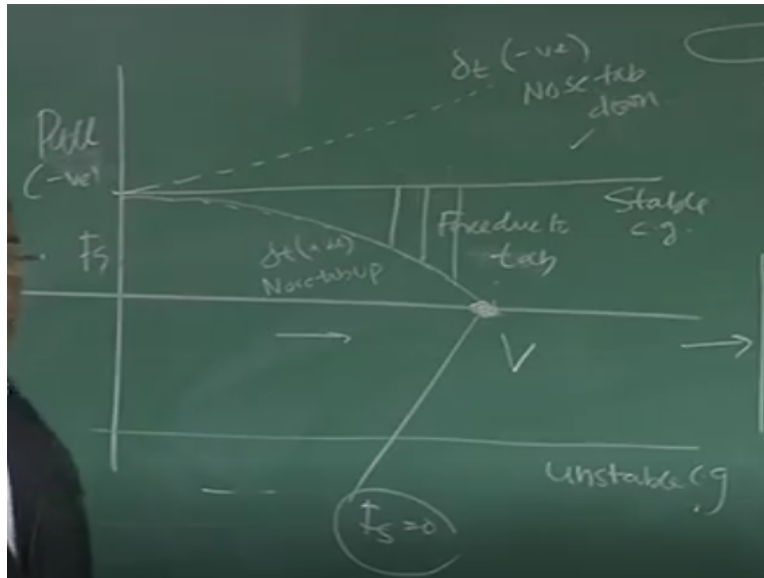
So again K half row V square has to multiply with this term, if I want to take it out but here the CL was setting here so CL I can write as, $2 W$ by S by half row V square, as simple as that right. Put that CL here multiply with this term, you will get this term that is $K W$ by S , $C_H \Delta$ by $C_M \Delta$ DCM by DCL free. So this is after lot of struggle, we got a Neeta expression okay.

What does it tell you? See all those expression α T ϵ 0 , what not? Finally what you had seen here it is very surprising that stick force primarily depend upon this term, V square it goes here, but here to see no where speed is there. This is only dependent upon the stick free status of the air plane and what are the engine moment characteristics, what is wind loading?

But nowhere V is there, did you see and here V is there, V square, that's not, isn't pretty interesting. So, we will try to exploit these, what does this mean? So then now if you see DFS by DV, it will be K row $V A + C_H \Delta$ T into ΔT . This is another D expression we are getting, this DFS by DV in general, it has nothing to do with DCM by DCL free, which is given by this we could see from here that the second term has no influence on DFS by DV.

But try to argue why it is happening like that, this is happening because please understand DCM by DCM free is relevant about trim, all this stability we are talking about the slope at trim. So if you want to really bring this DCM by DCL free in the DFS by DV or out of that stick force gradient at V trim then you will find DCM by DCL free will again re-appear okay, let us do that before you do that, we will also try to give a more insight to this expression FS, what is happening in this? This is V and this is FS.

(Refer Slide Time: 14:44)



There is a constant term which is $CH \Delta T - CM \Delta DCM$ by DCL free W by S wing loading and K , this is a constant term, which is independent of V , what is the sine of DCM by DCL free? It is -, this is -, this is -, so - - + is - and you know K is also -, do you remember the expression of K if you see that expression of K you will find there is a - sign sitting there, so this is also -, So -, - + and -, so this is a constant force FS constant, will be there and what is the sine of the contribution, this contribution is negative, so it is as well as convention it is a pull.

I repeat here again watch out for this term, second term okay; it is this. This is one, this is independent of V . Second thing what we are seeing, it depends upon of course wing loading, if I see the sign this is -, this is - this canceled sound, this is - and K is also -, if you see this expression of K , K is - hence solved, this sitting here so this whole contribution is negative so a negative constant force will be there in this expression and that is negative means a constant pull force will come.

So what you will do if I write on the top this is pull okay, which is negative so we will find a constant pull force, because of stable CG which come from DCM by DCL free bringing Negative, if it is unstable airplanes statically unstable that is DCM by DCL free was positive, then this sign will become positive for that was founded, should have been here for unstable CG, Let us focus about statically stabler. This we know fine.

Now what is happening, see the first term, this term, what is this term doing? This is $K \frac{1}{2} \rho V^2 A + C_H \Delta T$ into ΔT , assume that A is not that significant, and you could see that let me draw it and then I will explain. Let me write this, if I for time being, if I neglect this contribution of A , which is small then you could see that, if I put ΔT positive that is I will put the tab like this if ΔT is positive, okay.

Then what is happening I know $C_H \Delta T$ is negative and this is positive, positive means nose tab up, what is the meaning of that? That is this is the nose of the tab, it is up, so you know this is as for as we understand that tab has gone like this okay, so the nose has gone up. If it's that trailing edge and this is nose is up position. That is relative positive which is same as elevator ΔE positive same I mentioned. So if ΔT is positive this is negative that gives me a negative, and K is negative here so that will give me a FS positive.

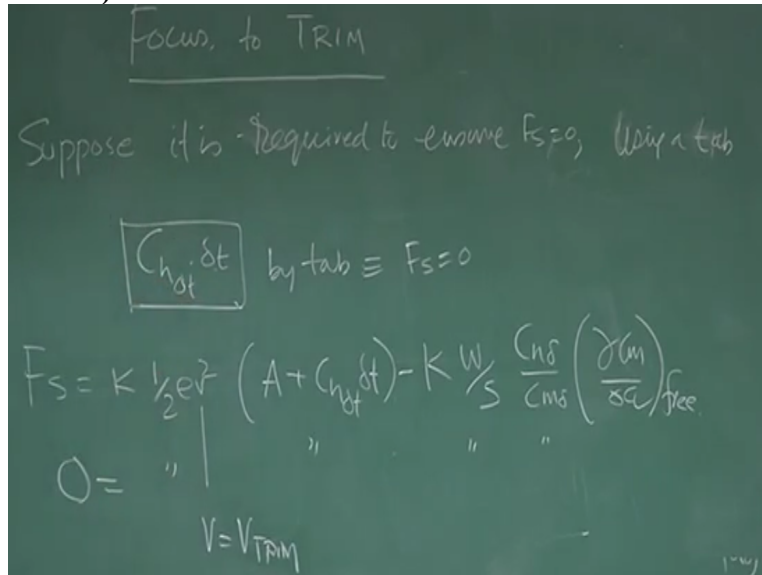
Please see this. FS because of tab will be how much? $K \frac{1}{2} \rho V^2 A$ into $C_H \Delta T$ into ΔT , if I put this positive which is nose up and this is this is what you call nose tab up, there it is positive so this is positive, this man is negative and K is also negative we know by definition, so what is happening? Positive, positive, positive into negative okay.

So, negative into negative positive. So, FS I am getting positive so this will give a push force, like I mentioned, is this clear? This is positive, this is negative and this also negative so, total sign become positive, so it make this time as V is increasing, you will find it will contribute towards opposite of it pull force, right.

So that is why If I go on giving ΔT positive, and as the V is increasing on a particular ΔT than set, as V increases see this contribution more more of positive gives, it gives and finally it nullifies here and if you see this difference is, I write it force due to tab, right. Okay which is you could see very clearly if I put tab in this configuration, then that is giving a force, stick force FS which is positive right, and this stability term was giving a negative force because, like a full force, so there will be a point where net FS will become 0.

So this is the point where how you can ensure that if it goes to 0 root tab, this is very clear, that is how I was telling you can bring a equilibrium using a tab, by proper tab setting if you understand this. Now we also need to discuss about, why DCM by DCL free was not explicitly seeing.

(Refer Slide Time: 21:26)



Now will be focusing our attention will focus attention to trim, it's very important because, we need to know what is the force gradient I need to apply and that is we have to apply at that trim okay, so let's assume that suppose it is let's say it is required, it's required to ensure $F_S = 0$ using a tab. We have seen how can you make F_S a 0 using a tab, that if I expand this statement I would say assume that trim is always deflected in such a fashion that automatically $F_S = 0$ comes right, trim tab takes care of it and we say that airplane is trim using that tab, okay, clear.

If I do that I need to know what is that $C_S \Delta T$ into ΔT contribution by tab, ensure $F_S = 0$. That you can easily find out because I know $F_S = K \frac{1}{2} \rho V^2$ into $A + C_S \Delta T$ into $\Delta T - K \frac{W}{S}$, $C_H \Delta T$ by $C_M \Delta T$ into DCM by DCL free. Question is, what is $C_H \Delta T$ so that $F_S = 0$? As we have been discussing here, let us assume that the Tab is rolled appropriately all the time, to ensure it is a trim that is $F_S = 0$ and corresponding to $F_S = 0$ will be V trim okay, with that we are now trying to find out what will be its contribution?

As you understand we want to deflect the tab such that it's a trim if $F_S = 0$, so I will put 0 here and I put this expression applied to this, so what we are looking for? We are looking for what is

this contribution of CL Delta T into Delta T that is contribution from the tab. So that it ensures that FS = 0 right, it has the trim. If I know do that i have to put FS = 0, and this expression so from this we will get see yourself that you will get an expression.

(Refer Slide Time: 24:26)

$$C_{h\delta t} \delta t = \frac{2W/S}{\rho V_{trim}^2} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{\partial C_m}{\partial \alpha} \right)_{free} - A$$

$$F_S = K \frac{1}{2} \rho v^2 \left(A + C_{h\delta t} \delta t \right) - K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{\partial C_m}{\partial \alpha} \right)_{free}$$

$$F_S = K \frac{W}{S} \frac{C_{h\delta}}{C_{m\delta}} \left(\frac{\partial C_m}{\partial \alpha} \right)_{free} \left[\frac{V^2}{V_{trim}^2} - 1 \right]$$

CH Delta T into Delta T = 2 W by S by row V trim square, CH Delta by CM Delta into DCM by DCL free - A okay. You could see here, this = 0 so I take this here and then divide by half of V square all these things you will get okay, please understand when I am putting FS = 0, I am assuming that tab is roll to ensure that it is at the trim that mean then V will become V = V trim right okay. And that is why you find here this V trim clear, no issues, okay.

Now you see so, what will be the FS now? FS will be, FS expression you know K half row V square into A + CH Delta T into Delta T - K W by S CH Delta by CM Delta into DCM by DCL free okay, this already you have derived. What I have to put? I have to only put this CH Delta T into Delta T by this expression okay, this will come here and then you can easily write FS = K W by S CH Delta by CM Delta into DCM by DCL free V square by V trim square - 1, fantastic expression, what we have done? CH Delta T into Delta T.

We have just substituted this expression, how you got this because, you made an assumption that, the tab is rolled automatically so, that FS is 0 so that trim is achieved. So we found out how much tab is required for different V trim, for different wing loading, that we put in the general expression, so you got this expression okay.

(Refer Slide Time: 27:01)

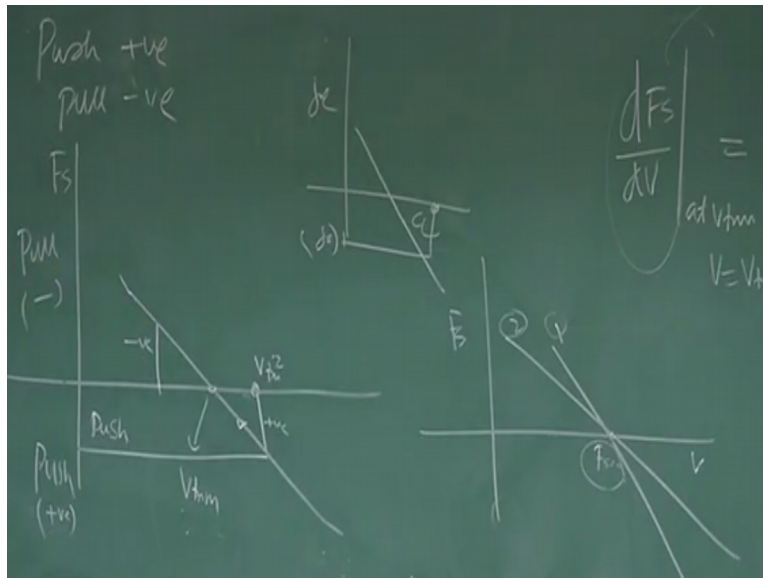
$$\left. \frac{dF_s}{dV} \right|_{V=V_{trim}} = 2 \bar{K} \frac{W}{S} \frac{\bar{C}_{ns}}{-\bar{C}_{ms}} \left(\frac{\partial C_m}{\partial C_L} \right)_{free} \frac{V}{V_{trim}^2}$$

$\frac{dF_s}{dV} > 0$

Now you see if I differentiate we will get DFS by D E at V trim that is I evaluate this V = V trim and I will get it to 2 K W by S CH Delta by CM Delta into DCM by DCL free. Now have a closer look, here one thing I have seen DFS by DV, now you see DCM by DCL free coming right, and please understand fundamentally we are calculating DFS by DV at V trim at equilibrium, so DCM by DCL immediately appeared here, which is correct slope right.

If you see here this is the sign negative, this is negative, this is negative okay, this this become positive this negative here K is negative so, actually it is D FS by DV, by this expression it become greater than 0, is it clear? Please understand that K is -, this is -, this is -, this and this, Now I will plot this in a different manner that is where you should be careful.

(Refer Slide Time: 28:22)



We have taken push force as positive and pull as negative, that we will plotting it now differently put pull here that is negative and push here, so naturally we will find DFS by DV will become now negative okay, this is clear, and this is the V trim. Please note down YX pull has been put in the Y axis push below this X axis right, so naturally slope will be different so this is the DFS by DV when you plotting like this.

Pull as negative and push as positive here right, so what does it say, it says FS like you had Delta E VS CL trim graph like this. What the graph was used to stick fixed stability concepts? That if I want to fly at this CL, this was Delta E required right, and now what is to be referred from this graph, that if I am flying at this V trim, if I want to go to this V trim 2 I have to apply what type of force? Push force, what UN auxiliary flight, what is the meaning?

That if I am flying at say 100 meter per second if I go to hundred twenty per second. So, it say you have to apply push force, how much push force it will be dictated by this scale. So it is consistent, I want to increase so I am going giving push, if I want to reduce the speed then I have to give a pull force, negative, okay.

This is positive push clear also it tells you that if you push the stick and take V trim to this point, and now withdraw that stick, it's an reversible control so again the speed come to this V trim, similar to is here, one thing you can understand, if you could see the gradient DFS by DV for 1 is more than the two if it is large gradient one thing is ensured that, if the pilot leave the stick it will

not, it will always oppose any change from there $FS = 0$, so you can fly safely, you can fly relaxed manner.

If the gradient is too low then for small disturbance, you know it will pilot has to control it right, but if it is highly the gradient is high, then it tells you it require large DFS by DV to go from 1 V to another V, so inner wing will be resisting right, so pilot can fly at ease, this is also extremely important okay, okay I will end it here and I know that lot of expression have come in one of our Mann Ki Baat session we will take up one of the topics and again we revisit. Okay.