Aircraft Stability and Control
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Lecture- 37
Forces and Moments

Let us also now realize 1 thing.

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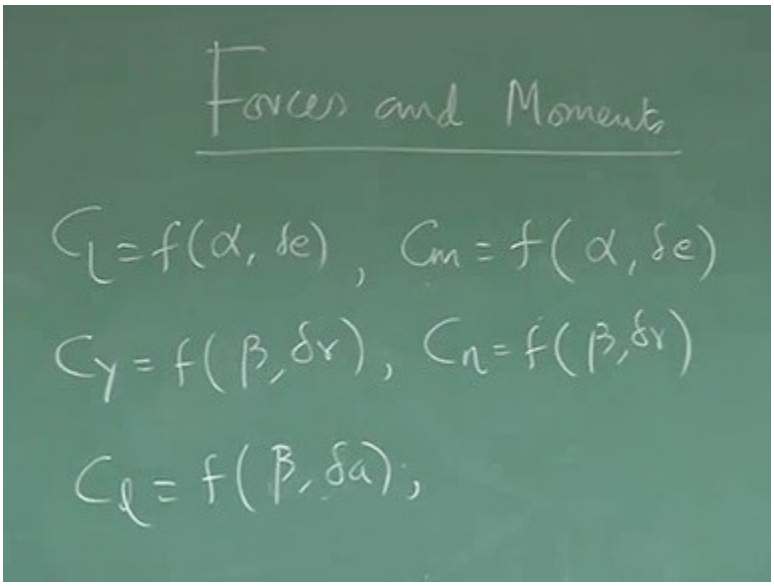

When we talking about forces and moments we are talking about forces and moments along force is along X, Y, Z is an moments about X Y and Z whatever you have learned so for that I note that CL and its function of Alpha and Delta E then as per moment is consumed cm have a function of Alpha and Delta E. Let say then come to CY side force procession again it is function of beta and Delta R and CN also function of beta and Delta R and CL only moment sufficient function of beta and Delta A.

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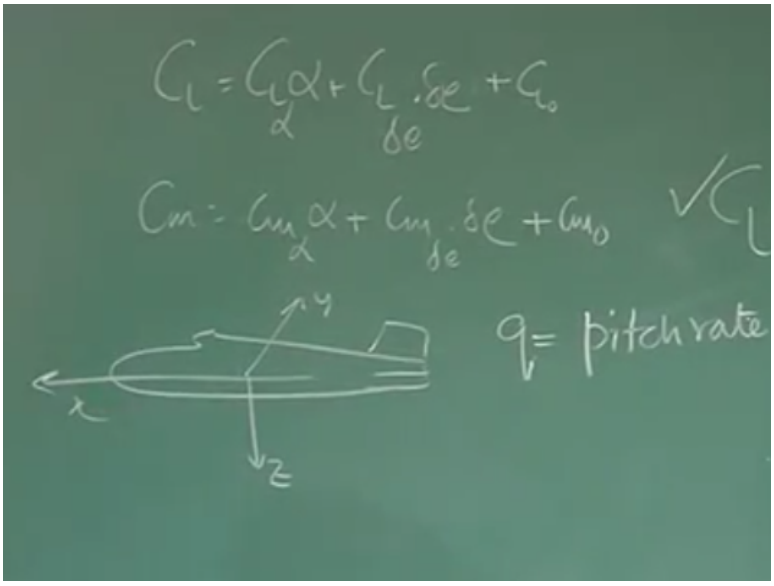

Let me explain this the first when I talk about CL we wrote CL = CL Alpha into Alpha plus CL Delta into Delta E. similarly for CM we wrote CM Alpha into Alpha plus CM Delta E into Delta E. We are now going step by step getting the first case. Now I can erase this part. what was the understanding here of course there should be CM 0 there should be CL R what is the understanding here the understanding is this is the lift coefficient this the moment coefficient any condition where aircraft is having Alpha and an elevator deflected by Delta E.

There no dynamic state it is a steady state type okay m But when I am writing equation of motion the whole airplane I am giving a Delta e actually airplane will have some transient then it will go to steady state. So what happens if an aircraft is disturbed is given by an elevator. It stuck oscillating in this direction pitch up pitch down pitch up pitch down or its oscillation about Y axis. Right.

And that is we see this is my X axis this is my y axis this is Z axis. So there be oscillation about Y axis and we say that that is Q will the pitch rate. What is the meaning of pitch rate at what rate the altitude of this airplane is changing oscillating about y axis pitch rate. So now we will see because of this dynamics.

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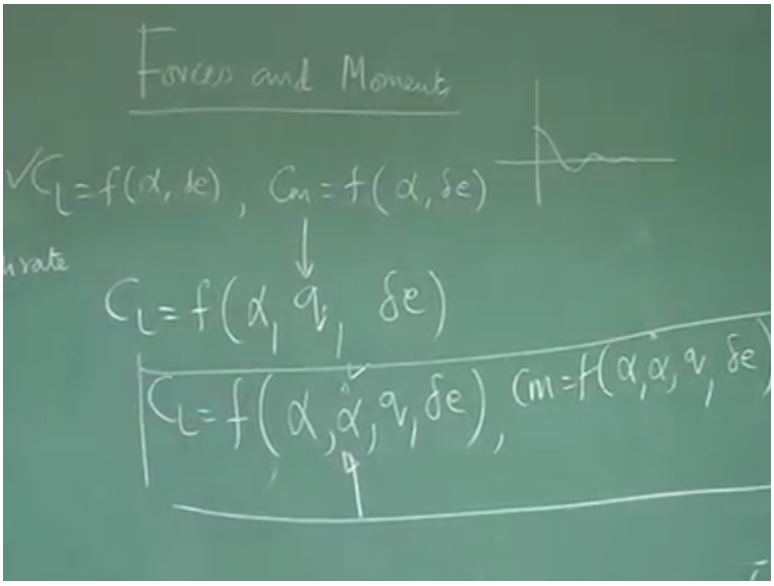

CL will not only function of Alpha also the function of Q and Delta E. Q will we will show you how this pitch rate also will change CL right. Similarly will find the further analysis will find it not only depends upon Alpha Q and Delta E it also depend upon Alpha dot. At what rate angle of attack is changing okay. We will explain why this happened but I am just building up so that we just develop expressions.

So the CL is like this then the CM also will have the function of Alpha dot Q Delta E. Now see from the steady case it's just function of Alpha and Delta E similar CM is function of Alpha and Delta E. But the moment of talking the dynamic case when alloying the oscillations like this there is a Q the pitch rate so that we are saying the CL also the function of Q not only Q as airplane does like this the velocity vector and the axis there angle changes angle between the velocity vector and axis changes.

So there is a induction of Alpha dot. CL also be function of Alpha dot and since CL in the function of Alpha dot Q and Delta E. CM natural be function of Alpha, Alpha dot Q and Delta E plus CM is nothing but CL is multiplied by length Okay. Now challenge is here in fire for the highly dynamic aircraft highly maneuverable high performance aircraft, this will here be function of even Delta you not in what rate we are moving the elevator then also became interesting variable model CL but we are not a such high performance aircraft.

We are going for conventional aircraft where the CL is module like this and CM is module like this now let us take CL what you are are seeing we go very, very slow in this.

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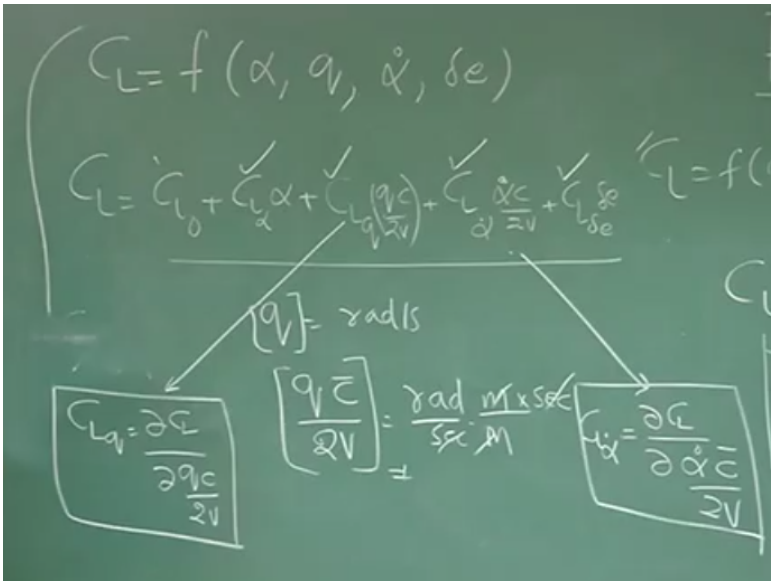

CL function of Alpha I will write Q Alpha dot and then Delta E okay. Now I can write CL = CL not plus CL Alpha plus CL Q let me write QC by 2 V I will explain you why you write like this plus CL Alpha dot Alpha dot C by 2 V, plus CL Delta E into Delta E. Normally if I give you expression like this we will write CL = CL not plus CL Alpha into Alpha plus, CL Q into Q plus, CL Alpha dot into Alpha dot plus, CL Delta E into Delta E.

But you have seeing here for Q we are replace QC by 2 V for Alpha dot replace the Alpha dot by 2 v okay, Why? Just understand this are all of Alpha and Delta e but if you see the Q as a what is the unit of Q radius per second. This is a dimensional what is Alpha dot it is the radius per second so time is there so this are not dimensionalised but all other are dimensionalised. Control variable are motion variable so you want to write CL using all non-dimensional motion variable okay.

So, what is that if Q is radius per second the unit wise right. What we have done multiplied by chord mean dynamic chord and divided by 2 V many literature they divided by V and what is the dimension of this is it radian per second then meter per meter then second then this all cancel

again it is became dimensionalised radiant dimensionalised are in it right? So, now what is happened by changing Q to QC by 2 V we can make this dimensionalised.

And you have modified CL to if I use the first definition here the CL Q will be DC L by DQ but in this way we are define please note that CL Q is DC L by D QC by 2 V correct Not this. So this is non-dimensional quantity okay, This is CL Q means D CL by D QC by 2 V physically meaning is what? what is change in CL because of Q. if the Q part is pitch up or pitch down how much CL is going to be changed that is the this derivative is tell us that gradient will tell us.

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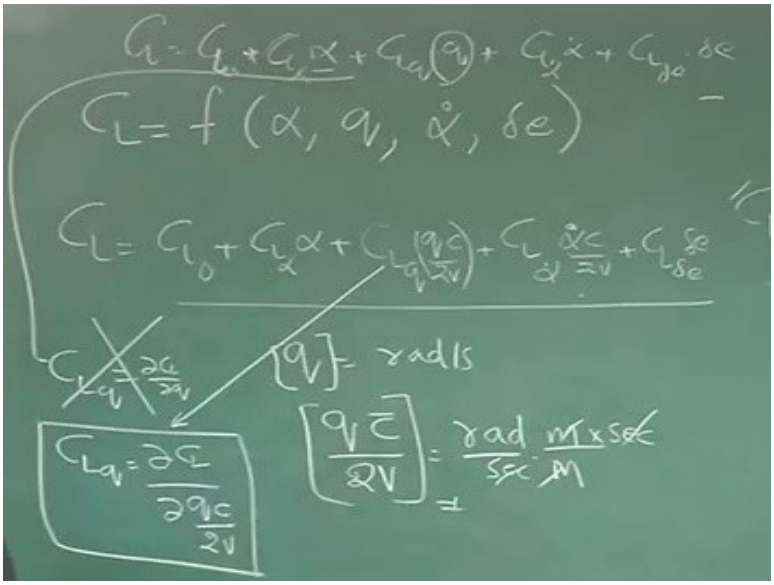

Similarly what is the CL Alpha dot CL Alpha dot will be D CL by D Alpha dot C bar by 2 V please note down this again this is met non dimensionalised really this is the way you are defining you define us CL q us D CL by D QC by 2 V CL Alpha dot us D CL by D Alpha dot C bar by 2 V okay? This should be cleared now you need to know each of the derivative what is CL Alpha? What is CL Q? What is CL Alpha dot and what is CL Delta E?

If I know this then I am writing the equation of motion if I know what is the value of Alpha? What is the value of Q? What is the value of Alpha dot? And what is the value of Delta E? How know how CL derivative is? As simple as that that the beauty of this modeling I repeat again if I know the values of CL not, CL Alpha, CL Q,CL Alpha dot and CL Delta E then whenever using

equation of motion then to get the CL I need to know the value of Alpha, Q, Alpha dot, and Delta E.

Please I know other thing I know what is the CL the airplane is experienced correct okay? That is the purpose of this in this passion and also to put an effort to know can I calculate CL Q. Can I calculate CL Alpha dot. can I calculate CL Delta E. All the derivative based on the to the plane condition.

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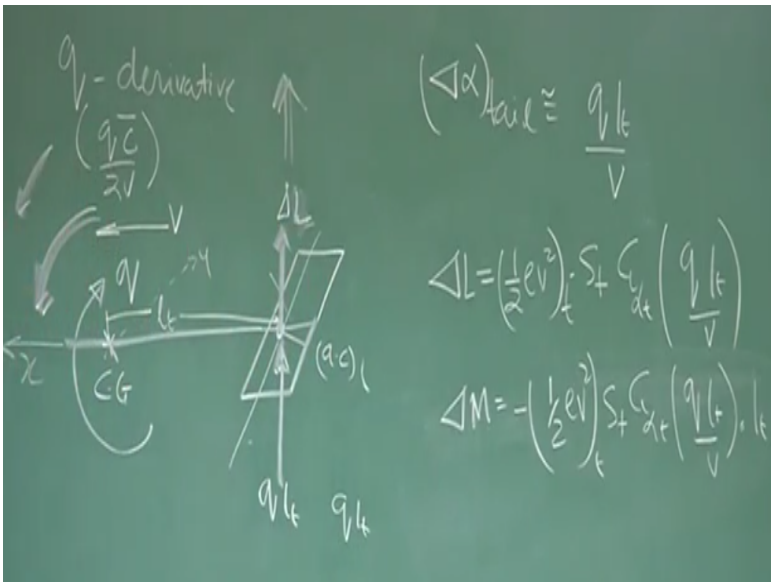

Let us say let us focus on Q derivative means you know backup in your mind it is basically you are talking about Q C by 2 V right, That is a non-dimensionalised let us see here there will be any CL because of Q or not okay, So how do I visualize let me take the horizontal tail part of the airplane. Let we consider horizontal tail of an aircraft and let say this say the airplane is going positive Q. positive Q are why because the pitch rate it was the pitch rate which moves on positive.

As we know this is an X and this is Y rotating about Y nose up it is the positive pitching moment. Now the moment it's rotates about Q what happens? This tail goes let's say A.C of the of the tail somewhere here let's say I concentrate this will be going down with a speed Q and this length you know by now it is LT, so if this is going with Q, what is the speed at which point will go down ? This V = omega R so it will Q into LT it is clear or not for example.

If I take this example this is tail and this is going to pitch up positive Q, its pitching up positive Q this tail is going down for relative this tail will go with what speed? With Q into LT omega R V= omega R. And as it goes down I can always say the relative air will be coming in the same magnetic field with the opposite direction. So what will happen here since it is rotating like this AC at this point will go down with a speed QLT, so I can say relative air will come with an speed Q into LT Roll out.

I repeat again as this airplane rotating with Q about CG this tail is going down with a speed Q LT. Especially the AC of the tail and since the tail is going down like this, I can say relative air is coming with the speed Q into LT. Remember this airplane is moving also with a speed V so Delta Alpha introduces a tail will be how much? Will be QL T by V approximately. The vertical component and horizontal component.

If this is the increase in angle of attack what does that means? It means it will generate lift here because of Delta Alpha, so what will a lift Delta L will be half row V square at tail into S tail into CL Alpha tail into Q LT by V. no objection this will give what type of moment about CG? Because of Q the lift is here upward so what type of moment will be give? It will give nose down moment, right?

Delta L it is more precise if you there increasing lift because of Q. and this lift Delta L will give moment which is negative. So Delta M I can write as half row V square tail S tail CL Alpha tail Q LT by V into LT. Do u think this is right expression? what I to take care remember whenever writing moment and all you need to careful about the sign what is happening because of positive Q Delta L is like this that is giving a nose down moment.

So Delta L is giving a Nose down nose down means negative moment, so I have to ensure here I have to put minus sign now it is perfect. So we have got 2 expression now let us see how can I generate 2 derivatives.

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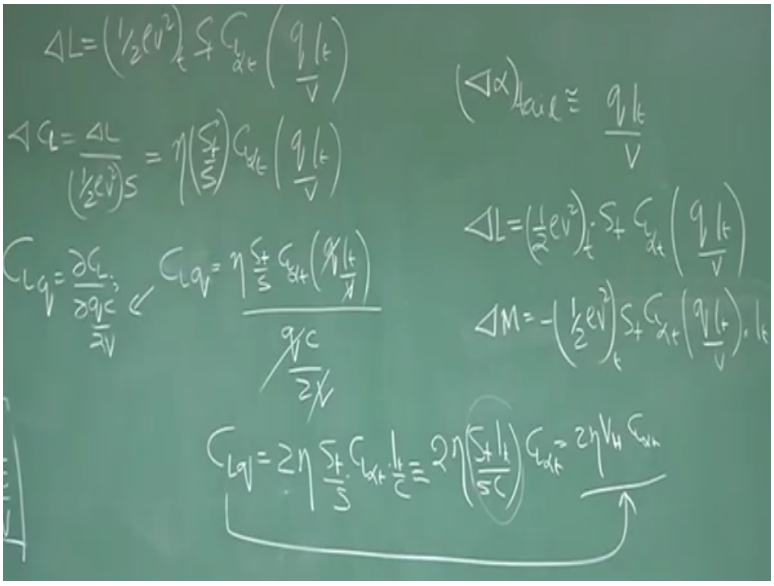

So Delta L = half row V square tail S tail CL Alpha tail into Q LT by V, so what will be Delta CL? will be Delta L by free stream dynamic pressure into S reference that is Delta CL or I write Delta C capital L because small L is for rolling moment.

So this will be now = this divided by this your expert this is Neeta ST by S right CL Alpha tail Q LT by V. what is your aim we want to find out the expression for CLQ. What is the definition of CLQ? It is DCL by DQ C by 2 V so from here if I divide this expression by say Delta CL = Neeta ST by SCL Alpha tail QLT by V so you want to find, CLQ which is DCL by DQC by 2 V because it is linear So I can divide this by QC and 2 V and left hand side I will get CL Q as per the definition here.

So what happens here? Once I put this I see this goes off V and V goes off, so I have expression for CL Q as 2 Neeta ST by s CL Alpha tail LT by C tail by now you are expert you know the Way to write this, ST LT by SC into CL Alpha tail, with 2 Neeta are familiar with 1 of the popular expression, this is the popular expression and that is 2 Neeta into VH into CL Alpha tail VH is the Tail Volume ratio so what is the message?

Message is if I know the tail volume ratio if I know CL Alpha of the tail Neeta is around 1, I can easily calculate CLQ because CLQ is = this expression. Why I want to find CLQ because I want know what is the CL during the flight. When there is a Q if so I know CL Q and if I measure the

value of Q. I can find how much is of the contribution here right? Now this is the first expression now see this Delta A what is happening in Delta A.

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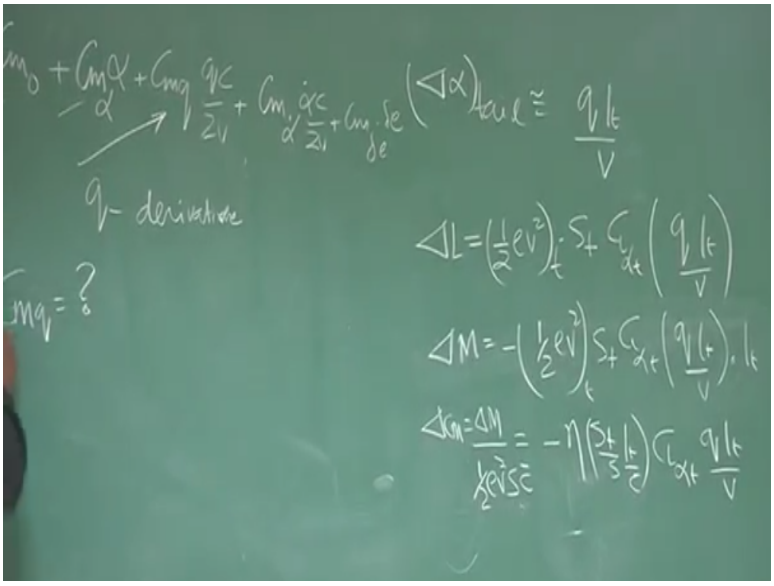

When I expand CM the similar way you know this will be CM 0, plus CM Alpha into Alpha CMQ into QC by 2 V plus CM Alpha dot Alpha dot C by 2 V plus CM Delta E into Delta E, by now you know how to calculate CM Alpha and what is CM O, we also know CM Delta E they also static case now we are trying to go for dynamic case,. Now you are trying to find Q derivatives okay that is what the change in the force is is a moment because of Q pitch rate.

We have already find the expression for CLQ, we will now find the Expression for CMQ, what will be CMQ? what is the Expression? Can I estimate it? Compute it approximately using some expression right. So we will now come back here we have seen from Delta L to Delta M is this expression, So if I want to have Delta CM that will be Delta M divided by half row V square S C bar, okay So this will became again you are now expert Neeta ST by S LT by C bar CL Alpha Tail Q LT by V. okay.

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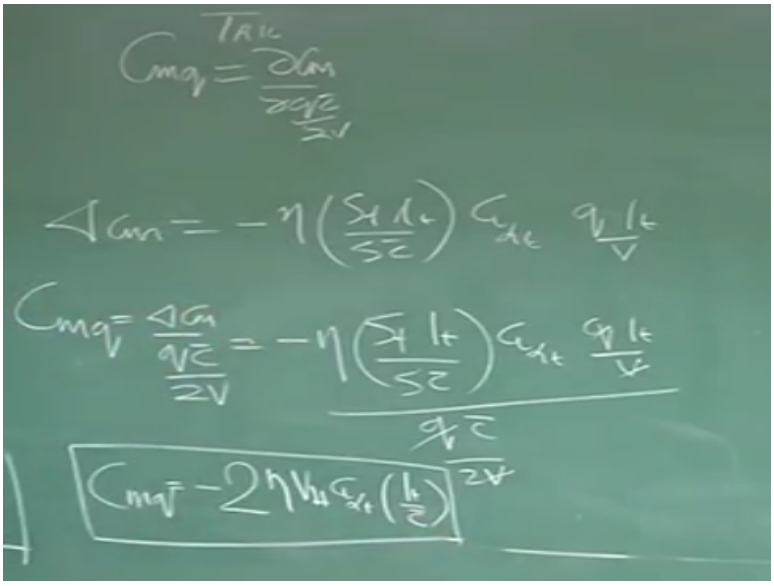

What is the expression you got for Delta CM? Write it here do not forget our aim is to find CMQ right? Our aim is to find the expression CMQ which is nothing but DC M by DQC by 2 V C bar by 2 V, so we have got the expression of Delta CM is = minus Neeta ST LT by SC bar into CL Alpha TL into Q LT by V. Please remember whatever CLQ expression we have derived and now CLQ we are deriving these are primary only for horizontal tail contribution.

We have to similarly find out the contribution because of Fuselage and Wing, but we are trying to find the first expression for tail contribution, because we know that is the primary component which will add this derivative. so this is of course I repeat here all now focusing on tail into horizontal tail so now our aim is to get CMQ, So CMQ is Delta CM by QC bar by 2 V this is = minus Neeta ST LT by SC bar CL Alpha tail Q LT by V divided by Q C bar by 2 V. So what is the final expression?

You could see V V gets cancelled Q was here so, you have CMQ as Minus 2 Neeta VH CL Alpha Tail into LT by C. First you need expression what is CMQ? CMQ is Minus 2 Neeta VH CL Alpha tail LT by S. You could see 1 thing very interestingly that you could change LT if you changing VH that means if you have change VH CMQ is largely affected and also if you change LT this also will change CMQ.

Suppose if you want to increase CMQ, CM Q means what damping so this will tell you that, if I want increase damping increase LT or increase VH are both okay. This is extremely important, what call derivative I am now focusing only on derivative.

I will come to the physical significant of it as we are clear. What has the derivative mathematically or empirically looks like right? With little bit of inside you could see that, why I am saying this CMQ is damping derivative is I like to explain you immediately now, because now at least you know what is the expression of CMQ. Before I explain what is the nature of CMQ I will let me explain first CM Alpha right.

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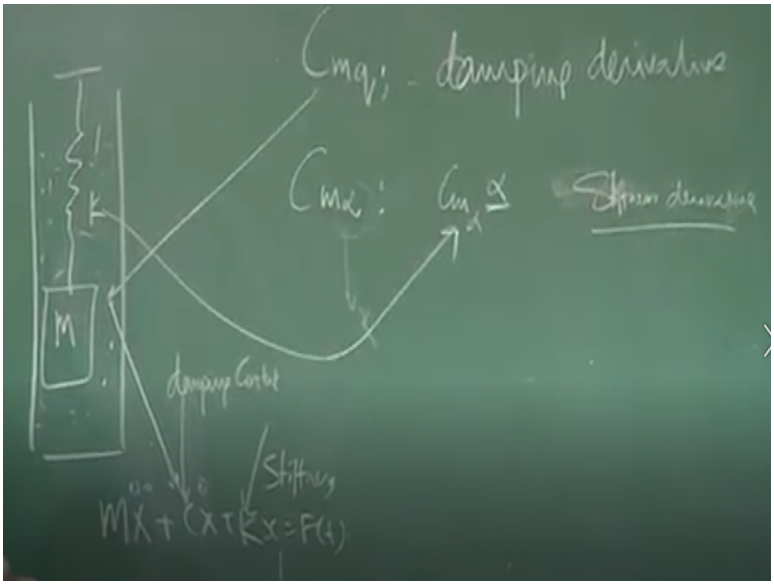

And I will take you back to all those examples of mass spring remember have been giving this example, So we wrote MX double dot plus CX dot plus KX = F of T per this was stiffness and the C was damping constant right. How to define damping constant C the restoring force or moment is proportional to the rate of change of displacement so, this constant is the damping constant and the constant.

Which tells you the amount of force or moment which is proportional to the displacement only, that is stiffness this is rate of change of displacement and this is displacement here you see when we are talking angular stability Alpha is like X here, and Q is like X dot here it's a rate okay? so this moment whatever Delta CM is coming because of Q it is proportional to the Q, I could see

that what is happening here how this moment was generated this was a Delta L and Delta L has a Q here if Q is there Delta L is there.

Delta M is also again this Delta M is coming because of Q right. So, if Q is positive Delta M is negative so, this restoring moment is proportional to Q to the rate that is how CMQ we call it damping derivative CM Alpha restoring moment is CM Alpha into Alpha right? so, it is proportional to Alpha so, this is stiffness derivative right. Stiffness now understand how stiff the spring from is analogous to K has stiffness as you call spring constant, And this is Analogous C which is because of the medium are the flight.

Now we understand if I want to make an airplane dynamically stable, not only need something like K also need something like C that it need to have stiffness, as well as damping and that is why we are now develop this expression and we will ensure and see that or airplane has both static and dynamic stability. In general for normal case if an aircraft dynamically stable that is statically stable also right. But will be doing our job in a manner which will not only make as understand what is static stability what is dynamic stability on how the air coupled okay, in terms of response.