Aircraft Stability and Control
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Lecture- 44
Small Perturbation theory continued

We have seen perturbed equation of motion using the stability axis system and last lecture we have seen it is derived. One equation along X axis now today we will be doing along Z axis.

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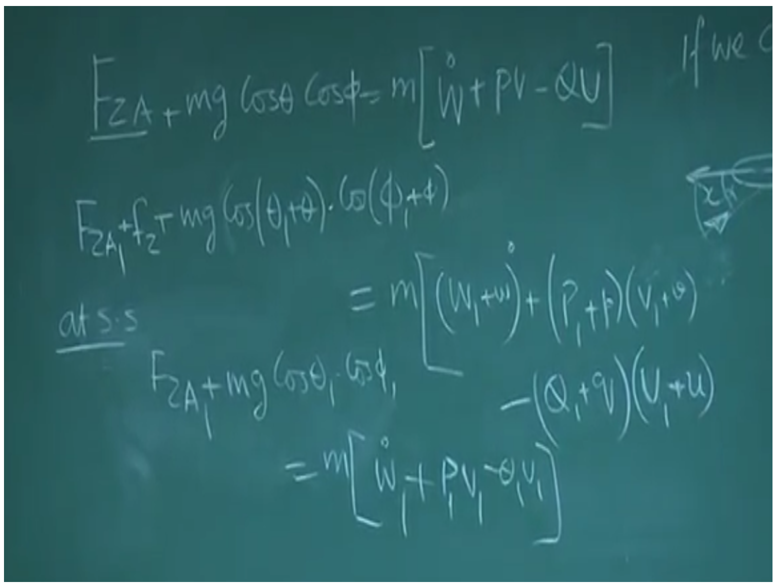

Z equation because we are talking about longitudinal motion that's why we are talking about x z and motion about y. So the equation is Fza or what should of I write mg cos theta cos phi = m w dot + P v –Q w ok. Now you are expect we introduce perturbation. So how do I introduce perturbation? For FzA what I write Fza 1 that is aerodynamic force acting along Z direction at steady state plus because of perturbation there the force + fZ + mg.

Here it is cos of theta1 at steady state plus perturbed theta into cos phi1+ phi= m again here the w1 +and the small w dot+ p1+p very mechanical ok. v1+v- Q1+q into V1+v. Please understand that we are this is not V it is Qu sorry. So this is v the u1+u very boring you could see from my energy level here I am getting tired of writing all this. That is why you are the younger generation people if you take pen and pencil and derive it ok. And you have to do it once and forget it.

Remember the final result understand the physics of it and go for designing an airplane or analyzing an airplane. So this is the perturbed equation introduced certain quantity again we will see whatever we did last time. At steady state FZA 1+ mg cos theta 1.cos phi 1= m w 1dot + p1 v1-Q1 u1 use this and expand this and use this and also use product of two small perturbed quantity can be neglected and if you see in the mechanical way you will get the second relation as I will write it here.

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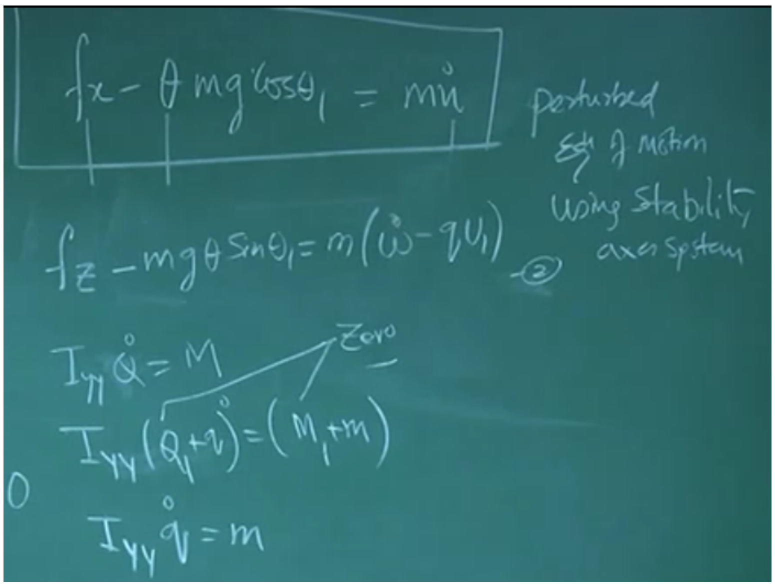

That is Fz- mg theta sin theta1 = m into w dot- q u1. So what we will get is Fz- mg theta sin theta1 = m into w dot- q u1 so this is your second equation and to understand what are the equations means Fz means the perturbed aerodynamic forces along the z direction. The body fit direction. Theta is the perturbed quantity an theta 1 is the steady state value at equilibrium and w dot is perturbed acceleration q is perturbed quantity right. So similarly equation you could see the third equation is the pitching equation is I y y Q dot = m pitching moment.

And this you could easily see that I can write it as I y y q1+q dot= m1+m. What is q1. Q 1 is the pitch rate at steady state but at steady state what is q is zero because airplane is going like this. There is no pitch rate this man will vanish and here m1 is the pitching moment at steady state. Because of trim m1 will be zero but perturbed quantity will not be zero because as you disturb airplane will do like this and try to comeback or diverge.

So this man will not be zero but, at steady state m pitching rate will be zero that is how we define trim. So If I do that if I get I y y q dot = m simply this is zero and this is zero. Very simple. So what did we get you see how beautiful equations we got from all those juggleries and what is important is you have to remember the equation and try to develop field for the equation. But as I always as mentioning once you have to write this equation and expand it class tenth and eleventh level you should be able to get this.

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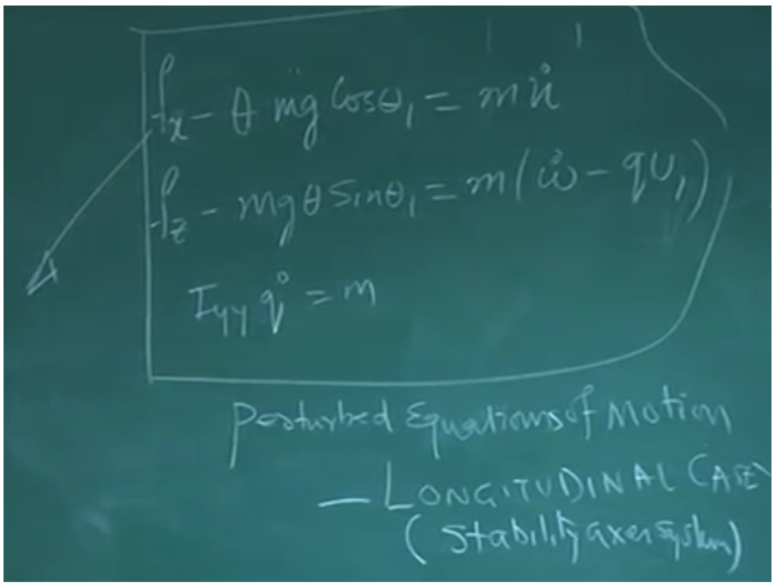

What are the equations you have got Fx- theta mg into cos theta 1= m u dot and then fz- mg theta sin theta 1= m w dot- q u1 and also of course Iyy q dot =m these are what. These are perturbed equations of motion longitudinal case. Also remember while deriving the first equation we have assumed that base axis system x axis pointed towards the velocity. So you must suppose to note down the stability axis system. This had made this equation simple.

Now this is the equation. So what is our aim? Our aim is to see how to analyze dynamic stability of an airplane? You could see here f x aerodynamic force perturbed dynamic force is and theta again is perturbed quantity and all is perturbed quantity. What does it mean? That if there is disturbance through fx and fz or the moment. How perturbed quantity or q or w or theta or u are going to vary if it comes down to zero then you say, it is dynamically stable. Suppose I am going like this say there is a aerodynamic force because of the angle of attack disturbance came.

There is sudden aerodynamic force will come fx Fz and moment. This will create a response in terms of perturbed theta perturbed u and perturbed w if I need solve this equation if I find perturbed quantity vanishes comes back to zero and the disturbances are withdrawn then I know it is dynamically stable otherwise it is dynamically unstable.

So what is called for this? I need to solve this equation I need to module fx, fz and m and how to calculate fx, fz and M. Once I know that it is simply solving the differential equation and simultaneous differential equation right ok. So please understand we have come very close to developing an equation. Equations of motion where we can have our own control. Yes now I know this airplane configuration is going to dynamically stable or not.

Now I have to change the handling quality or if I have to change the natural frequency if I am change the dumping ratio I know how to do it. This will give us so much of information. That is why you are putting so much of time to write all this step and you do not like at least I don't like writing all those doing equations. We have developed perturbed equations of motion longitudinal case and we have assume stability axis.

If you could see that this equation is very much simpler for stability axis assumption. W1 will equal to zero. Now again I repeat why we have developed this equation. Because we want to see whether the aircraft is dynamically stable or not. And we need to solve this equation in terms of u the perturbed u and perturbed theta and perturbed q and you could see that how this u theta q perturbed q and u varies if it vanishes and then you say it is dynamically stable the airplane dynamically stable.

And what is the FX, fz and m is the perturbed force along x direction perturbed force along z direction and perturbed moment about y.

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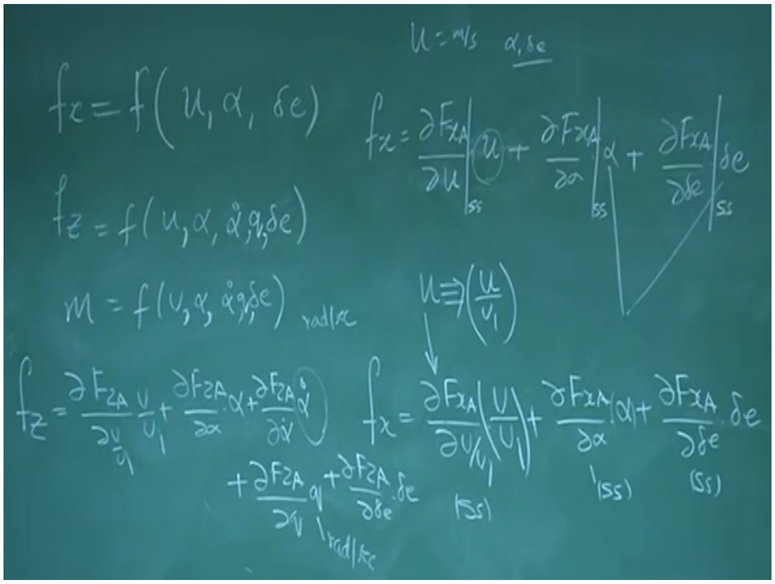

Let us see this perturbed force fx that will depend upon what. That will depend upon what you could see that u alpha and delta e very simple because if u is there then it is dynamic stability half rho u square so it will fx will depend upon u. Fx will depend upon alpha because of alpha there will be cl an cd and component form of x and delta e of course as we deflect the elevator that will also change the little bit of cl little bit of drag.

So it is not a bad assumption to write f x function of u alpha and delta e. Similarly Fz also you can write function of u, alpha, delta e and sometimes you know that we see that it will be also be the function of alpha dot and I will explain you why alpha dot and sometimes it is neglected also and we will discuss in progress. And m = function of u, alpha, alpha dot and delta e. Let me write delta e at the corner so that it may look like simpler.

Why we are writing in the corner because we treat delta e as a control input. And u alpha, alpha dot they are all. What we call motion variables. But if you see here we missed one very important derivative in moment ok. You have seen that is not only alpha at steady state there is alpha will give moment but it is moved like this because of q the damping comes from the tail.

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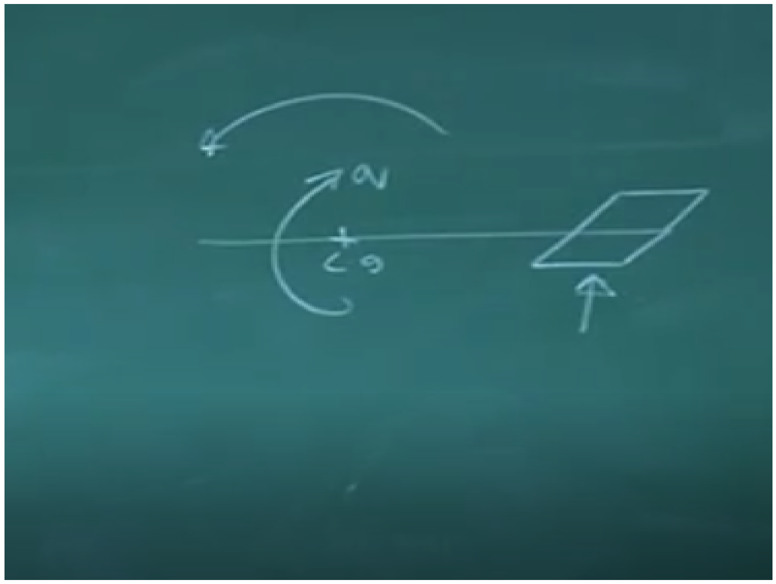

We have seen that if I take this tail you have seen that cg. If I take this tail there is a cg and q from then relative air will come and that will give you the moment and the force. If there is a Q there will be force experienced by the tail in the opposite direction as it goes down so the force as well as the pitching moment and the pitching moment will be functional of q predominantly.

What is the modified thing modified thing is fz is a function of u and pitching moment and perturbed alpha, perturbed alpha dot and perturbed q pitch rate and delta e and pitching moment is also function of perturbed u perturbed alpha perturbed alpha dot perturbed q and delta e ok. So this is fairly a good modeling for small disturbances small perturbation.

Now the question is since we are assumed it to be case of small perturbation. We want to take the advantage of linear dynamics and then and then how should I expand fx. f x as df and we will expand fx taking advantage of linearity fx= df xa by d alpha by alpha let us say first u. First I write d f x by d u into u + dfxa by d alpha into alpha plus d f x a by d delta e into delta e. what is the meaning of this.

The perturbed force can be modeled if I know derivative dfxa by d u at steady state partial derivative I can get fx. If I know dfxa by d alpha at steady state again if I know that dfx by de delta e at steady state this is important right. So this is typically advantage of linear expansion. But another thing to observe here that alpha. What is the dimension of alpha and delta e that dimensionless varies, but what about u. u is meter per second but alpha and delta are dimensionless.

But we want to write in the whole equation in terms of constant in terms of dimensionless in motion variable or control variable. So what we do the trick is you write u as u by u1. So what is u? Whatever u is here it is place it by u by u1. And then you could see I could write fx by changing the as d f xa by d u by uq into u by u1 + dfxa by d alpha into alpha+ dfxa by dfe by delta e you see that everything alpha e and alpha e are dimensionless. Let us comeback to this.

What is the meaning of this term dfxa by du1 this is the partial derivative evaluated where evaluated at steady state. You see what is the meaning of this to once we try to develop expression to calculate d f x a by d u by u 1 ok. Similarly for f z what I can write is fz will be dfza by du by u1 into u by u1 you are now expert + df za by d alpha into alpha+ dfza alpha if I write d alpha dot into alpha dot again you will see what is happening this is again alpha dot q they are dimensional quantities. So how you are writing like that.

I agree your point is correct. We will do some correction. So mechanically if I expand it, it will be d f z a by du by u 1you have seen it would be dfxa by u and you agreed that by u by u1. So this becomes this alpha is fine but the moment I have got alpha dot. D f za into alpha dot into alpha dot I find this man has got radians per second. Q is also radian per second which is also the dimension I have to make both of non-dimensional to be consistent.

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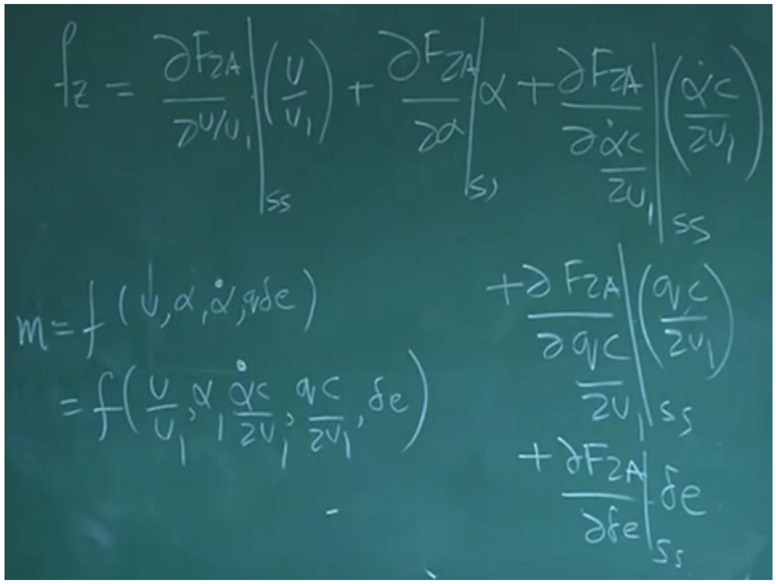

So what we do? We do the simple trick fz equal to dfza d u by u1 into u by u1 + dfza by d alpha into alpha no problem everything is fine but the moment I come to dfza by d alpha dot. I write it like this d alpha dot c by 2 u1 into alpha dot c again you see alpha dot. I have replaced alp-ha dot by c, c is the chord but 2 u 1 that is the non-dimensional quantities. And similarly I write dfza by d for q c I write 2u1 into qc by 2u1 again take care of this dimensional aspects and then we have d fza by delta e into delta e.

Again if you come back to this expressions what is the meaning of dfza by d u1. This is the partial derivative evaluated at steady state ok. For now this is clear to you but we are trying physical interpretation of this derivative. And we find how simple and how wonderful it is. For m has function of u alpha, alpha not delta e. Now you have become smarter. F is u by u1, alpha, alpha dot c by 2 u1, QC by 2 u1, delta e right what you have done. We know final value will be changed alpha dot QC by u1.

So now we mathematically replace u by u by u1 alpha dot c by 2u1 and q by q c by u1 into delta e to make all that simpler. Now if I use this.

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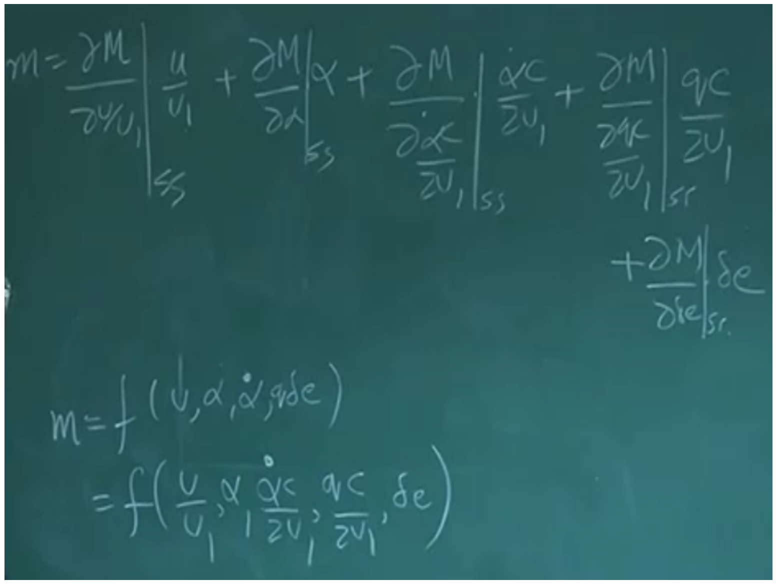

If I write expansion what do I get m equal to dm by d u by u1 into u by u1 + dm by d alpha + dby 2qc by u1m by d alpha c by 2u1 into 2v1+ dm by 2u1 into qc by 2u1+ dm by delta e into delta e what is the meaning of all these partial derivatives and these are evaluated at steady statea t equilibrium for which we are doing perturbation ok. You may suddenly start thinking why alpha dot by 2u1 you are fine sometimes they take u1 for all these conversion I mostly agree this I have done like this.

So this is the expansion what is the importance for understanding all this how do I evaluate this partial derivative. So we will start with the process of calculating this partial derivative so that it gives physical feel for you then writing some expression and let's do that.

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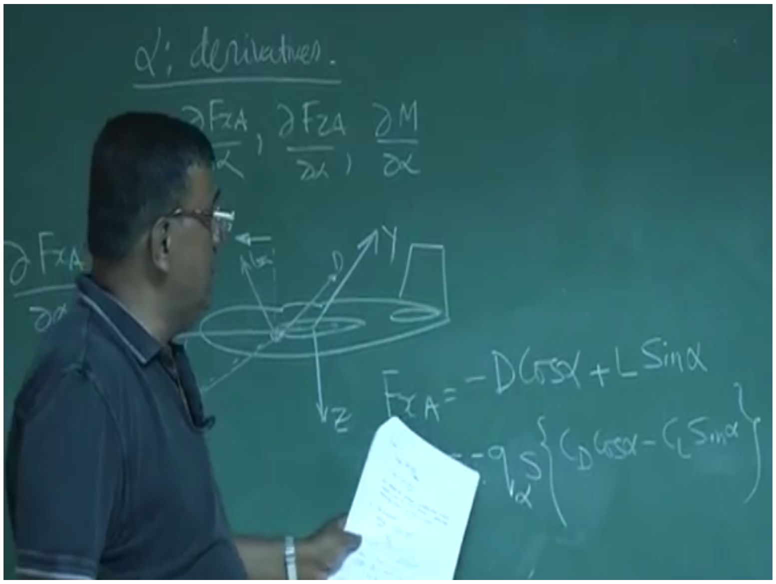

Let me segregate **alpha** derivative. What exactly are we talking about dfxa by d alpha, df za by d alpha, dm by d alpha let us see physically what is this means and how to evaluate that. Let us start with dfxa by d alpha let us draw an aircraft this is the axis system and let there be an alpha ok. So what is fxa it will be –d cos alpha + l sin alpha. You could see that this is the drag will be here so general I could see so minus because of d.

So l sin alpha ok. There is no issue on that this is the d cos alpha and it will come in this direction. Why minus sign because it is in opposite direction of x. Now if I find out Dfxa by d alpha x =-q alpha s c before I do this make sure that we are not getting lost. I write this in aggregated state – q infinity s and I put it here cd cos alpha – cl sin alpha. So this is cd and cd cos alpha plus here so minus is here plus here so minus here.

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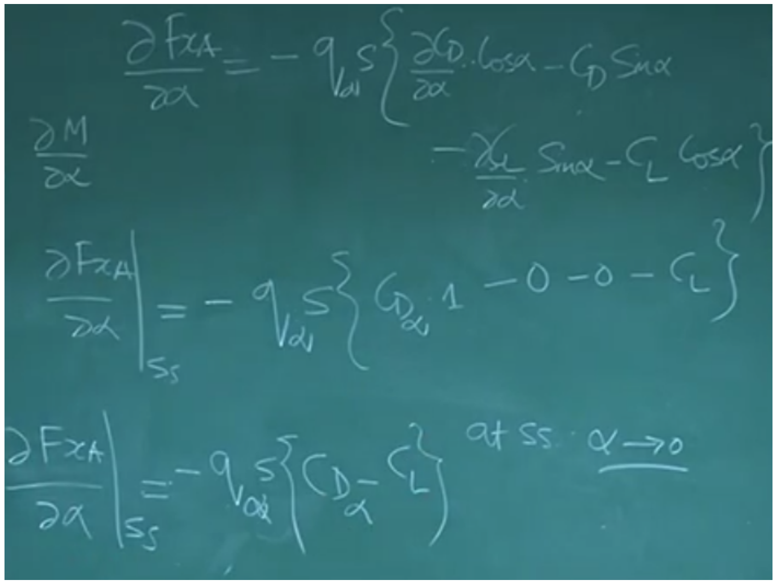

Now I take derivative dfxa by d alpha what are this meters will be = -q infinity d cs by d alpha s did by d alpha into cos alpha See derivative of cos alpha is minus sin alpha then minus is come here so dl by d alpha sin alpha – cl cos alpha . As simple as that dc dc by c alpha d alpha sin alpha – cl cos alpha minus there already. So what is then story is we have to evaluate dfs by d alpha at what condition. At steady state.

So I have to calculate and evaluate this thing. So I have to calculate dfxa by d alpha at steady state that will be what. That will be at –q infinity s alpha s c d alpha at steady state what is the value of perturbed angle of attack it is zero. So this cos alpha is 1. So at steady state alpha value I is zero. So there you go alpha and this goes and it is minus zero so and again minus zero and minus cl.

You see at steady state the perturbed value of this one goes zero and this one goes zero and this one goes zero and this one is cl. So you have dfxa by d alpha evaluated at steady state q infinity s cl1- cd alpha. At steady state alpha is zero and cl will be corresponded to cl at steady state. So I get this expression dfx by d alpha as this. So that is cl alpha. So steady state you know half rho v2 this is s you understand cl1 will be what. Cl at steady state is nothing but Cl1 is2ww by s by rho v2 cd alpha is what it is cd +kcl. Dcd by c alpha these are derivative.

So these are all note for us. So simplified expression we get for d alpha. Why we are all evaluating this? Because we have to port value expression here dfx by d alpha I have to replace it by q infinity s cl1 – cd alpha evaluate every all derivatives and try to substitute them by such expressions.