Aircraft Stability and Control
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Lecture- 40
6 DOF: Angular Momentum Components

Good morning friends, we were discussing about the steps required to formulate 6 degrees of freedom equation of motion and how did you we start is a that if I want to write equation of motion.

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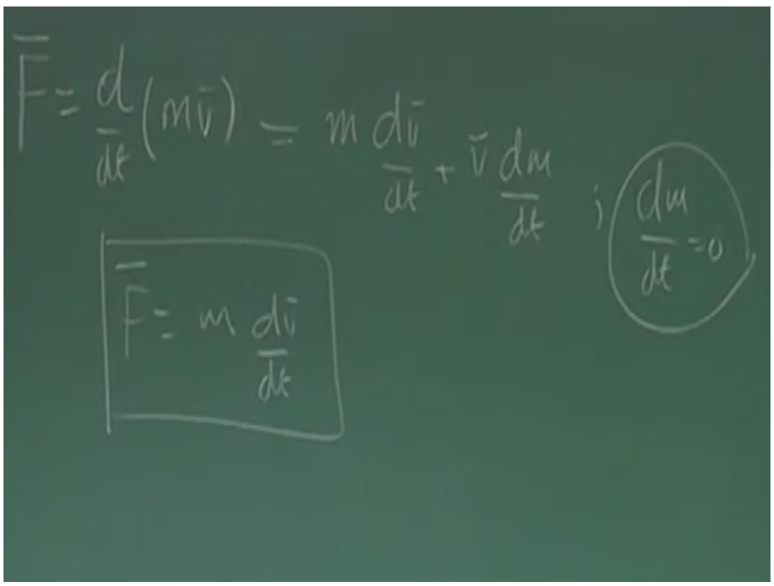

Then I need to use this D by DT of MV that is external impress force that is responsible for changing the momentum of a body right? And also we need a simplification is this can recognize MDV by DT + VM by DT right? And what is simplification. We need we said DM by DT is 0 this negligible change in mass please understand we are developing this equation of motion for his specific purpose.

For purpose of dynamic stability analysis and when you do this dynamic stability analysis we are not doing for a longer time we do for a small time may be we need only data of 3 4 seconds 5 seconds 6 seconds that's all it should be able to immediately characterize generally for stable airplane whether it dynamically stable or not okay? So, because of the duration is small we said we are assuming that whatever change of mass has happen because of will consumption that is neglect okay?

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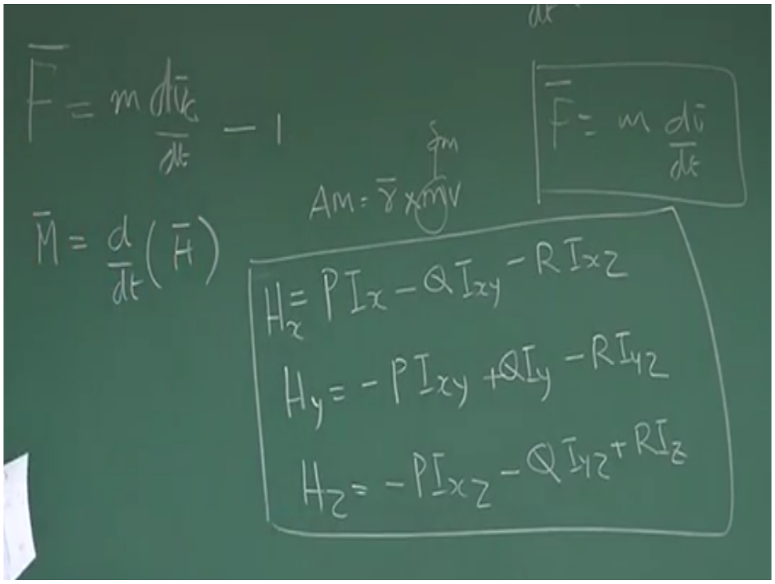

So, who we wrote F = M DV by DT and based on this we develop further equation we wrote F = M DVC by DT understanding was whatever external impress force is acting on the body air can modulate for rigid airplane as if this is as simple as this external impress force will change the acceleration of the center of mass what is center of mass automate center of gravity we say on earth it is the point where all.

The mass is can be assume to be concentrated recall we started developing this by assuming elemental mass then the summation right? So, this is 1 understanding second we got that external impress moment that is again that will cause rate of change of angular momentum this was linear momentum and this angular momentum and from there we got equations that this H we try to develop how to derive an expression for angular momentum.

We again followed that definition of angular momentum has R cross MV right? And we again to Delta M then the summation last lecture you could see so, after doing that we got the expression H = P IX - Q IXY - R IXZ then XY this is HX the component of H along X axis. I will explain, what is the X axis I will try to recall has so, that it goes deep into your mind let me first write and HY we got has - P IXY + Q IY - R IYZ and HZ component is - V IXZ - Q IYZ + R IZ that you we exactly where we ended our last lecture so, I will go little more into this so, that your clear? What exactly happening.

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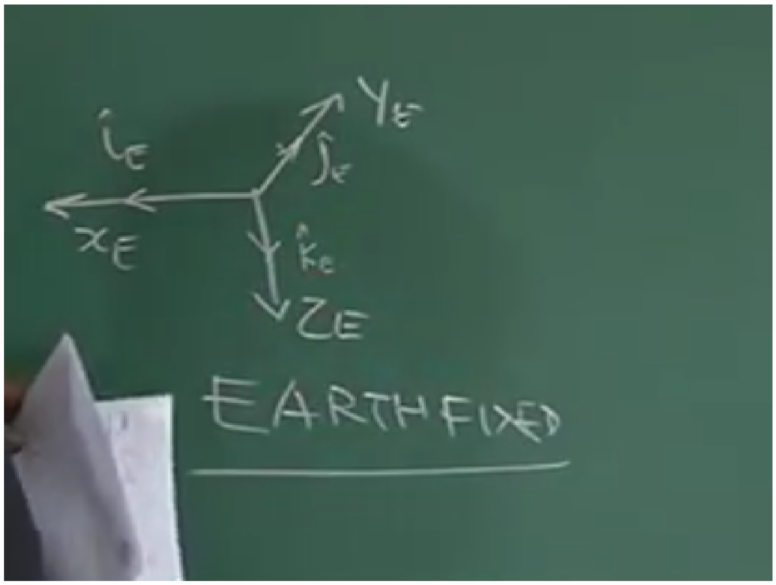

1 thing we should not forgot since we are applying Newton's loss of motion we are defining everything velocity angular momentum all this thing we refer to inertial frame right? And you know our study, what is the inertial frame is earth fixed inertial frame so, this XE YE ZE why we are choosing earth has inertial frame because we are assuming here that us acceleration is negligible to the context what we are doing stability analysis.

So we are assuming ours as know acceleration or the acceleration is to insignificant to do anything with the stability analysis that's why we say we are assuming earth fixed and twitting it has inertial frame for the purpose of this steady okay? And what is HX? HX is angular momentum of the airplane along X direction that is along the unit vector IE which is fixed this is not rotating right? What is HY? HY is angular momentum along earth fixed there direction unit vector similarly K direction for SZ and how did you develop this equation that's also important.

We trying to go back and see how this expressions. Where obtained HX HY HZ is the component of angular momentum along inertial frame which is earthy fixed and unit vectors are IE JE and KE right?

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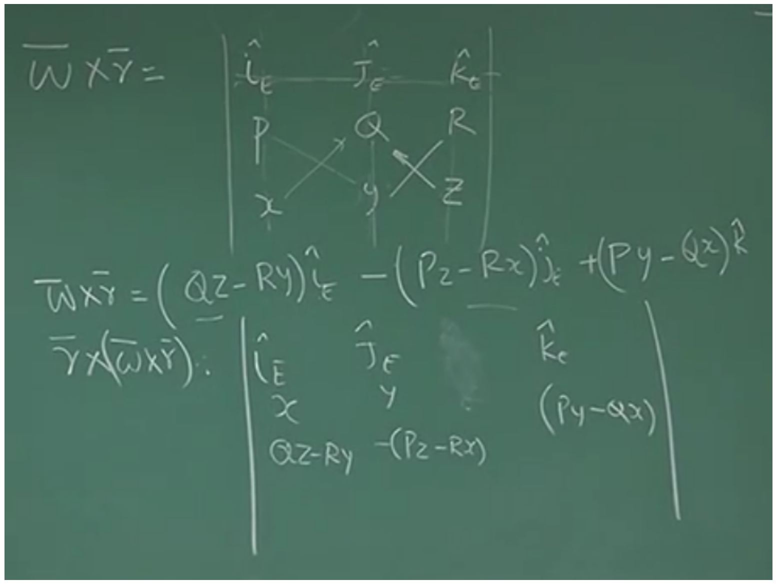

To recall what was the approach this H we are shown to be = summation R cross omega cross R DM right? What was R again see if this first the airplane this is the wing just recall this is the center of mass and what was R? R is the distance of mass element Delta M with respect to center of mass okay? Or the center of gravity for a purpose what was omega? Omega was the angular velocity of this Airplane about inertial frame correct?

What was omega? So, omega was what we assume was angular velocity about inertial frame which are having I Y Z can KE as the unit vector. So, if I write omega in terms of refer to inertial frame I will write so, omega X component X of P into IE + Q Y component into JE unit vector + R KE unit vector okay? What are PQR? PQR are the component of angular velocity omega along inertial earth fixed XE YE ZE axis right? Okay? So, this is clear? So, now what will do? Will try to see from here how do get this expression I will write the steps and you should do yourself.

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First of all let’s find what omega cross R omega R is cross would be what is R? R is are you know his X let as X IE + Y JE + Z KE okay? Now, omega cross R is what now I (Refer time: 09:23) I write IE JE KE and omega omega is P Q R R is X Y Z so, if I try to take the cross product omega cross R this will give me omega cross R = let me write QZ - RY I - PZ - RX JE + PY - QX Q bar how do I get this we are trying to explore I try to understand.

How did you get this expression HX HY HZ which are the component of angular momentum of the body along earth fixed a XE YE ZE axis system right? Please go back to my lecture notes the expression of H was given us summation R cross omega cross R DM and you know R is XIE YJE and ZKE now if I take the cross product omega cross R now you know IE JE KE P Q R X Y Z and if I take omega cross R I will get this 3 term like this you know very well.

How to do it take this so, the I so, QZ - YR - J right? Cut this it is PZ - RX so, will for K if I do like this this is PY - QX okay? so, this is omega cross R what we are suppose to find R cross omega cross R so, for R cross omega cross R is it be again very simple again you write IE JE KE unit vectors and then it is R cross so, X Y Z and for omega we write Q Z - RY then which is - PZ - RX and let me write this should be separate here KE unit vector this will be PY - QX okay?

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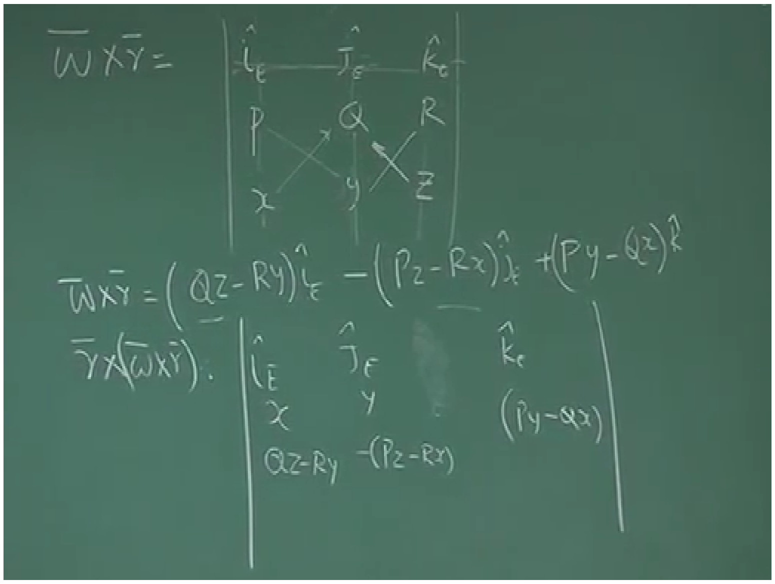

Then you do this cross product and then we will see that you will get an expression again by mechanically just it does not require and a skill of simply we have to do similar way and then you will get whole expression

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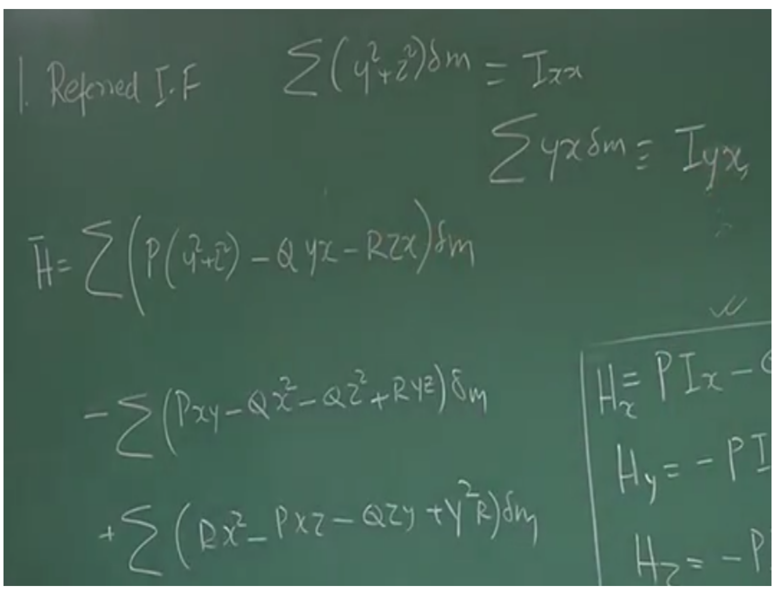

H has let me write this summation of P y square + Z square - Q YX - R ZX DM then - again summation of P XY - Q X square - QZ square + R YZ DM and third 1 will be + summation R X square - P XZ - Q ZY + Y square R DM. Clear? See for example if I take the first term this I will take it out so, this will be I and this into this this into this - this into this this is the procedure this into this - this into this so, immediately you could see you get PY square here right? So, accordingly you can develop okay?

But later given interpretation what is summation of Y square + Z square DM we all know what is it this is nothing but IXX right okay? Similarly you find summation of YX DM this will be cross moment of inertia IYX. So, accordingly you can develop so, what we have seen we now know what are the steps to get this HX HY HZ which are the component of angular momentum are about the inertial earth fixed frame okay?

And you are were smart enough to know how this terms have come please remember you have to do 1 solve this things then you play around with the result okay? So, put your time to see the how what are the steps how we got this thing that clarify so, many things when we will be going for implementation. But remember 1 thing your all working so for in inertial frame right? What is the issue now.

I will be discussing then we will try to justify why we need to work in body frame that will make a life more comfortable right? That is the part now coming so for we are working in inertial frame.

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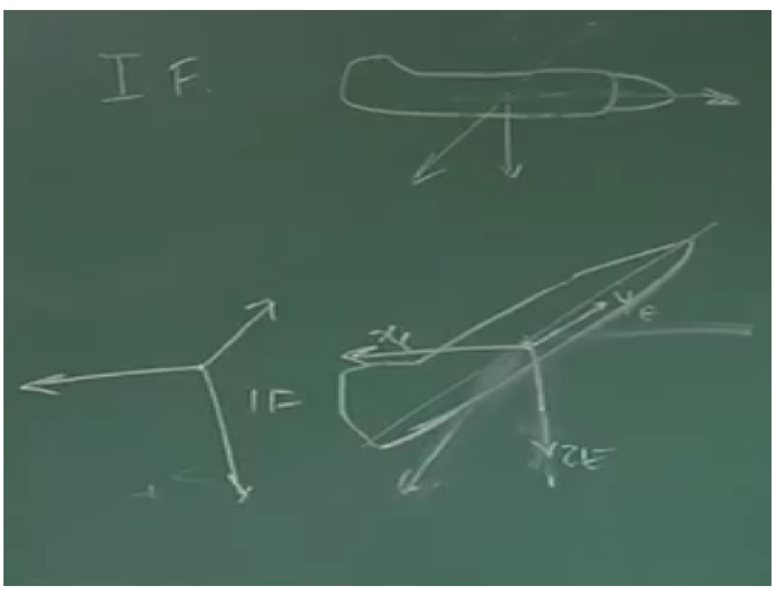

What are the issues see suppose this is the body okay? This is the airplane and are working with respect to inertial frame. Now see we have been using moment of inertia of the body. When your estimating deriving the expression for angular momentum like IXX IYY IXY IYZ so many but imagine suppose this is an inertial frame and I fixed I also imagine this inertial frame is same were here. Whatever way you think that is this inertial frame I can always see inertial frame at this point which is fixed that orientation there is no change in orientation okay?

Now what will happened if the body moves rotates like this as the body moves now see the moment of inertia about fixed axis system will go on changing right? This is the body and let say axis system is this which is exactly what is in on the earth whatever we are drawn here if the body rotates what is happening whatever X axis was earth is here now the position of particle flow the axis X axis is changing so moment of inertia will change similar thing will happen.

Other axis also as the body rotates and if I am doing all measurements to the respect to fixed axis.

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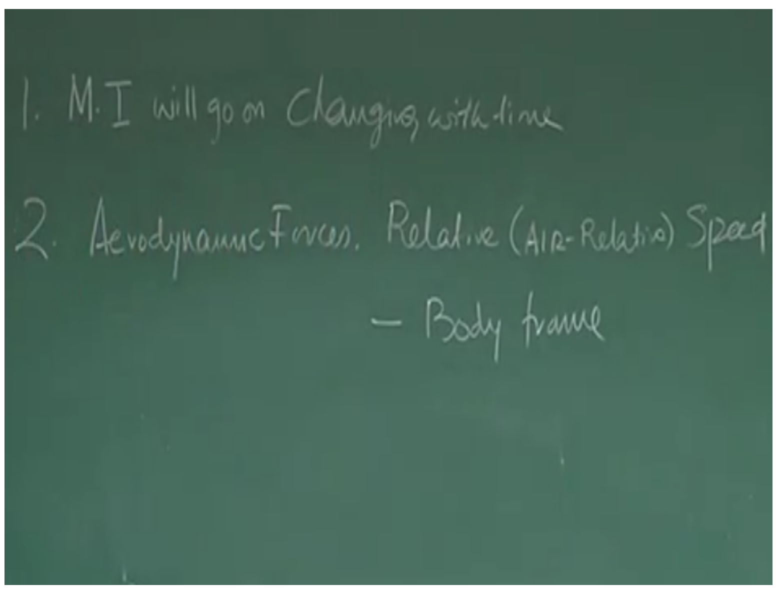

The moment of initial of the body will go on changing with time moment of inertia will go on changing with time. Right? Explained here suppose the body have rotated like this okay but its earth fixed axis is stationary okay? This is the X this is R to be more precise lite system the way we are doing it lets a this is X earth this is Y earth this is Z earth right?

As the body has been rotated that distance of the particles from the X Y Z axis also changing that is if it was initially like this if you think of a particle here it has some coordinates with respect to earth fixed X Y the axis it rotates same particles now different coordinates. So its moment of inertia will change that why is I can change as the body rotates if I working with inertial frame
Its moment of inertia will go on changing with time.

Which creates an lot of complication for us for competition this is 1 second thing is you all know the Aero dynamic force dynamic forces is be depend upon relative almost precisely AIR relative Air relative speed or velocity okay? So it has nothing to do it what is a speed with respect to the ground are its system.

I have given system example aircraft that could belong the ground how are you there is a wind it will generate or it experience an aero dynamic forces. So as per as aero dynamic forces you concern I will be more comfortable if I see what is happening with respect to relative to any frame that is fixed the body okay? I will not in comfortable in terms of if this velocity is respect to ground because the force is depend upon the alternative velocity into the body.

So, that also associates it is better towards the body frame okay? 2 things I repeat as I am rotating the body are the body rotation free space the moment of inertia body about the fixed at fixed axis system changes so it is a moment of inertia will go on changing an time that gives complication second point because the aero dynamic force is depend upon relative air speed relative air with respected to body not respected to ground.

So, I will be more comfortable towards with a body body frame and that is where what is d1 is we prefer working in body frame.

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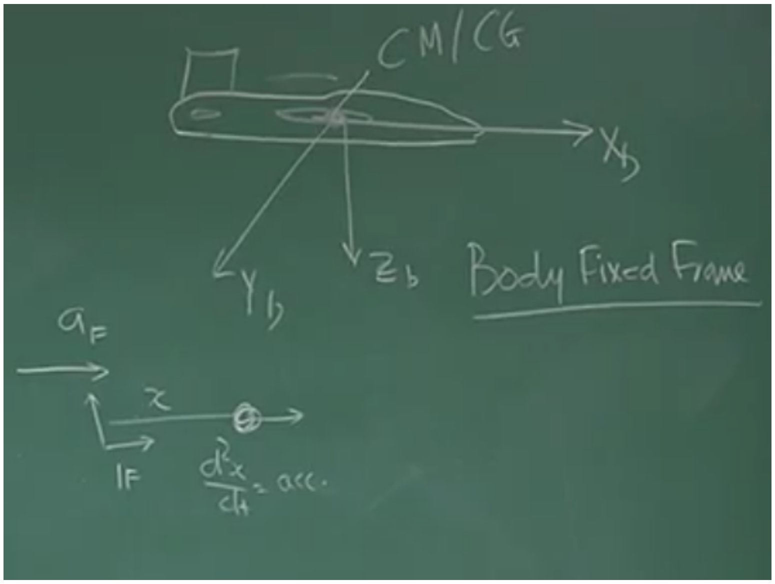

Lets this is XB YB and ZB this is the body fixed frame that means as the body rotates the frame also rotates so 1 thing in the frame rotates in the body frame the moment of inertia will not change with time right? Because axis also rotating if it was respect in inertial frame there are the body rotates inertial frame is also rotating so, there will be changing moment of inertia.

If I work in body frame as the body moves axis also moves so moments of will remain same no issues second thing if I working in a body frame I can get what is the relative speed of the air plane with respect to the medium okay? So, this 2 things makes like comfortable and of course it goes outside it is located as center of mass or center of gravity right? We have a desire to work with a body frame it will make all like very comfortable why is the problem?

Let us see it is good we want to work in body frame that make our life comfortable but what is the problem?

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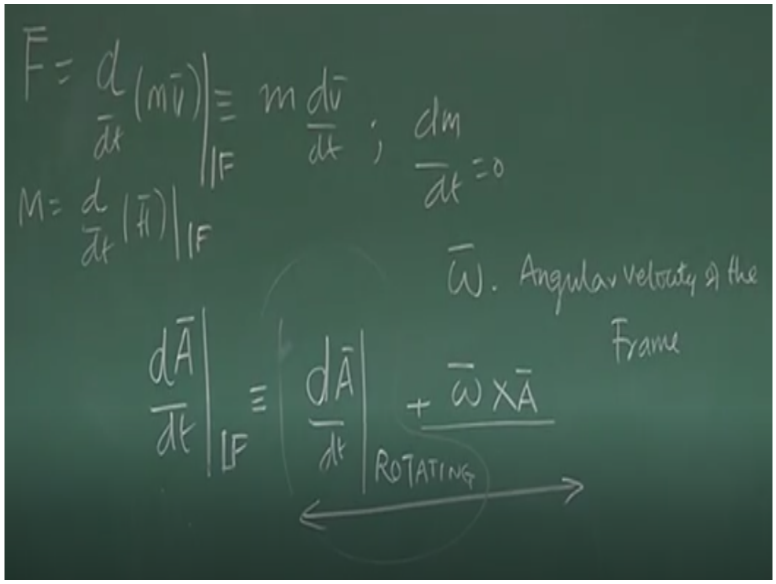

Problem is when we are applying F = D by DT M DV by DT or M = D by DT H rate of change of momentum and we have neglected your DM by DT so this was to be more precise this was D by DT of MV rate of change of momentum and that will wrote as M DV by DT because DM by DT was 0 so, need to have vector differentiation in which frame this should be in inertial frame.

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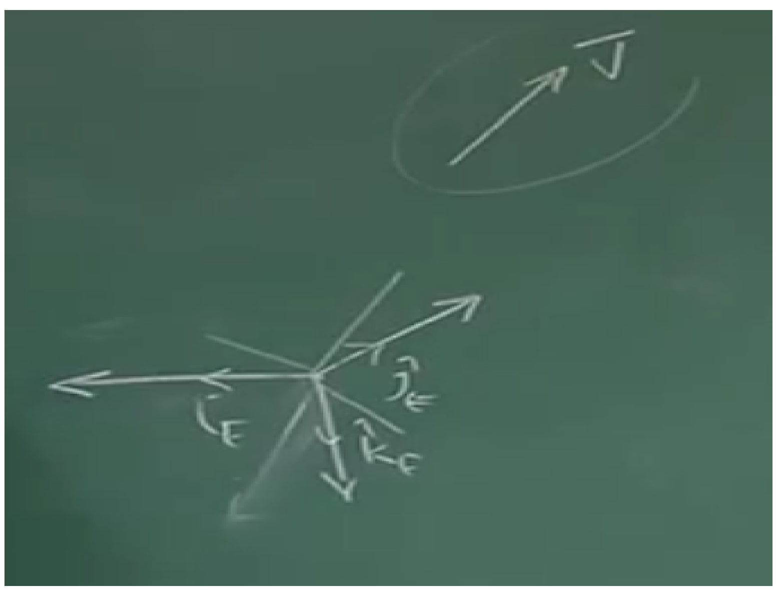

Please understand 1 thing suppose if you are in a frame so this is vector in this frame okay? And assume that this directions are not changing the frame is not rotating okay? Its unit vector direction IE JE and KE are fixed right? Okay? Now see if this body rotates like this what will happen what will happen is un less it is body language velocity V then the component of V along

X Y Z will move on changing right? You need to say the rate of change of momentum along X Y Z

We can easily find out what is the component here? What is the component here? What is the component here? And take that derivate of that component the scalar component that will be actually component towards the rate of change of the momentum are rate of change of velocity okay? However this is the velocity vector right? And velocity vector let say which fixed here and the frame is rotating then what will happened 2 things are happening when frame you rotating.

What is the component is changing? Scalar component because the unit vector is changing that means that is change DI by DT DJ by DT DK by DT, that is the rate of change of unit vector why because the direction is changing if the frame is rotating the direction of I J K are changing so vector is different vector so direction are different so there is the rate of change of vector for our purpose right? So what is to be d1 is that if I want to really do this derivative.

Let's derivative of a vector in inertial frame, I will ask a question can I still use this as an effect but operate it in rotating frame that is I want to work in this body frame which you rotating but I will ensure that it is equivalent. Equivalent to differentiating the vector in inertial frame so what is the way to do it okay? That is the question you are asking.

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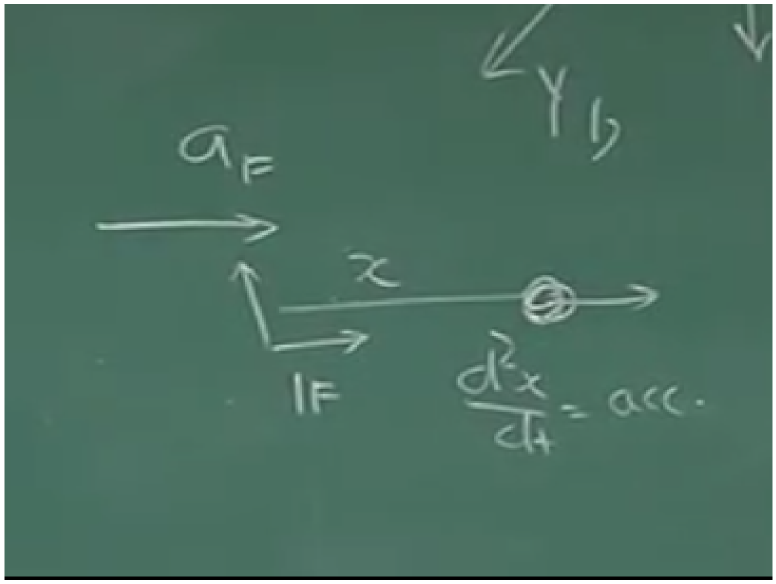

For example if I measuring acceleration of a body let say some axis system is there I say it is a inertial frame so I know that whatever the X difference is there D square by DT square is the acceleration correct as longer. This is inertial frame. But now if frame itself is accelerating respect with some AF. Then whatever acceleration we are measuring with respect to this AS to this frame will not to the true acceleration.

So what is the best way to get the true acceleration? That if I know frame acceleration. Some of I should observe in this acceleration and correct this acceleration then I should get it okay? With this philosophy, will now let see how mathematically this problem can be handled by operating through the definition of a vector in rotating frame so what do you want DA by DT but I want to work in rotating frame and I will just show that it is very while standard result and that is this DA by DT inertial frame equivalently.

Can be handled by operating by differentiating in a rotating frame if you put appropriate corrections and that is DA by DT in rotating frame + omega cross A. That is you find DA by DT assuming there is no change in I J K direction okay? Only scalar part I repeat here. What is the message here the DA by DT inertial frame is equivalently, I can use an operate on rotating frame, if I get DA by DT in rotating frame and add this omega cross A omega is the angular velocity is the angular velocity of the frame let us try to understand it more.

And I would suggest that let us see how this equation was and the expression was derived. Once you do that oh this is the thing the message is what you need not differentiate the vector its inertial frame you still can work in body frame rotating frame if you instead of this you do this in a rotating frame + add this that all okay what exactly it means when we derive this relationship Thank you very much.