Aircraft Stability and Control
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Lecture – 43
Small Perturbation Theory

Good morning friends, we have so far developed equation of motion and we have so far developed equation of motion expressed equation of motion in body fixed axis system and we call u dot v dot w dot p dot q dot r dot equations and you all know why we are developing this equations of motion. We are very focused. We are developing the equation of motions to study the dynamic stability of the airplane.

What is dynamic stability? If a body is disturbed from the equilibrium and it totally has tendency to come back to equilibrium but I can also finite time to come back to equilibrium then we say this is dynamically stability system okay. So we now use these equations and use small perturbation theory we have early understood some part of it in last lecture. To repeat this we are actually talking about small perturbation for simple reason that.

We do not want much of nonlinear aerodynamic information of picture and we will be operating at steady state and we will assume that at steady state the aero dynamic co efficient can be expanded it u see linear approximation right. So we will be focusing towards small perturbation and you know this small perturbation there are many basic examples that product of two small perturbed quantities can be neglected and you see as you develop you will be using those concepts.

Let us not forget we are trying to understand the dynamic stability of airplane. And try to develop a model to study the dynamic stability of an airplane to evaluate dynamic stability characteristic of an airplane through its natural frequency through its damping ratio and time double the time to half, all these things you know. But to make thing simpler, what we will do? We first do dynamic stability for longitudinal case.

Perhaps in the course we will be focusing on longitudinal case. In a case one advanced dynamic stability goes place in lateral direction in itself it is formed. This course is the first course on dynamic stability. We will not unnecessarily make life miserable. What we will do? We will take simple things and understand what is the approach how to interrupt's so that we can understand it for much complex cases okay.

Remember that we have u dot v dot w dot p dot q dot r dot equations since I am doing the longitudinal case that means I am talking about motion in the vertical plane. So I will be bothered about u dot, not v dot okay. I will not talk about bothered about roll I am talking longitudinal so see u dot this is w dot, plunging and also the pitching right. So three equations will take for consideration.

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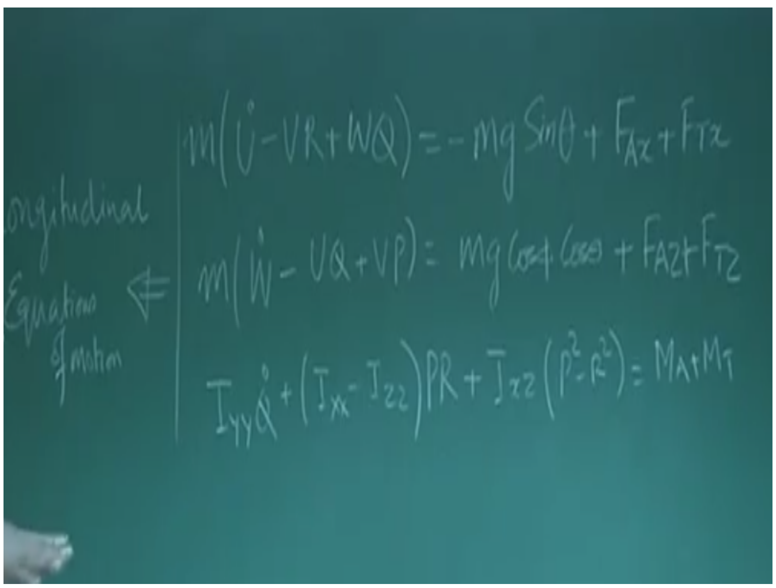

And one is mudot - v r + w q is = - mg sin theta + fax + f t x I will explain all this term and we are already doing this. Still I must explain let me write this equation m w dot - u q + v p is = m w dot - u q + v p is = mg cost phi cost theta + f a z + f t z. And the pitching equation that is q dot so this I y y q dot + I x x - I z z PR + I x z into p square - r square = ma + m t.

So these are the equations and we are naming it as longitudinal equations of motion. Why longitudinal you understand longitudinal means the motion is restricted to vertical plane so this is u dot how the speed is changing, how the plunging motion is changing and how the pitching motion is changing so that is u dot. Here u dot w dot and here it is q dot okay. You know this phi theta these are all angles you are very well known.

You know that the particular order of rotation sin theta phi to locate orientation of the modified axis system with respect to at initial frame with reference right. So here when I have written force is along x direction or along u direction that is fax is aero dynamic forces.

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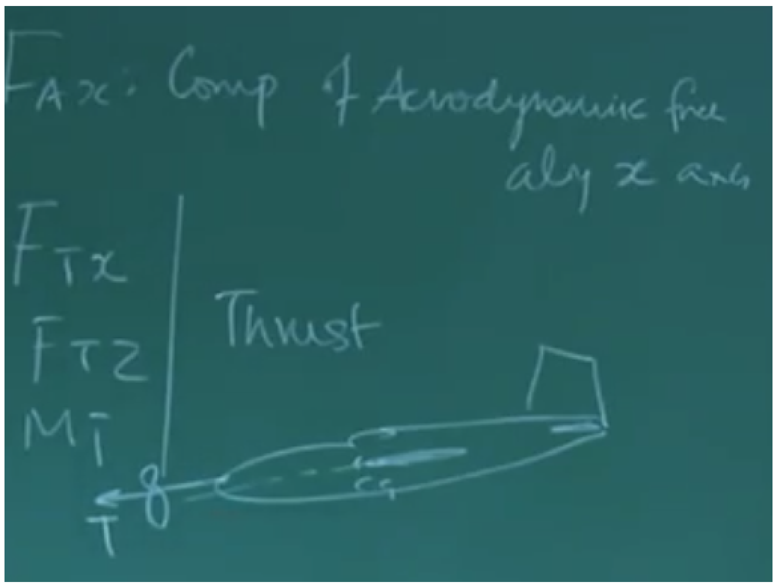

fax I mean component of aerodynamic forces along body fixed x axis right which is very important. Similarly f a z is the component of aerodynamic forces along fixed z axis and m a is about moment about y axis the aero dynamic motion. And what is f t x. f t x is because of thrust. F t x f t z m t all are because of thrust okay. Suppose this is the airplane suppose the engine is somewhere here so it will give the thrust also c g somewhere here you can see to give the moment also right.

So we are focusing more on longitudinal stability so that is why we have picked this three equation for your understand right. And also we understand that we will be using particular what we call perturbed, small perturbation we will be using small perturbation theory.

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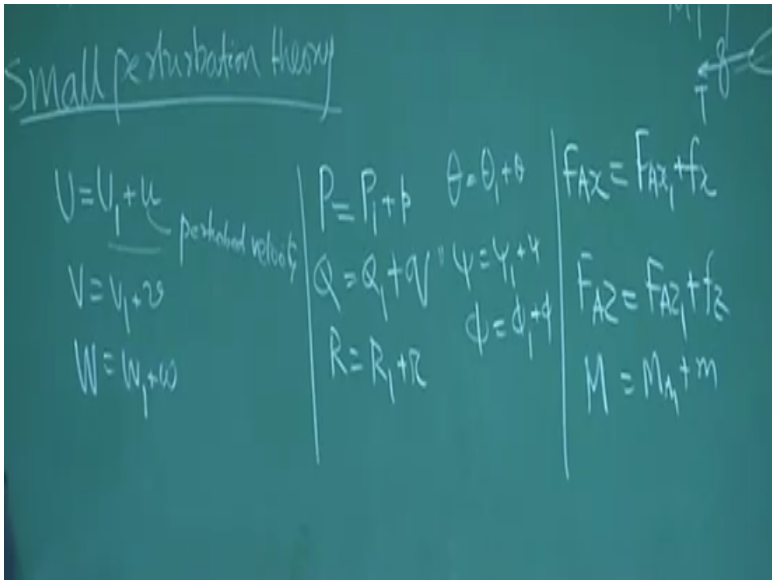

We also have very good idea now small perturbation theory. The key point is. Is this perturbation will be introduced where? See what we are going to do. We are going to see the dynamic stability of the airplane. So as per the definition we are going to disturb it at one point, at the equilibrium that is at the steady state. So we will introduce this perturbation at the steady state that is the catch point. And see how perturbed quantities are behaving.

So let us say we represent u total velocity after giving perturbation after giving disturbance is u1 that the speed at the steady state + the small quantity perturbed quantity u. so this is perturbed velocity so you could see that we have using the linear approximation. Similarly we can write v will be = v 1 + small v w is = w1 + small w. what is w1. w1 is the steady state component of total velocity along local z or body axis z direction. For example if the airplane is moving like this.

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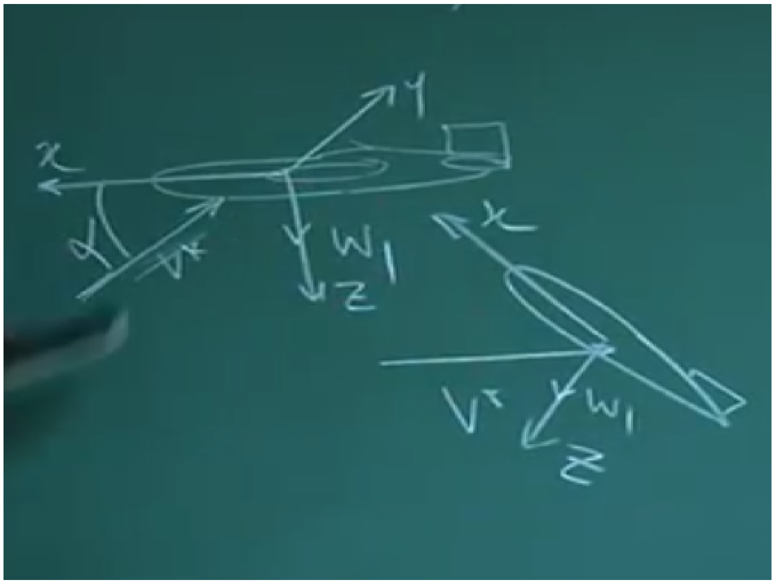

This velocity is v star as said and the x axis is here y axis is here and z axis is this and this is alpha and w1 is here. w1 positive in this direction and w1 have a component of v along w1 along z direction local z direction or again you see the better way if this is the velocity v star if this is the airplane and this is exaggerating for understanding. This is x axis and this is the z axis so what is w1. w1 is the component of v star total velocity along z direction at steady state. Clear, so this should be very clear in your mind.

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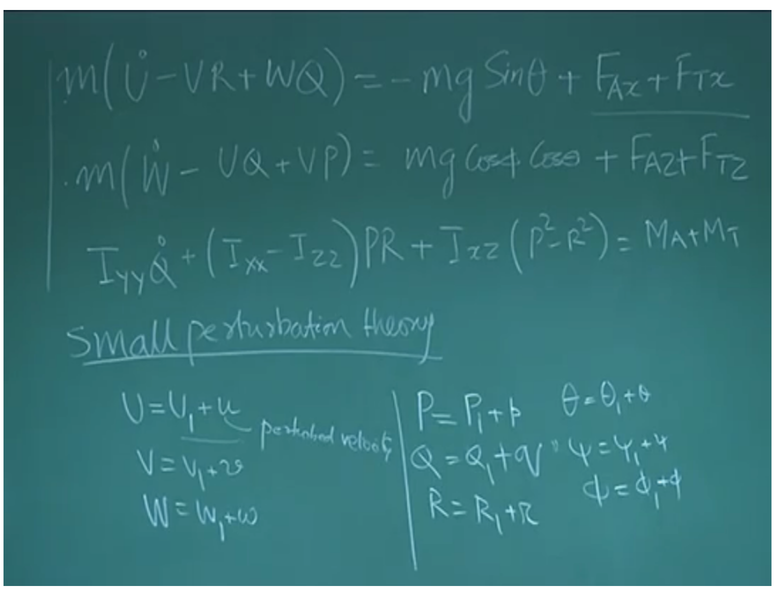

And now we will go on doing mechanical things so u be have written as similarly we can write p is = P 1 steady state + small p + q is = Q1 + small q R = R 1 + small r then theta = theta 1 + small theta then chi = chi 1 + small chi and phi = phi 1 + small phi.

All this smallest quantities pick you u v w theta chi phi they are should be understood as perturbed quantities okay. Now we will substitute this in this equation and see how equation of motion in perturbed case developed into okay. So we will take this case now here we will do some little bit of adjustment.

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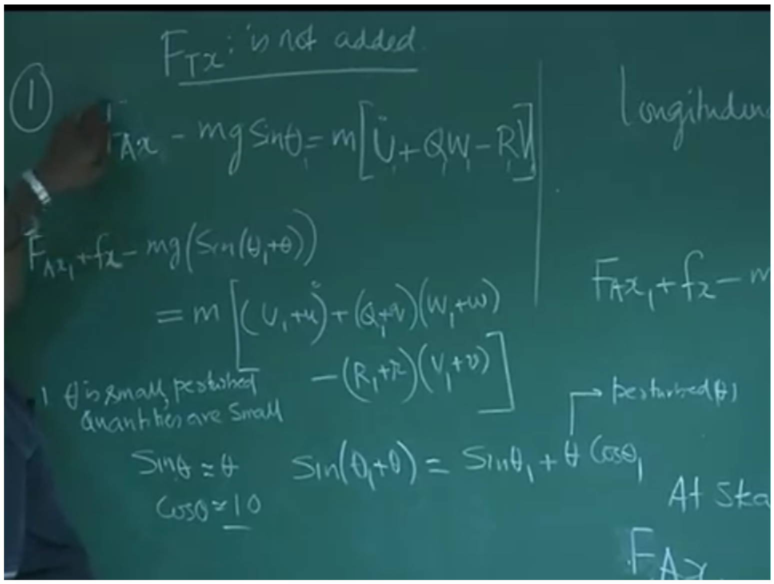

We will write the first equation as fx a - mg sin theta = mudot + Q W - R V same thing okay we have written this equation. Now notice here that we have not include f t x because we understand that if you know how to handle f t x mathematically f TX will be similar so you can add f t x term that the way fx have negatively stable okay. Here f t x is not added as you understand that if I have face fx and I know how to handle fax I can able to handle f t x also okay.

When I try to make blackboard looks little cleaner and more focus on understanding rather than all those mathematical manipulations okay. Now what is to be done? Yes this is fax I can write fax 1 + fx I will explain it what is it. F a z like f a z 1 + f z and m = m a 1 + m. what is this 1. Once I am writing the quantities with substitute 1 that is these are the condition at steady state. Why that is important after all we are giving disturbances about steady state which is the equilibrium point for us and these are the perturbed quantities so what is the meaning.

If I write fax total after perturbation if fax 1 that is whatever force is at steady state + that perturbed quantity, perturbed aero dynamic force okay. So we can some time also just to avoid confusion we see we will be stressing the conventions the science in the manner you will feel very comfortable. Many books will write fx as represent fx as fax assuming that this is only for because of perturbation only because of aero dynamic force.

For example let is physically what is that mean? Let us say the plane is moving like this and at the steady state this is the dragly the same alpha is very small. The moment I disturb it so there is a perturbed alpha so some component of lift also will come along this direction additional component. So that is what is because of perturbation right. Which is too added over all that of the aerodynamic forces at steady state and we are again taking the advantages of we are assuming is very thing is linear that is why small perturbation okay.

This is very important that is why small perturbation we are using. Now you see I will just substitute here. I will write for fax 1 will write f what is written as fax 1 + fx - mg sin theta we will write sin theta 1 + theta very mechanical don't get upset with so many terms this very simple. So u 1 + u dot + what will happen here? For qI write Q1 + small q. So I write Q1 + small q for w I write w1 + small w - R for R I write R 1 + small r for V I write v 1 + small v right this is the equation while we have introduced perturbation meaning that I what.

Airplane was moving so that fax was at steady state if fx a 1 - mg sin theta 1 is m 1 u 1 dot + Q1 w1 - r 1 v1 that was the condition at steady state all these quantities were at steady state fax 1 sin theta 1 u 1 Q1 w1 r 1 v 1. But the moment when I give the disturbance fax 1 now has become fax 1 some perturbed force aero dynamic force has come because of perturbation right.

Now I can expand this sign all this multiplication then what I do is I will put the approximations number one theta is small or to be or perturbed quantities are small and then I will explain it. I write the expansions what I get is let me write for you to complete the equation so that you are happy so let me erase this part. It will take longer length of the blackboard so I can erase this also. So theta is small so this is again perturbed quantity.

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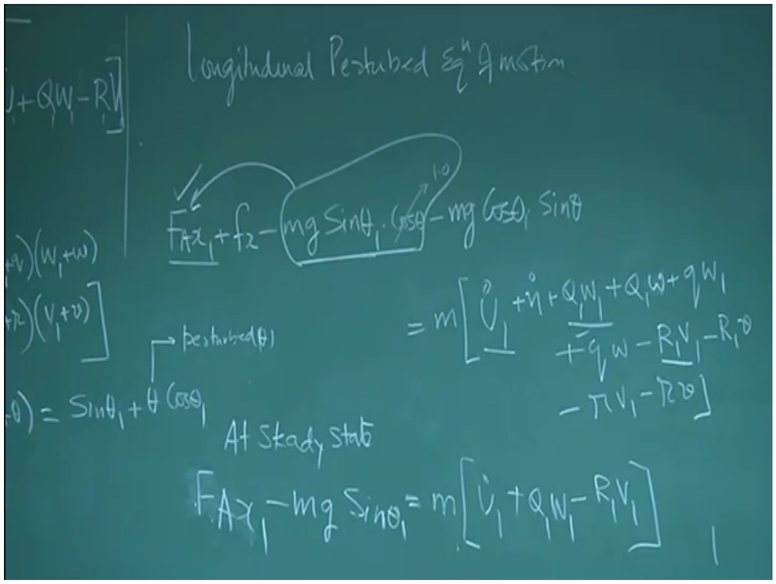

If I expand it what I get we see fax 1 + small fx - mg let me write this mg sin theta 1 cos theta you should also do it if I commit a mistake you should be able to correct it. Cos theta 1 into sin theta this is = mu 1 dot + u dot + Q1 w1 + Q1 w + q w1 + q w - r 1 v1 - r 1 v - r v1 - r v. You could check yourself this is sin theta 1 + theta I have to expand sin a + b so I will get those term here u 1 dot + u dot is here q Q1 into w1 so this physically at steady state so write as Q1. So it is Q1 w1 that term is there Q1 small w is also there so this is small w. I will write like this small w.

okay and then small q w1 yes that term is there small q w1 and then small q w q it is there fantastic so for correct. Then - r1 v 1 here is r 1 v1 r 1 small v very correct. And then - r capital v 1 yes, - r v1 very good. You could do this wonderful expansion okay very good. Now we are assuming theta is small so I will say sin theta is theta and cos theta is 1 okay. So if I explain we see sin theta 1 + theta which will be nothing but sin theta 1 + theta cos theta 1 you understand this approximation is.

If this theta is small theta 1 is small that is why theta sin was here so we have make it theta, theta is perturbed quantity. This you should be clear this is perturbed theta okay. And here sin theta 1 cos theta was there. Theta is small so we have used cos theta is = 1. So theta small is theta perturbed theta okay. So once I do this I get this shot of relationship now yourself you see. Now what we are doing? We are doing longitudinal perturbed equation of motion right.

And this perturbation is introduced where at steady state. So I try to write at steady state what were the condition and that is fax 1 we will use this equation and we write this is a x aero dynamic force at x. so what is steady state if this is fax 1 - mg sin theta 1 right = mu 1 dot + Q1 w1 - r 1 v 1. Let us see what is the condition at steady state is. When we write steady state fx fax 1 - mg sin theta 1 - mu 1 + Q1 w1 - r 1 v 1.

And if I carefully see this expression I underline this u 1 dot Q1 w1 r 1 v 1 okay. This is actually = fax 1 and mg sin theta 1 cos theta so this becomes 1 of cos theta being small so this term also gets coupled with this term. Now I can say fax 1 - mg sin theta 1 fx a 1 - mg sin theta 1 is = mu 1 dot Q1 w1 - r 1 v 1 so I can remove this term so they are automatically they are equal right. So if I do that if I take over this term what do I get? Once I know that fax 1 - mg sin theta 1 cos theta is 1 perturbation this term this term = this term so this this and this so what is.

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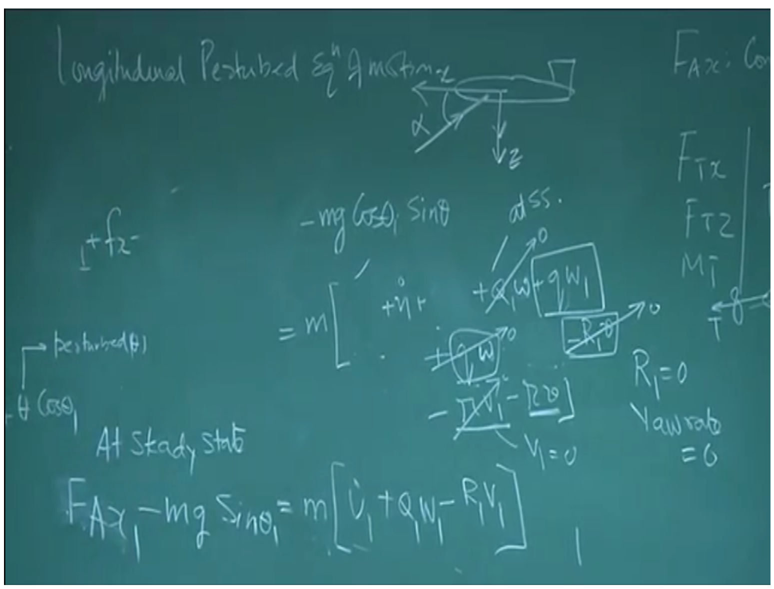

What relation here is fx right - theta mg cost theta 1 is = mudot + q w1 could you see this I have removed this so let me do for u this gentleman goes okay. Because they are = this this this and r1 v1 where and this okay. I am erasing this. Now we are left with what on right hand side mudot Q1 w then q w1 like this.

Now remember at steady state what is the value of Q1 at steady state when the airplane was going like a cruise, at cruise is there any q is there any pitching rate. No so this also vanishes correct. But I can I talk same thing about this term q w1. What is w1 w1 cannot be 0 because didn't be 0 because this airplane axis is s here z here and some angle of attack alpha.

So the component of velocity along z direction is w1 at steady state so this will n or vanishes and q is perturbed quantity it is there it will not vanish. Then we have q into w okay both are perturbed quantity small quantity so I can say they are product of two small perturbed quantity can be neglected so this man also goes.

And here you see r 1 into v when the airplane is at steady state cruise flight we are talking about cruise we are giving perturbation there you know art there you know where you are it right. So r 1 is 0 this man also goes. This goes as r 1 is 0 vow rates is 0. Similarly r v 1 is to this term. What is v 1? V 1 is the static velocity but I am talking about cruise and we are talking about motion in the vertical plane so naturally there is no v 1 so this v 1 is 0 so this man also goes.

And r into v is again product of 2 small quantities so they also vanish. So what is happening what are the terms we are there for you one is u dot another is q w1 so I get equation.

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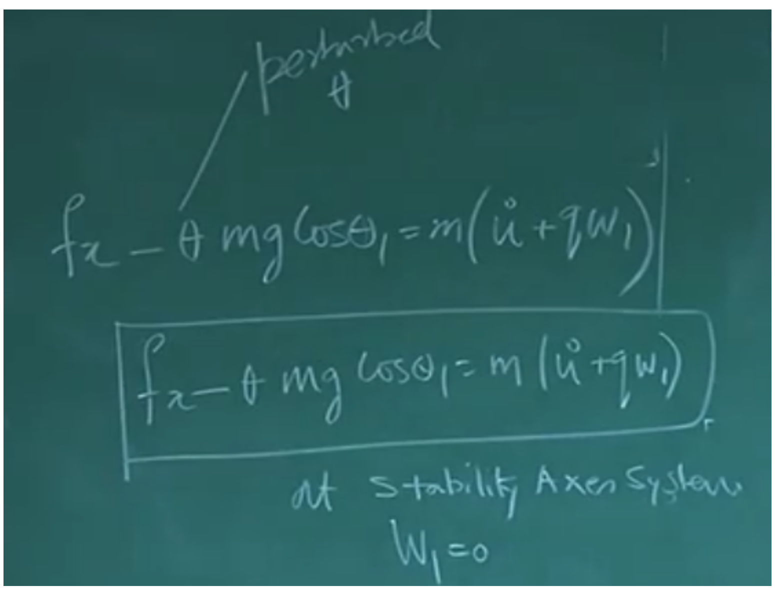

fx - theta mg cos theta 1 = mudot + q w1 what are the theta perturbed pitch I guess perturbed theta okay. So now I can write fx is = - theta mg cos theta 1 is = mudot + q w1 so this is one equation of motion which is written along the x direction. What is fixed x direction we will lose some more modification or more expansion so that things look pretty simple but for all of you I must tell you do it once and then forget it when you use the final equation and then you understand the physical meaning for this term that is more important.

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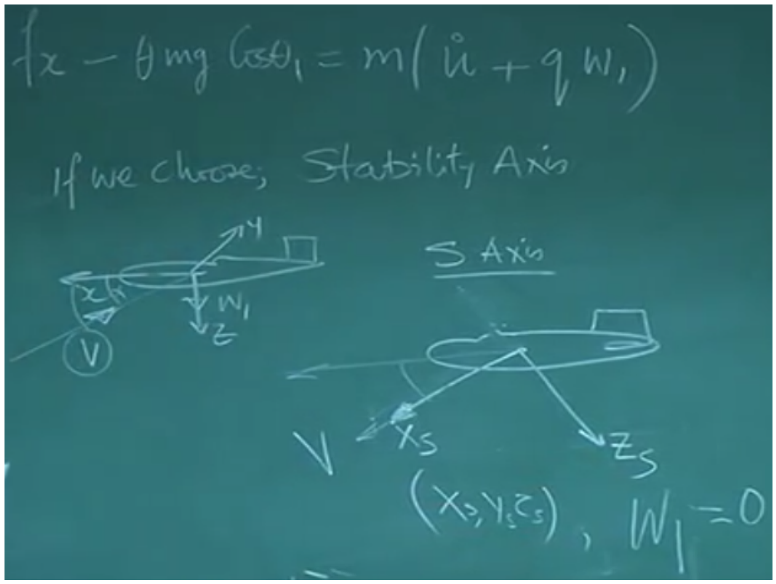

fx I have the equation like fx - theta mg into cos theta 1 = mudot + q w1 this is the perturbed equation of motion longitudinal one of the equations and I know by now fx is the perturbed aerodynamic force along x direction theta is the perturbed pitch angle theta 1 is the steady state pitch angle, u dot is a perturbed acceleration u dot q is perturbed pitch rate w1 is the component of velocity in along z direction at steady state okay. Now there is the trick we will do. If we choose stability axis. What is the stability axis? That is this is the airplane okay.

This is the x this is the y and this is the z and relative velocity is something like this that why we put alpha. Whenever we have to find force is along x y z we have to take component right like this. But now if I choose stability axis and define it axis which point towards this is the velocity vector. So that x axis is such that let me do it like this. This is the airplane and this is your relative velocity vector so line by x axis along the velocity vector at steady state so this become - so I write z s x s clear.

What I have done. This was angle alpha correct. Between the velocity vector and x axis. Now what I am doing. I am defining stability axis system such that this x axis is alive along the velocity direction so x axis is here. The moment I put x axis here I call it x s that is x is stability axis z is stability axis what is the advantage see this is the velocity direction the lift and drag will be perpendicular to velocity and parallel to velocity.

So I am very happy very easy for us to handle the aerodynamic forces we need not take all the component every time here and there. That is the advantage of stability axis that point x axis modifies the x axis in a direction which point towards the relative velocity air velocity redirection and so that another advantage, when it is x s ys z s axis choosing that is when I am choosing the stability axis system.

Now the velocity vector is along x s direction right. So there cannot be any component along z direction because they are perpendicular right. So stability axis system one another advantage at computational point of view that is w1 is 0. Please get this point. If I write when this was the stability vector and this was not axis system there will be component of v along w1 or along z which is w1. This will be noted as the component of velocity along z direction. What is the z direction.

But the moment I put axis system such a way that x axis is along the velocity vector direction then this velocity cannot have the component along z direction because they are perpendicular. So automatically at steady state w1 will be 0 that is w at steady state will be 0 that also simplify the equations and if you are operating at stability axis system, then you know w1 will become 0 so this equation gets modified to.

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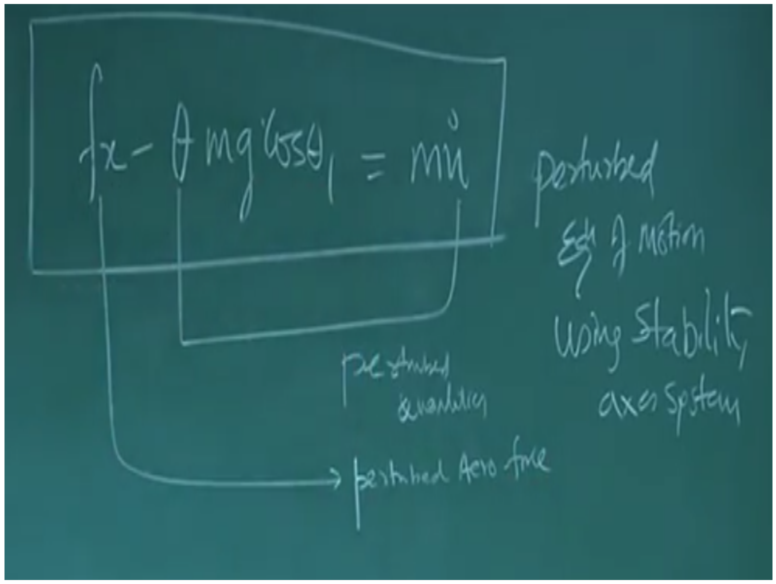

What will happen then it will become fx - theta mg cos theta 1 = mudot so this is the perturbed equation of motion of course it is along x direction using stability axis system correct this part is clear. So what are the understand ding here that x axis is pointing towards the velocity vector so that component of velocity along the z direction w1 is 0. So now perturbed equation of motion along the x direction looks like this fx - theta mg cos theta = mudot.

And please understand theta and u are their perturbed quantities along with fx which is perturbed aerodynamic force along x axis.