Aircraft Stability and Control

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Lecture- 39
Six Degrees of Freedom of an Aircraft

Good morning, so today we will be spending time in developing 6 DOF or 6 degrees of freedom, 6 degrees of freedom equations of motion, and by now you know what are the 6 degrees of freedom.

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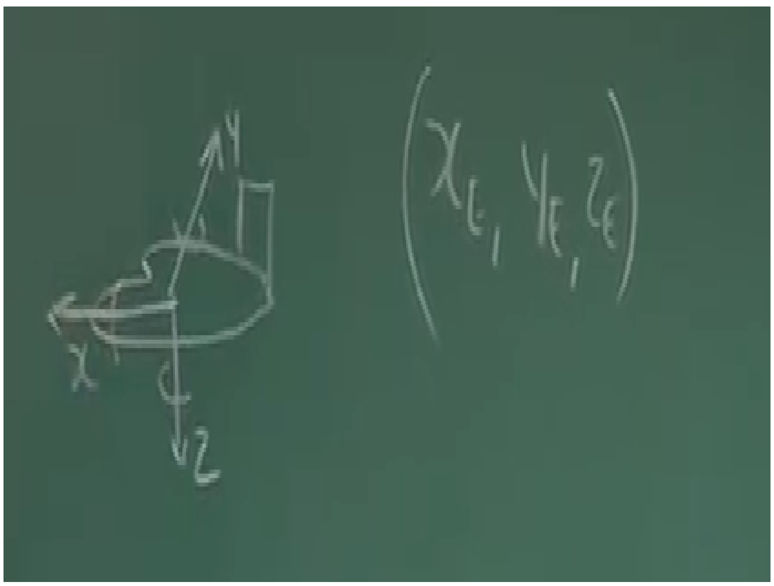

If this is the airplane, we have X Y Z so, 1 motion translator motion along X along Y along Z and rotation motion about X Y Z that is, xyz these 3 motion and this, this and this right? These are 6 degrees of freedom what is the meaning of that? Meaning is there by what is our interest? Our interest when we develop 6 DOF 6 DOF equations of motion we say the airplane is moving, I want to give a disturbance the airplane and see how its responding in terms of all these 6 degrees of freedom.

And from that response we like to utilize it conveniently to understand whether the aircraft is dynamically stable or not, our aim is for dynamic stability analysis just not forget that that is our basically aim.

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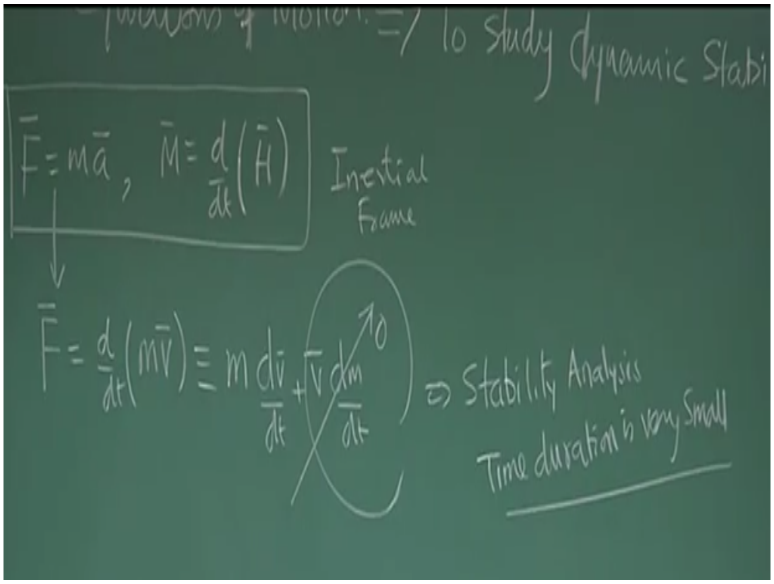

Why we are developing this 6 DOF equation of motion, our primary aim is to study dynamic stability of airplane aircraft, 6 DOF equation of motion is can be used for so many things, if you want to know up to what range it is going, how it is moving in different directions you also can do that but, we are very clear about this this equations of motion and we are developing with clear understanding of mind, will be studying dynamic stability of an aircraft.

We have already completed static stability of an aircraft right. And now how to develop the equations of motion, you know F is = MA and like moment is D by DT rate of change of Angular momentum okay. But the constraint is when you want to apply these, you have to very very clear that this should be applied with reference to inertial frame, that is important and what is a inertial frame.

A frame which does not have any acceleration, and for our purpose we have assume earth fixed access system which is we are calling that that is our inertial frame, because you are neglecting the rotation of earth okay. So, for an aircraft is good enough of assumption, let me tell this XC YE ZE with the axis system.

For earth fixed which is for us is inertial frame of reference, so what I can do is this equation if I go between I can write this is nothing but D by DT of M into V, and this is = MDV by DT + VDM by DT right? And you are assuming that DM by DT is 0 the mass is constant why this is okay for us we are doing stability analysis, or M is to do dynamic stability analysis so will be studying the response of airplane for a short duration right? For example it is in the cruise.

Now you give some deflection and see what is happening but this study will be for a very short duration, with respect to the trim condition and during that short duration it is fair enough to assume that fuel consumption is very negligible, so this assumption is okay. Why this assumption is okay. Because we are doing it for stability analysis stability analysis, and time duration of study time duration is very small, so it is fair enough to assume that fuel consumption is 0.

Or wind negligible, so the weight remain the same so DM by DT is 0 fine, so I write F which all of us know the external is MDV by DT correct, now let us go little bit deep into this expression, remember we are developing this equation no 1.

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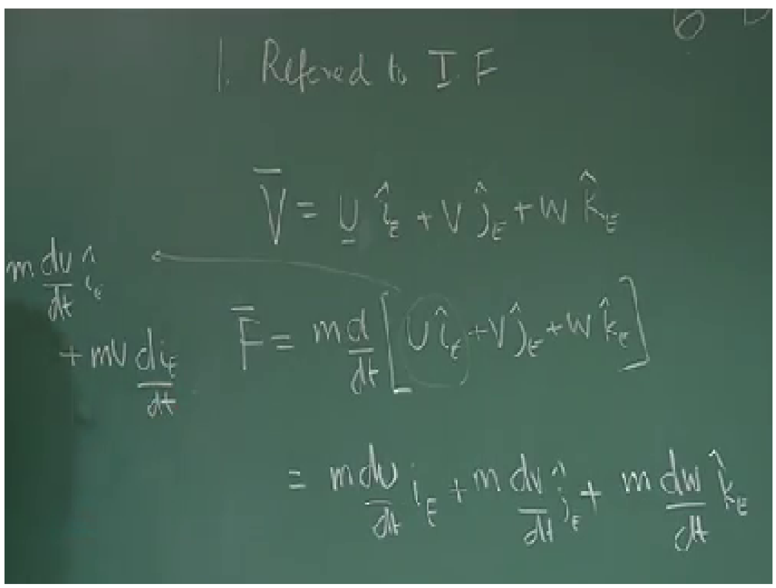

With referred to initial frame of reference, which is earth axis so now I can write V as U Earth into I of Earth or let me write U for clarity U IL + V J earth + W K earth what is the meaning of this? This is I am resolving V total velocity along I J K of initial frame of reference Correct okay. If I further try to understand suppose this is your IE vector this is your JE for your inertial and this is ZE, and suppose this is velocity vector V.

Then I am taking the component of V along IE and That is U component of V vector along JE is V, and similarly component of V vector along ZE is W. So if I put F is = MDV by DT, So it will be D DT of V for V I write U I E + V JE + W KE like this.

This is the total velocity V, Rotation wise do not get confused, this is the component of V along inertial frame Y Axis right. So now if I take derivative I get MDU by DT into IE + MDV by DT JH direction + MDW by DT in K Earth. Please see here when I was taking the first term derivatives I really should be MDU by DT so let me write DU by DTI + U DI by DT right. Here if I expand these this should be MDU by DT I + MUDIE by DT but I know.

It is a inertial frame of reference so IE JE KE are fixed so, they are not changing the direction so this is 0 I have not put those term here so from here now we can easily see F vector is having FXIE + FY J + FZK and what is FX? FX is nothing but MDU by DT what is FY MDV by DT what is FZ MDW by DT okay. This is there is no problem on that.

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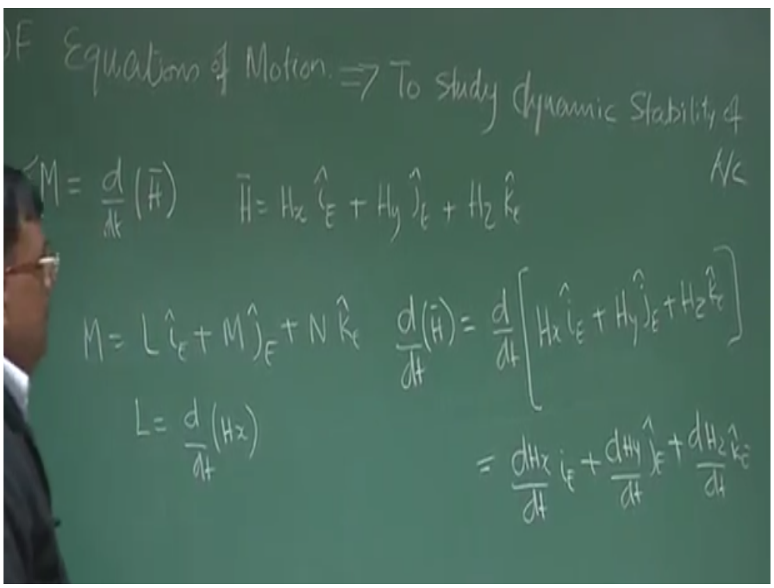

Similarly if I do this M summation of a FD by DTH. I may right its angular momentum as HX X component along Earth initial frame and HY into JE + HZ into KE, so this is a compliment of total angular momentum along inertial frame with its earth is IE JE and KE. If I put this thing here I can write.

M as LIE + MJE + NKE so what is L? L is the nothing but HX. So L = D by DT of HX this is clear? M is D by DT of H so D by DT of this vector so if you're not clear let me repeat here, so D by DT of H it means this is = D by DT of HX IE + HY JE + HZ KE. So if I expand it I get value DHX by DT IE + DH Y DT by JE + DH Z by DT into KE. No problem clear about it so for this I am writing as L. for this L for this M and for this N so I can write.

This as LIE + MJE + NKE, See whenever I am deriving some Expression please understand, since I do not have any introduction whenever I get inside that I am a bit fast you may miss the point I go on repeating it. In fact you find many lectures I repeat on a next day. that is what that sort of a taught comes to me ,when I review it when I see you when I think, what I am done today, so please your open forearm very aggressively, whenever you not able to understand just right me drop a line. Okay.

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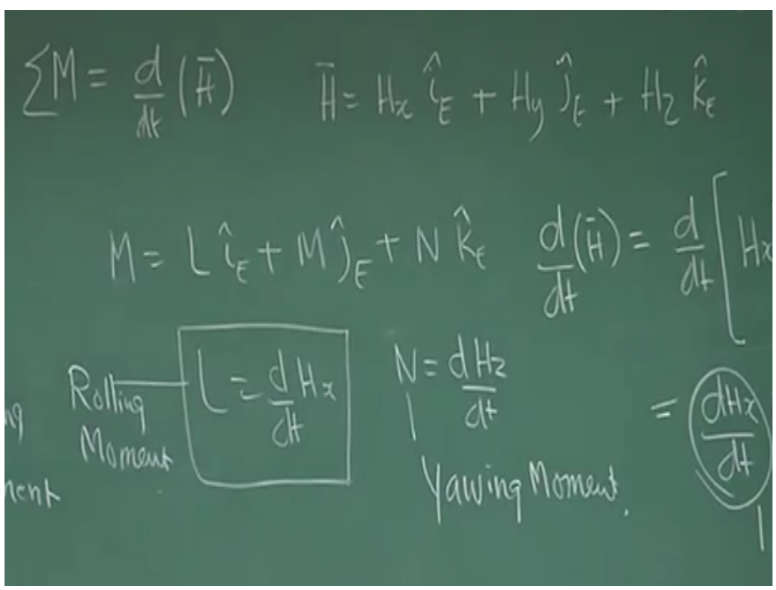

Now let us come back to the Physics, what is L? L if you see here is D by DT of HX what is HX? So let us ask a question that what was H? H was H vector was the total angular momentum. And total angular momentum about which axis? Waste we are all working in inertial frame, so it is about inertial frame IE JE KE axis. Correct. So what is DHX by DT this is the component along X direction, Right. So component along X direction. Please try to develop the understand the physics.

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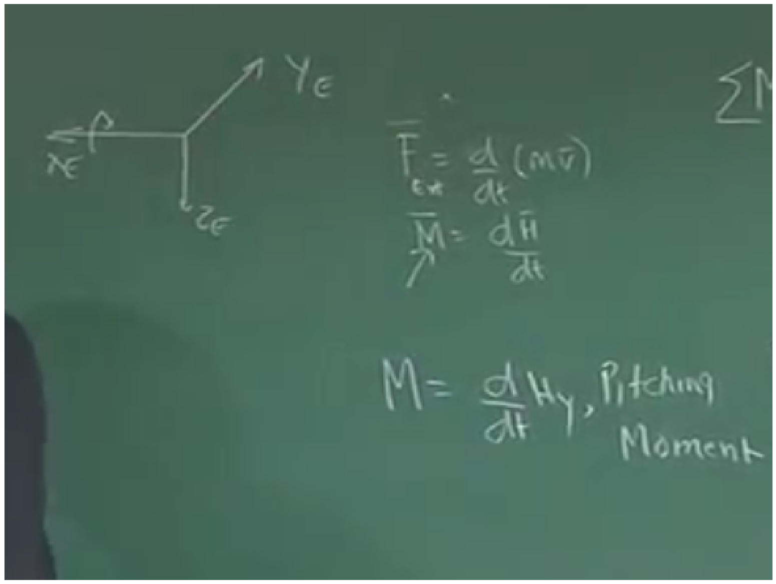

So that you are not lost in this term, and very simple compared along X axis, this is Y this Z .ZE XE since we are working with Earth fixed inertial frame, so this is a moment rate of angular moment who causes the change of angular moment, that is some External applied moment was with there like who causes the rate of change of momentum in linear case, it is external in impulse force, so who will causes the change in the angular momentum? That is some external impress moment.

So this is nothing but about X axis so it is rolling moment L is about X rolling moment right? So this part is rolling moment. So this is about X axis clear, I repeat like you have F = M or D by DT of MV what is our understanding? That this external force which causes the rate of chance of linear momentum Right. Now if I right M is = DH by DT. What is my interpretation this external moment causes change in the angular momentum vector.

It is a external moment applied on the body. And this angular momentum in vector has 3 components along IE JE KE axis, and this rate means rate of change of angular momentum, along IE similarly along JE along KE and this part is about X axis. So this is like a rolling right this moment is called rolling Okay

So this is called rolling moment L is rolling moment what about M is similarly D by DT of HY is about U axis, so this is air plane this is the X this is the Y so why axis which moment will be there? This is pitching moment. Right. It is pitching moment. And then if you see N here N, N = DHZ by DT so, this N will cross this DH by DT DHY by DT is the component right. It is a scalar components and direction is given by I J K. So, I have removed this vector SIN here right.

In ideally I should write M which is basically about Y axis pitching moment similarly N if they come here HZ about does not about Z axis so, this is nothing but yawing moment okay. And I know your smarter you know rolling moment positive is when right wing going down yawing moment is positive is we are right wing going back and pitching moment positive is going nose goes up all this things we are clearly understand.

So this is the component and let us see what is the next step? Let us not forgot that we are writing this equation of motion will refer to inertial frame which is for this particular case is earth fixed frame okay. So, now let us do 1 thing we draw an airplane.

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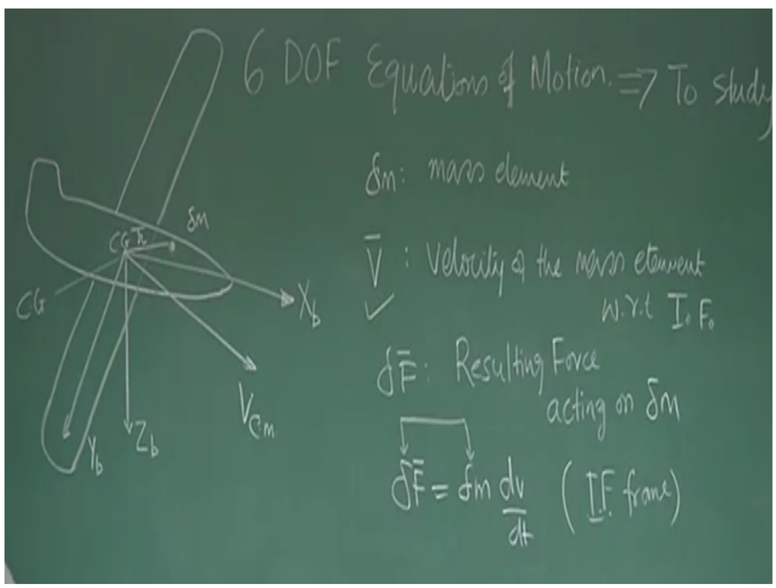

I always say when ever writing you expression do not forgot your working for an airplane so, that inside should not may be missed the airplane is the center of gravity I am taking any mass Delta M okay. And has you know now we are drawing body fixed axis system so, I will use the word XB then this is YB and somewhere here this into ZB B means body fixed axis system what is the beauty of this body fixed axis system that it rotates with the body fixed and located at the center of gravity here X axis I can align depending up on our requirement some time we align X axis in such a way that it becomes a plane of symmetric okay.

The vertical plane because plane of symmetric Advantage is if I put X axis like that you will see we can put the value of some cross moment of inertia = 0 so, that simplify our case assume that XB is along the fuselage reference line we are not putting any condition on XB will put it has an own required right? And let's this denote this as R bar R bar is and the position vector of this mass Delta M with respect to CG okay. So, let me clearly write this what is Delta M? Delta M is the mass element on the consideration.

That is total mass you know your distributing into small, small particles right. And remember we have assume that this airplane is rigid so, the relative position between 2 point physically remain same they do not change right. So, Delta M is the mass element on a consideration and V V is the let's they have to give V someone here that's the V what is V? V is the velocity of the mass element with respect to inertial frame velocity of the mass element with respect to inertial frame note is that.

I have drawn this the has the velocity of the center of gravity am I correct what did I write here? Velocity of the mass element to the respect inertial frame so, this is not correct representation this represent what? This represents velocity of the center of mass please understand this represents the velocity of the center of mass but we are talking about velocity of the mass element with respect to inertial.

This velocity of this element and you could understand the velocity of this element and this element did not the same because if a body rotating like this it's rotating about CG then if the evens a CG is not moving but this point having a velocity omega cross R right. So, this point velocity and this point because center of mass not same in general.

So, this is understand this is V velocity of the mass element and this is VC velocity of the center of mass this should be very, very clear okay. Once I am very clear about it then I write the Delta

F Delta F is the resulting force resulting force acting on DM so, what could be the resulting forces aerodynamic forces.

Gravitational forces right. For airplane this a 2 forces propulsive force will act in general through the proposal mechanism it there it is a propeller or it the get engine right. So, this is the total resulting force acting on D so, now I can use because I am working in inertial frame everything I am not developing inertial frame Delta F I can write as Delta M DV by DT.

Please note that this is DV by DV not DV C by DT because I am writing what is the effect on the mass Delta M because of external force Delta F acting on DM this Delta F is acting on DM so, I will draw a line like this so, that you clearly understand this is the external force acting on DM which is causing acceleration DV by DT or which is changing the moment right? In a chord is with Newton's loss of motion and definitely we are particular we know how to apply Newton's loss of motion.

So, we are applying all this thing in inertial frame which is earth fixed so, every measurement is made with respect to earth fixed right. That is if I am talking about this velocity V this is measure to respect to earth fixed axis system okay. which is fixed and inertial frame that should be very, very much clear to you now I and write.

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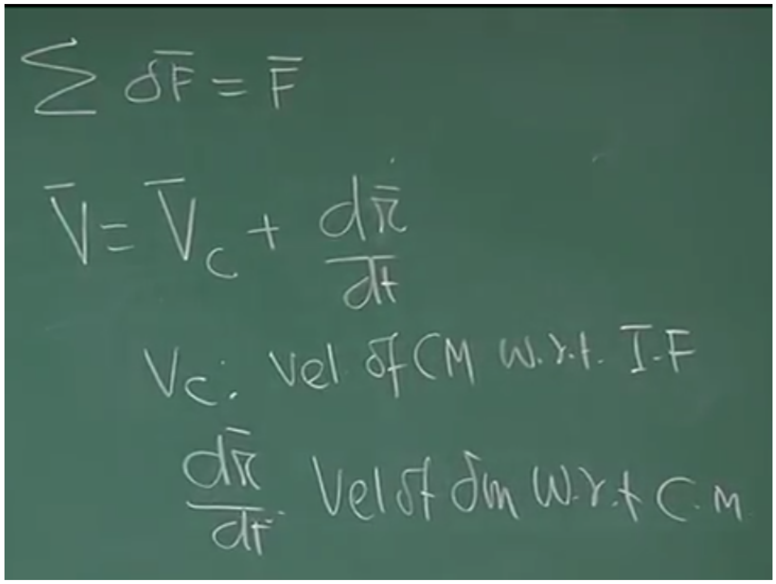

Let me write this 1 is summation of Delta F is F that is total force F vector will added and we know that V = V center of mass + DR by DT. No problem the velocity of this particle V because that is important for us here please note that we want to relate this velocity V which is velocity of the mass element with center of mass speed velocity okay. So, this velocity at this will be velocity of center of mass + DR by DT because if it moving like this it could move like this so.

That DR by DT will come here so, this is the expression then I can write because VC is velocity of center of mass with respect to this I inertial frame I will keep on writing. So, that it goes into your mind and DR by DT is velocity of elemental mass velocity of DM with respect to, with respect to what? DR by DT R is measured what is R? Let's understand R is the position vector that is it tells you where this Delta M is located with respect to what?

Is it with respect to inertial frame no it is respect to with respect to center of mass right. This R is respect to center of mass which I am writing a CG for general situation I write center of mass clear. So, what is DR by DT this is the velocity of Delta M with respect to since R was with respect to center of mass so, DR by DT is also respect to center of mass for as it is center of gravity all is similar concept is sent right. This is clear?

Then next will go are little bit of more involved equations very simple equations that more important please understand it should understand the physical meaning of it once we have understand that then this things are only your finger tips. Do not memorize anything here, any problem any where you have not able to understand use the forum aggressively okay.

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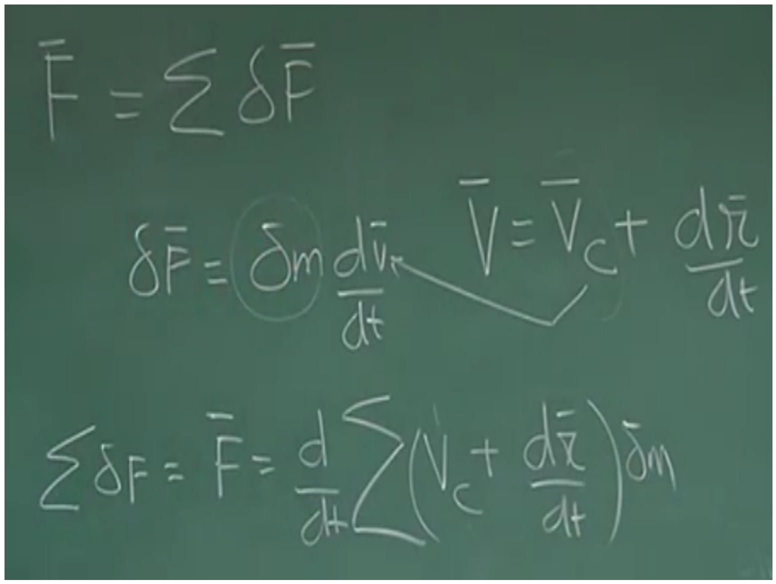

So let us see what is F? F external was summation of Delta F and by Newton's law of Delta f = M DV by DT it's for a Delta M mass element it is Delta M Dv by DT and what was V, V we have just now represent that as velocity of center of mass + DR by DT this was which respect to center of mass and this is we are respect to inertial frame.

So, I have substituted this here to get summation of Delta F = total F is D by DT of summation of VC for V it is VC + DR by DT like this and Delta M and Delta M is here is the total cross effect is was force on singular limit summation means total force so, after this what happens if I expand then I get.

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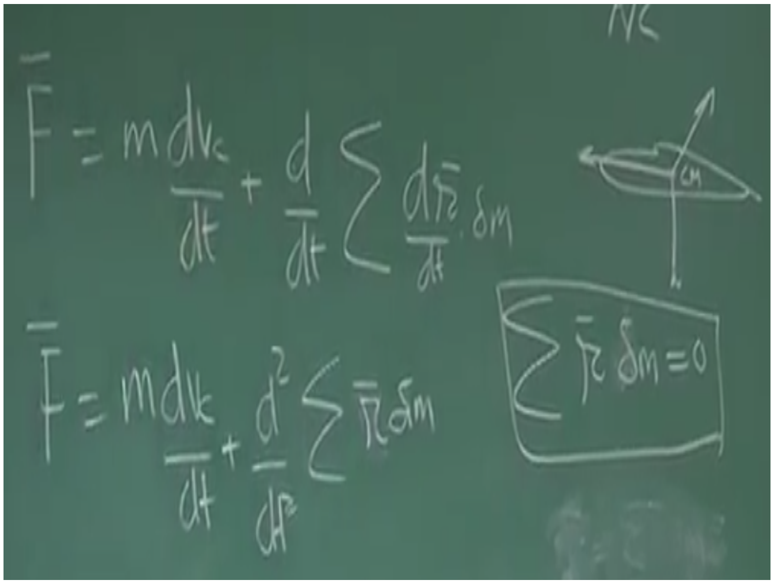

F = N DVC by DT + D by DT of summation DR bar by DT into Delta M so, again I do little manipulation I write F = M DVC by DT + D square by DT square summation R bar DM or Delta M. It's clear. M DVC by DT M DVC by DT then D summation this all been taken out so, D square by DT square summation RDM what is summation RDM? What is summation RDM please note that we have ensure that the axis system is at CG location right?

So, if this the airplane when axis system is that center of mass so, by these definition this automatically become 0 why? Because you know how do you define center of mass any center of mass. Any center of mass location is summation of RDM by summation of DM now says are here the axis says axis at CG this value becomes 0, so naturally RDM became 0 this is definition of center of mass okay. I am you should be able to understand this. If this is 0 then what I am having?

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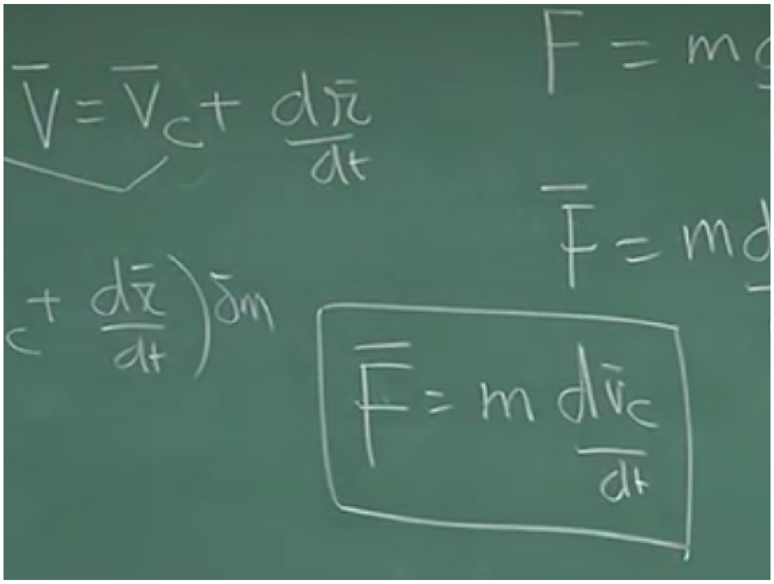

F is = M DV C by DT. What is the physical meaning of this try to understand that okay?

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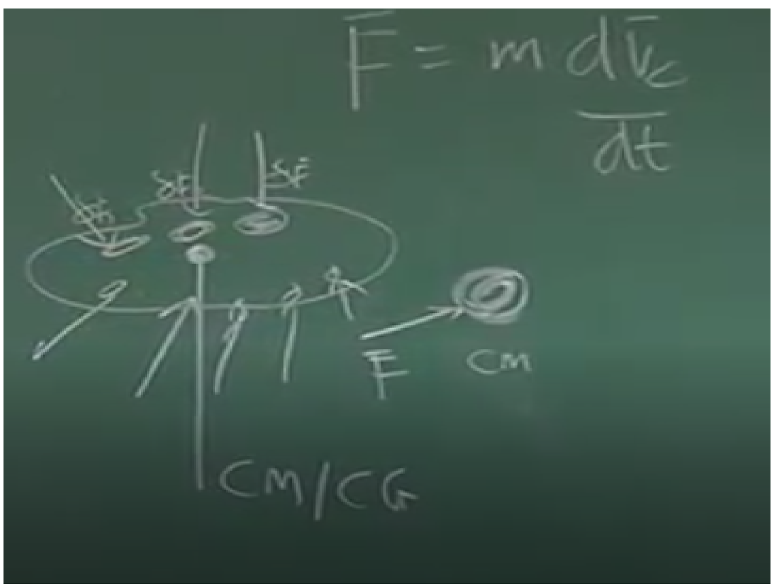

F = M DVC by DT. Which is the velocity of center of mass with respect to initial frame. What is the meaning of this? We started we this is the body we took element mark and this is element mask and you say this is Delta SF T another mask Delta F 2 another mask Delta F 3. So let say all the forces are acting in general direction on this body okay. Now how to apply Newton law? This terms to way to handle it as long as working in inner surface of a frame are says forget about the Delta mask and all these things.

Find out what is center of mass of whole of this Delta elements this is the center of mask of it is CG also okay. Once you have located that point then assume that whole mass is at the center of mass. And this is applied by the force F which is resultant of force. So now you apply it so this will tell you if there is external force M what will be the acceleration overall effect on the acceleration of the center of mass right? That is what you are looking for because we also want to track how the center of mass of the air plane is moving in flight right, Okay.

This is very useful equation will use 1. So, this talks about center of mass moving what is the velocity with what we also like to know what is the angle of velocity because it has gone not only 3 degrees of freedom 6 degrees of freedom U V W P Q R that is angular motions. So this equation is go to as for as velocity concern but you have to look for angular motion okay. And who cause angular motion like F cases linear motion and angular motion is causes by the moment right, which will also effect of force right.

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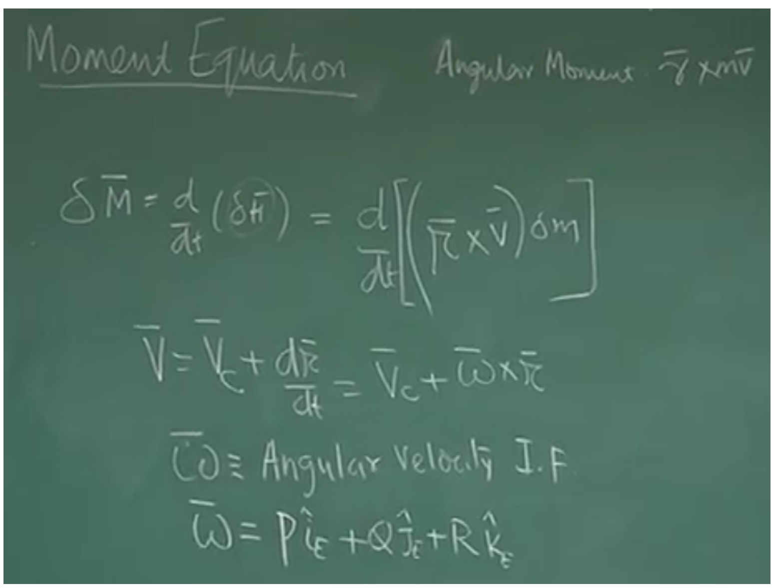

Moment so we will use the moment equation. What was the moment equation again Delta M = D by DT of Delta H we called Delta H Delta is the angular momentum. Elevator angular momentum and which is caused by moment acting on that element. This I can write by definition D by DT definition angular momentum is that is moment of the momentum remember? Moment angular momentum if want to defined angular momentum go back to class 12.

It is actually moment of a momentum and it is defined as R cross MV right, Okay. So now what I will do I will write here R cross V Delta M. R cross MV written in this passion right. This is rate of change of angular momentum and what is V V C center mass + DR by DT and DR by DT was with respect to center of mass but VC was with respect to the inertial frame and this I can write as VC + omega cross R where omega is omega vector is angular velocity with respect to what?

With respect to inertial frame do not forget that. This is respect to inertial frame then R you know is positional vector may here with respect to center of mass right. So omega will have again 3 components 1 P along IE axis + Q along JE axis + R along KE directional vector directional. IE, JE, KE are the unit vectors of the inertial frame which is as per earth fixed frame because neglected acceleration of the earth okay.

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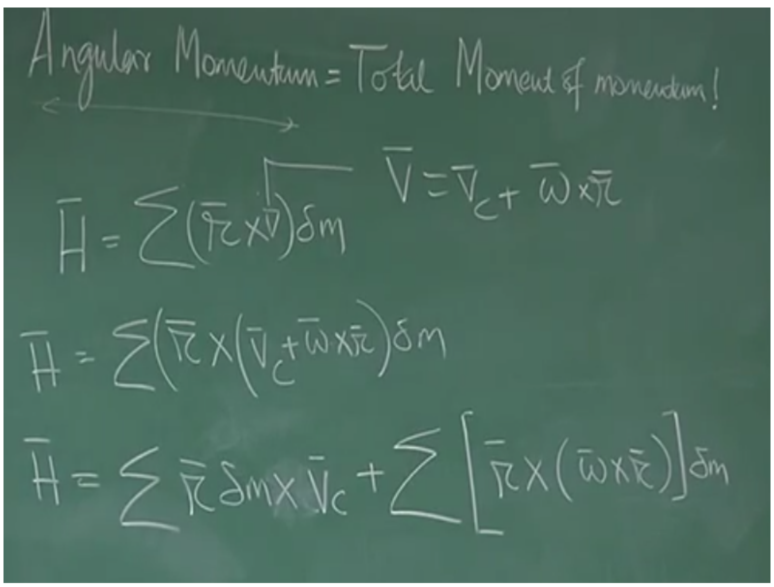

Now we are trying to define the expression of the F of angular momentum and just to recall remember angular momentum. We are interpreting class 10th, 11th, 12th was the moment of the momentum, moment of the linear momentum right. Okay so I thought which will help you to understand so moment is R cross force so, moment of momentum is R cross MV that R cross MV Delta M taken Out V is here so this V right. Remember V is nothing but V = VC + omega cross R. Now we will try to develop the question so that we can find out the angular motion right.

For that we know we need angular momentum like for linear motion for modeling equations to derive are to predict what to compute linear motion like U V and W, we use linear momentum angular motion will be using angular momentum and what is angular momentum? Go back to class 10th eleventh and all which is because to be defined as moment of the linear momentum right. So we have using that concern H angular momentum is moment of the linear momentum.

So V bar Delta is the linear momentum of a eleven all mass moment is R cross that and that is an summation it is the total vector some. And V you know VC + omega cross R for that is substituted here V = V C + omega cross R so I have the H expression has summation R cross VC + omega cross R Delta is here so again if I modify it will be summation R Delta M and VC first term + R cross omega cross R Delta M and by now we know if axis is at the center of mass of center of gravity.

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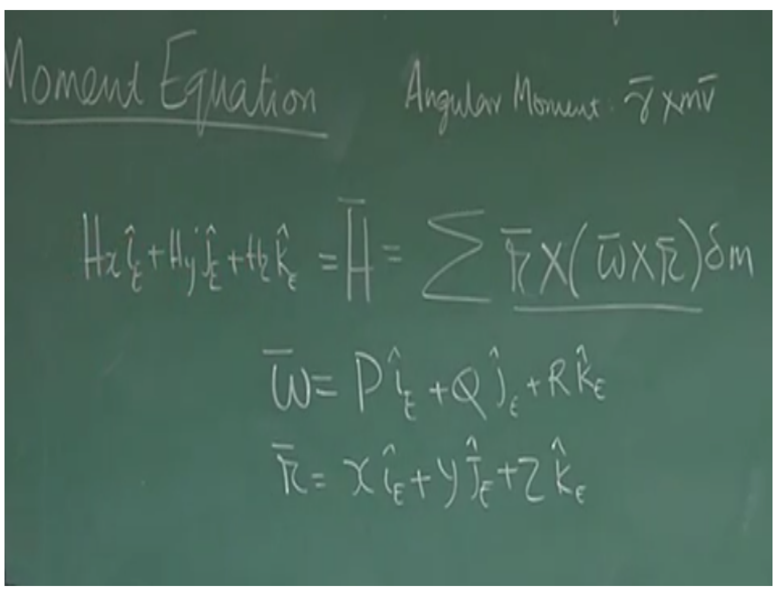

This will be 0 we have expression is H has summation R bar cross omega cross R into M right. Lets now forget this term when is because the axis is located at the center of mass. Okay. So this is very clean now? Assume I have 2 new solutions is because H has got component like HX IE + HY JE + HZ KE which are the unit vectors omega I will expand as P IE let you write this and explain Q JE + R KE what is the meaning of this expression.

This is the angular velocity which has component P along axis X axis Q along axis Y axis R along axis Z axis. Similarly R as X IE + Y JE + Z KE .what I do now I need to find HX HY HZ so I will operate this I will put R here I will put omega here and I will use the supportive of vector triple product and I will live it to you this should be able to derive it if I do the vector triple product expansion I will get expression like it. I will get if you write the expression I will write HX = P.

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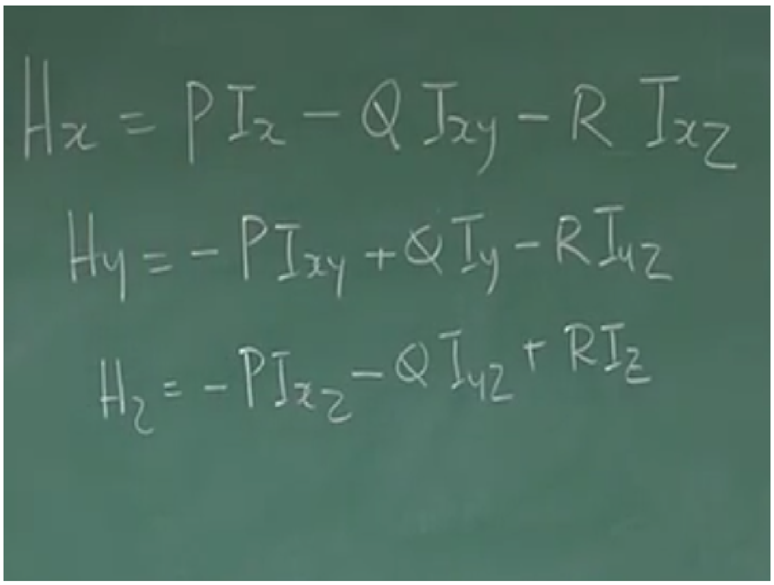

IX - Q IXY - R IXZ then HY is - P IXY + Q IY - R IYZ and HZ = - P IXZ - Q IYZ + R IZ .and you know that IX, IX is the moment of inertia about axis IXY IXZ cross moment of inertia and we should be able to easily derive this by using this information okay? I leave it to you at this stage and if you do not feel comfortable to come from here to here I am sure should not to be the case right.

You all engineering student now? This things you have done many a times I am writing all this things to familiarize to you so that you can go to the aircraft equation of motion and you cannot do that any way we will be putting after getting your feedback and will post it for yourself getting understand of it right.