Aircraft Stability and Control
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Lecture- 38
Aircraft Equations of Motion

So, good morning so far, we have been discussing the static stability part of it and from elevator required to stick force requirement as well as their gradients for example elevator per G the D Delta E by DN then the stick force per G all these things we have derived and at the end you realized that lot of equations were there but now once we understand the static part of it we need to go for dynamic stability and in that direction we have already made a beginning, we talked about point mass model.

That is how to write the equation of motion of a body in air moving in air and that was a very simple approximation however it is very useful you will find in all moments whenever they want to compute a range of a projector will be fired this point mass model very fine for simple reason those shells like a buffo shells or 105 mm artillery shell when they fly they fly at a almost 0 angle of attack for most of the firing angles so as it flies at 0 angle of attack.

So, naturally the force which it experiences primarily is the drag that too independent of angle of attack it is CD 0. However, when we are talking about dynamic stability we need to develop complete equation of motion of an airplane and we will see from those equations if I disturb the airplane how the airplane is responding and by studying their response or studying the air tendencies we will try to understand whether this aircraft is dynamically stable or not.

So, it is important that we develop complete equation of motion for an airplane in motion and you understand that airplane can have 6 degrees of freedom when I am talking about an airplane I am talking about a rigid airplane no needs with high tech materials coming will find that lot of flexibilies in the material for the wing in particular is, unavoidable sometime desirable so, I am not talking about though conditions.

I am already talking about the airplane, which is rigid that is the distance between two particle of any part of the airplane they remain fixed they does not move relative to each other right? So, that's the rigid airplane like rigid body so, with this assumption we will try to develop equation of motion for an airplane.

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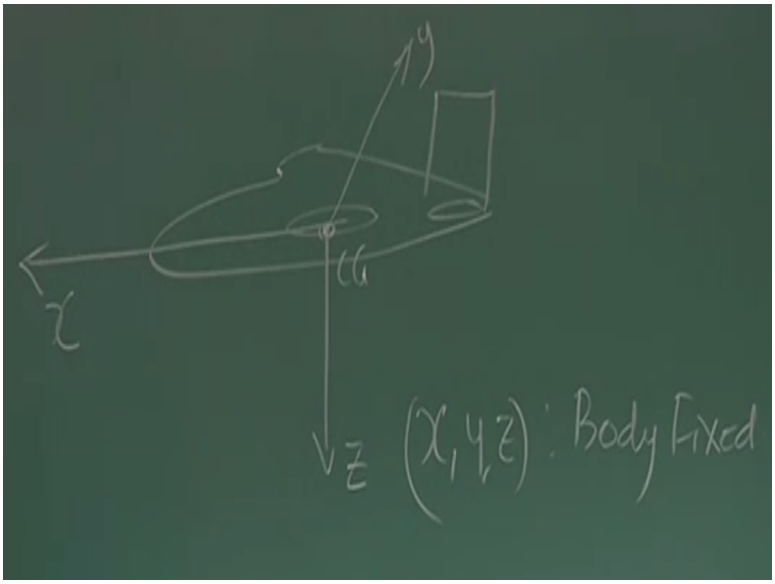

If I draw an airplane I preset the axis system and I am talking about an axis system is X Y Z which is body fixed X Y Z so, body fixed and located at center of gravity this point is at center of gravity. Now it depends on the purpose how do I align this axis there are many ways you can align this axis for many, many exclusive purposes but here to start in the simplistic manner, we are allying this X axis let say on the fuselage reference line this line on symmetric may not be guaranteed.

If I put X axis along the line of symmetric then you will see some cross product of inertia moment of inertia will vanish but we are saying okay at this stage with the X axis is the line of fuselage reference line here. When I said a body fixed axis please understand this axis is fix to the body to the airplane so, if the airplane moves axis also moves right? If I airplane takes an angle of motion like this then axis which are X axis it will also move with the airplane.

So, body fixed now if it is going like this everything is moving the same way as the body or the airplane equation is moving right? That is why I am calling it body fixed axis system.

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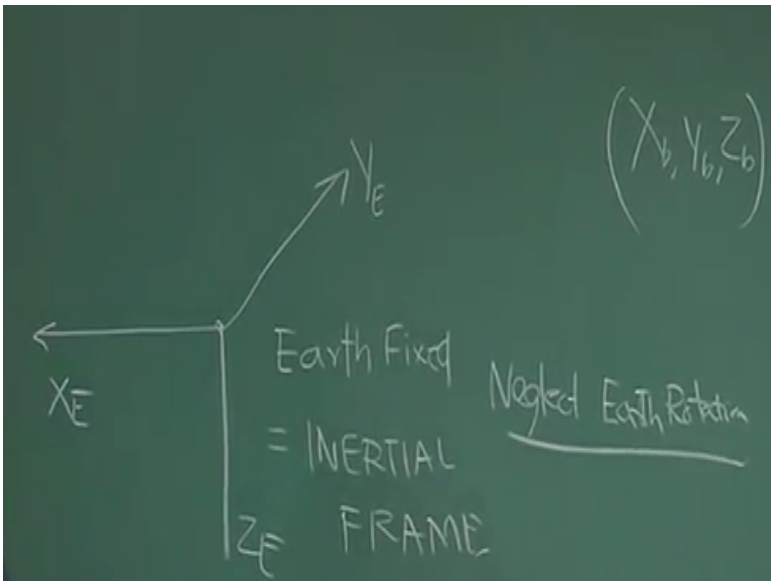

Now let me defined another axis system which is earth fixed this closely I can say this is inertial frame I will explain you what it let me write this then I will come back to this so, completion of this drawing let me write this is an earth fixed and inertial frame of reference I will talked about it do not worry. What is happening you see when I try to write the equation of motion if I through a body.

So, this will be acted upon by various forces right? one is of course of the gravity force another is as it's moving relative to air it will experienced aerodynamic force and if on the body if we are putting an engine then it will have propulsive force that is.

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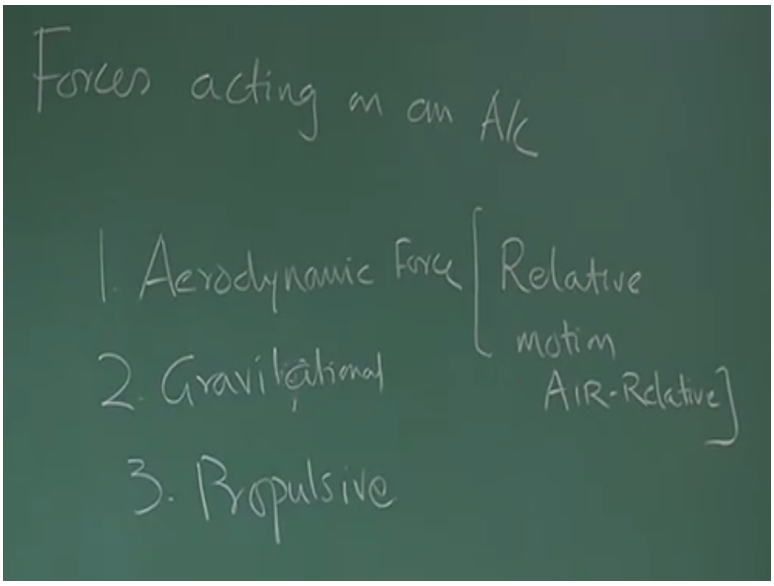

If I see forces if I write forces acting on an airplane or aircraft in motion one is of course aerodynamic force second is gravitational force of because of force of gravity and third is our gravitational and third is propulsive this is strictly true for airplane.

If it is the space vehicle then you will have all though interplanetary forces that will come to the picture. We are not considering those thing because those things are not significant as long as the airplane is flying within the atmosphere and airplane can only fly within the atmosphere because we have a wing and our airplane is a aerodynamic in nature that is forces lift to drag extra they come out of the relative motion between the air and the airplane that is important.

So, this if I try to write a keyword I will write relative motion or I call it air relative please try to understand this little more.

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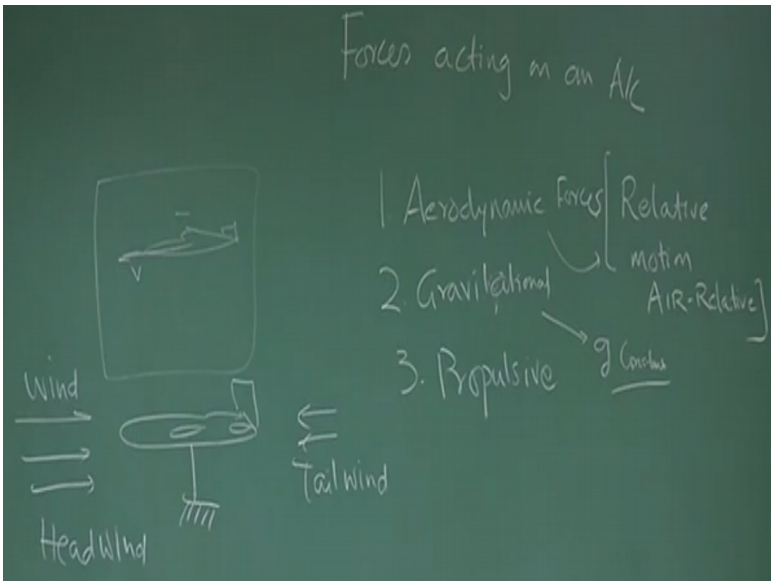

You all know that if this airplane is moving in air it will experience lift, it will experience drag because it is in motion and there is a atmosphere here but think about situation if this aircraft is grounded and it is toad like this. It is not moving with respect to ground it is not moving with respect to ground but if suddenly wind start blowing possible it could be wind is coming toward the head we call it head wind if it comes from here.

We call it tail wind and this is called head wind you could see if there is a head wind or tail wind it will again experience some aerodynamic forces although this aircraft is not moving in air its stationary on the ground that is obvious because we know this aerodynamic force or aerodynamic forces. They depend upon the relative motion are air relative okay? So, even if it is having 0 velocity with respect to the ground but with respect to the air it has a velocity so, it will experience a force,

So far aerodynamic forces it is important we keep our focus on relative motion with respect to the atmosphere clear? Gravitational you understand this basically we will assume here G to be constant because the height is not that large so, that we need to take care of variation of G okay? Propulsive we know it could be a get engine, it could be a piston plop IC engine but the another thing should understand when I am talking about these forces as far as stability is concerned the gravitational force will always act through the center of mass.

And we are evaluating the motion about center of mass like this okay? So, that will not have effect in this angular motion as far as stability is concerned because the moment because of the gravitational force about CG will be zero. However, we will have components in terms of moment about CG because of aerodynamic forces, because of propulsive forces depending upon where it has been kept this is in general we should immediately able to visualize when we close our eyes and think okay?

Once you identify the forces the source is where the force will available for airplane to drag we need to know how to write the equation of motion and when you talk about equation of motion.

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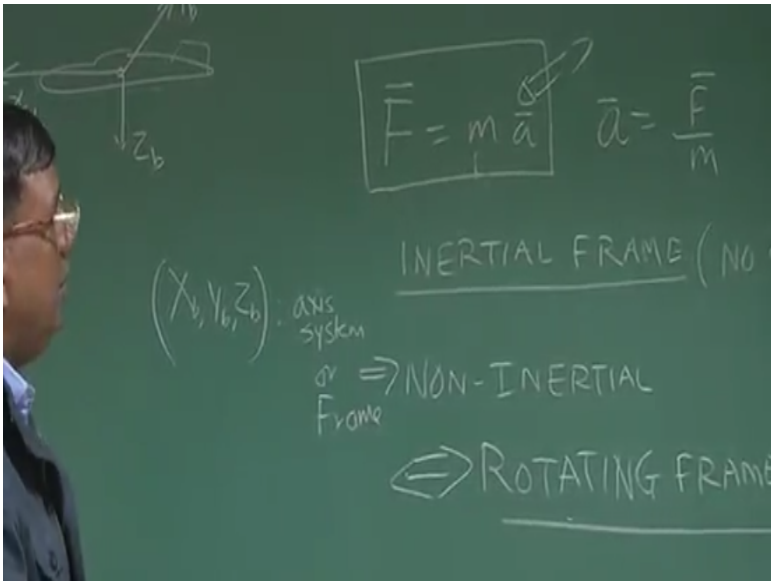

you go back to your again tenth, eleventh class and you will remember that famous F = MA that is if we apply an external force F on mass M then you know that F and acceleration are related like this or I can interpret it like this.

If I apply a force F on mass M I know how much acceleration it will generate what was the assumption here this is valid as long as I am applying this in inertial frame. What is the meaning of inertial frame that means this frame that X Y Z were from your measuring this frame should not have any acceleration clear?

So, no acceleration I could understand this is the airplane and you know this is X I write B for body YB for body ZB for body now they ask a question is this XB YB ZB axis XB YB ZB axis system or frame this is axis system or frame same meaning is it inertial? Is it a inertial frame of reference?

And if I am measuring anything with respect to XB YB ZB can I tell its inertial frame what is the question you will ask does this frame have any acceleration? If it does not have acceleration then we say it is inertial frame. But check here since it is the body fixed the body is moving body will have rotation body will have acceleration both angular as well as transliterating.

So, definitely XB YB ZB is non-inertial effect that is we find text book they write rotating frame so, there is a problem if I want to write F = MA, if I know from there and to find acceleration then a relationship is F by M but the problem is this is to be applied in inertial frame into whatever forces we are measuring whatever velocities we are measuring whatever distance we are measuring that should be measured with respect to inertial frame.

But unfortunately XB YB ZB are non inertial it is not inertial frame So, how to apply this under such condition question is clear? Okay? So, now we look for let as define some inertial frame unfortunately see the problem is our earth itself is rotating if I fixing an axis system on the earth it looks like yes it is fixed but it is also rotating so then that is an also non-inertial frame so, what do I do for space vehicle there could be a point many, many points we will be defined at infinity right?

And that way they can manage these situations in relative sense, but for earth there is a problem and my axis system is fixing on earth when the airplane is flying I'm seeing from here measuring everything with respect to the earth but there is a safe corridor of assumption where we can operate and handle this problem what we will assume for our airplane motion, the angular motion of the earth can easily been neglected but they are not very long duration things right? Okay? So what is the assumption we made neglect earth rotation the moment you put.

This becomes for all our purposes for aircraft performance and inertial frame that is why we are fixed XYZ or Z axis on earth that is why earth fixed this body fixed this is we are treating as a inertial frame of reference okay?

Once we have done that now let us again revisit famous Newton's law of motion, F = MA if I go to step back if I try to go back and introspect what is this equation representing we know that from Newton's laws of motion that F is = rate of change of linear momentum not aiming to.

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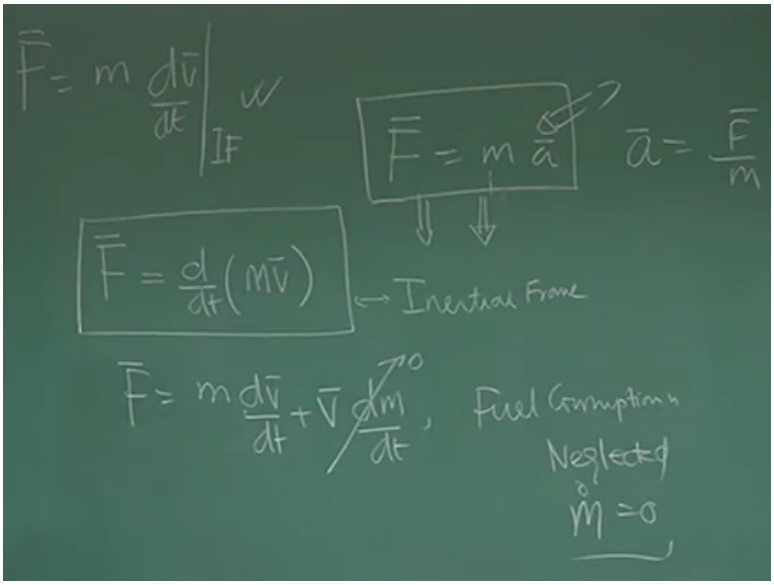

Linear momentum how to define linear momentum? Immediately, the answer come linear momentum is M into V but I would like you to think in terms of little bit of more philosophical definition when I say MV I interpret this as quantity of motion contained in a body like, V meter per second right? M into V is I will think like quantity of motion contained in a body now if I do a closer looker.

As far as Newton's law of motion is concerned this would be on the reference to inertial frame that is whatever derivative, we are taking that should be respect to inertial frame now if I expand this what do I get I get M DV by DT + V DM by DT.

The most of the cases we are dealing will assume, this part to be zero that is there is no change in the mass during the operation although you understand that fuel consumption will be there, but

we will assume that fuel consumption is consumption is neglected and that is how you are writing M dot =0 this is 1 but what is important, important is now F = M DV by DT and this should be done in inertial frame IF is inertial frame that means you've earth fixed frame as a inertial frame, so you observed what is the direction it is going.

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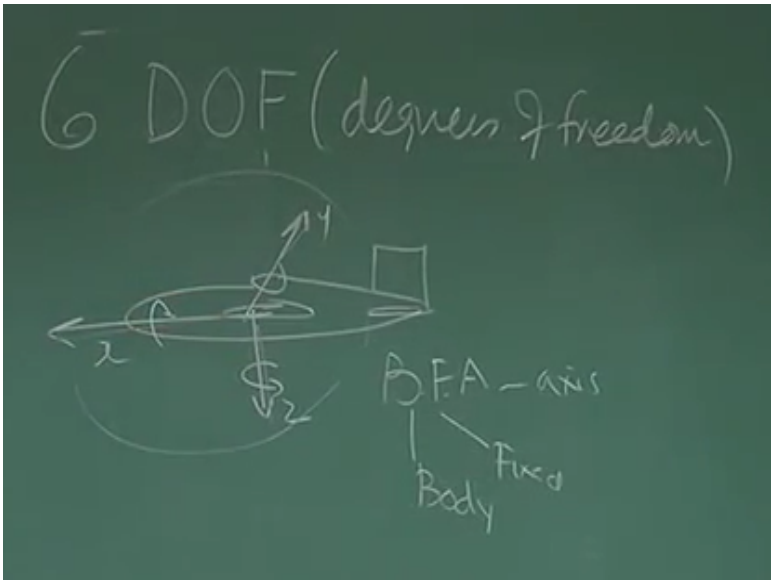

What rate it is changing with respect to an axis fixed on the inertial frame on the earth not with respect to an axis system fixed on the aircraft, because aircraft is rotating frame it's not a inertial frame so that is the catch point and that is the basis, where which will be used to develop equations of motion right? And typically we say sixth degree of freedom equation of motion or equations of motion and that is why 6 degrees of freedom we say 6 DOF, 6 degrees of freedom that is degrees of freedom why 6 degrees?

You will be clear about it you know it, this is X this is Y this is Z so 1 degree this motion along X along Y along Z and the rotation motion along XY and Z. So XYZ this motion and one motion this one motion this one motion this right? So, there are 6 degrees of freedom so what is the meaning there by we will develop an equation motion through which I should be able to solve a problem, that if I give elevator deflection aileron deflection how this airplane is changing in free space.

And that should be characterized captured by this 6 DOF simulation right? Okay? That is our aim what is the problem, we got stuck we got stuck that yes F = M DV by DT but we need to do with respect to inertial frame of reference this derivative however, if I am working with the body fixed axis system like this body fixed axis is BFA, this is body this is fixed axis the problem is if I operate on respect to this axis system this is not inertial frame of reference.

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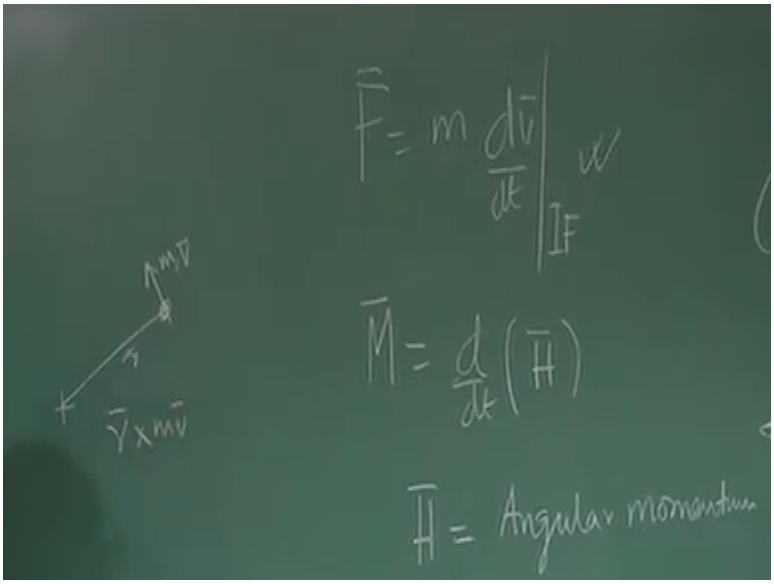

So I will not able to apply Newton's law correctly okay? In the similar fashion you know this moment and related like this rate of change of H, H is what? Any guess what is H could did you recall? H is angular momentum so, in your class 10^{th} you might have read if we recall that MVR right? That will help you see if this is R and this is the mass M, which is moving with velocity V then R cross MV is the angular momentum that used to study in class ninth, 10^{th} or 11^{th},

And one of my teacher told me if you want to remember what is angular momentum? Remember like this it is the moment of the momentum, so moment of the linear momentum, this is linear momentum and we I know moment means force into distance right? So, if you think in terms of moment of the linear momentum is angular momentum is my teacher used to tell me and that's to help me just for you are keep it back of you are mind if you try to develop physical field for angular momentum.

Thing like this is the moment is an effect moment causes an effect for rotation right? Which causes rotation so this moment of linear momentum is angular momentum and as F = MDV by DT, I can write moment = rate of change of angular momentum these two equations are to be utilized for one is I get acceleration from here linear accelerations I will get angular acceleration from here so I can get linear position and angular position by integrating all these solutions.

Right. So this is the basis only problem is I need to work with inertial frame okay? So this if I understand and keep back of our mind we will see how simple it is to develop fixed degrees of freedom equations of motion.

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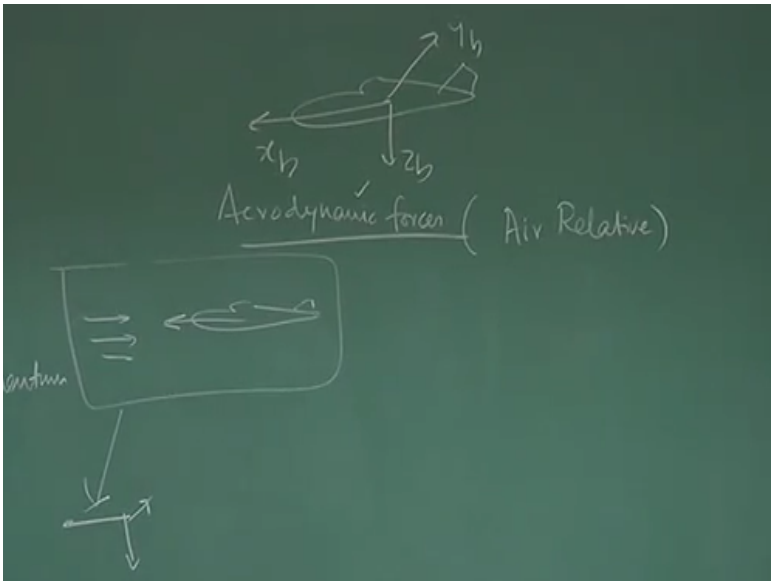

Now see the conflict, when I want to find aerodynamic force aerodynamic forces what did you discuss we discuss it depends upon the relative velocity okay? It does not depend upon the ground velocity that is the airplane if it is moving like this and this is the ground which is for our case inertial frame the aerodynamic force here will depend upon what is the relative airspeed with respect to the atmosphere not with respect to the ground as I give you an example.

If the airplane is stationary on the ground, where suddenly the wind starts blowing it will experience aerodynamic forces so far aircraft if I want to calculate aerodynamic forces these are air relative these are air relative that means I will be more comfortable when I operate with XB, YB and ZB axis system because this will be very useful to get me air relative speed along this

axis XB YB ZB right? Because we have seen ground (Refer time: 24:27) to ground which really does not matter.

It's all matter is what is the relative airspeed, that means the body is moving like this the atmosphere is stationary is moving like this so you know relative air will be this way that way and depending upon the orientation so, it is the relative airspeed, airspeed relative to XB YB ZB axis speed air relative speed with respect to the body means XB YB ZB axis but these are not inertial frame we cannot directly apply it so, what is the issue how to solve it that is the big question.

Please come back mathematically I want to find derivative DV by DT and DH by DT in inertial frame but we are finding this forces they depend upon the air relative speed so that is more because of it is easier to solve a problem in XB YB ZB axis because it is an air relative speed. Thank you very much.