Aircraft Stability and Control
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Lecture – 45
Perturbed Equation of Motion: Longitudinal Case

Good morning friends we were developing perturbed equation of motion longitudinal case that is you are restricting motion of the airplane in the vertical plane and we are talking about x axis and about z axis or along z axis plunging motion and also motion about y axis pitching motion right. And that is what exactly we are talking about longitudinal motion and we are try to develop perturbed equation of motion with what in mind that we want to use this equation to see.

Whether the aircraft is dynamically stable or not, or we will use this equations to characterize the airplane dynamic stability through may be natural frequency dumping ratio time to one or time to half. When we say time to half we were talking about the stable system. Time to double we talking about unstable system. So all those questions will be answered and we are trying to develop longitudinal perturbed equation of motion.

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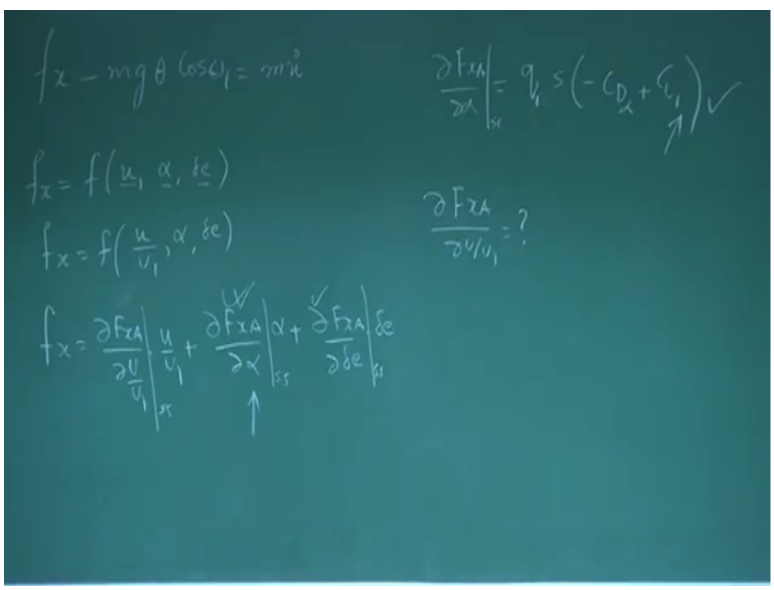

And if I remember we wrote equation fx equal to or fx minus mg theta cos theta 1 equal to mudot and this theta and u they are the perturbed quantity, similarly fx is the perturbed quantity. What is fx ? Because of perturbation, What is perturbed aero dynamic force?

fx is experienced by the airplane right. And in developing the model for fx and the assumption of linear because of small perturbation we realize fx will be the function of u alpha and delta e what are nominal aircraft right. We not talking about high speed or highly agile aircraft, when I am talking about high rate of turn extra. So it is fair good exemption.

With high maneuvering airplane there will be some of the terms may will come like delta e dot alpha dot so that can be handled as when as we understand the basics ok. And then we realize one thing that alpha and delta e they are non-dimensional but u is dimensional u is meter per second unit so we will write in the non-dimensional form we now write it like this u by u1 alpha and delta e.

And then we are using that at one test the aero dynamic is being linear so express fx as d fx a by du by u 1 into u by u1 plus d fx a by d alpha into alpha plus d fx a by d delta e into d delta e correct. It is very clear and what was this derivative? This is a partial derivatives number one and this should be evaluated as steady state because we are giving disturbance at steady state at the equilibrium and you want to see how this perturbed quantities are going to behave as the function of time. So that you can comment that the airplane is dynamically stable or not right.

Next challenge was how to estimate this derivatives? We have already last class we have already derived this expression d fx a by d alpha and we have shown that this equal to q infinite s minus cd alpha plus cl1 what is this 1? 1 is corresponding to steady state because these derivatives are computed at steady state and if that is true then to consistent instead of writing q infinite will write q1 so this is the dynamic pressure at steady state ok.

For example if it is flying at the altitude with density is rho and v is the speed is v half rho v square is the steady state value at that the point where the aircraft is in equilibrium right. This will have developed. What is next we developed is? So this is done now we are attacking this, d fx a by du by u1 right. Let us say d fx a by du by u1 before developing this please try to understand will there be any change because of change in u.

U1 it is only non-dimensional like that so the question we should ask ourselves. Will there be change in the force along the x direction because of change in the speed? As obvious we have the speed is change because of perturbation, dynamic pressure will change, so indeed there will be change. So we can do how to model this.

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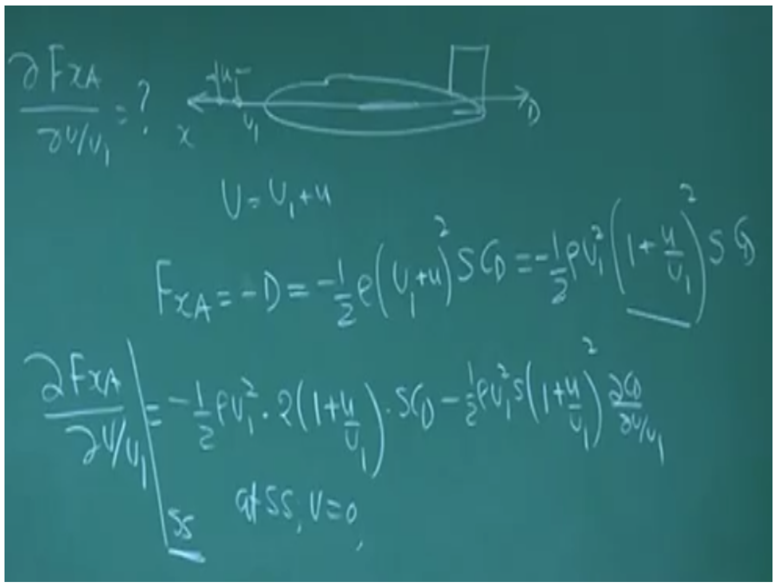

So this is airplane when we talking about the partial derivative hence we have liberty of others to assuming zero alpha or any other motion will be zero so we are saying like this, this is x this is as u 1 and this portion is perturbation small u total velocity of u is u 1 plus u no problem, again we are taking the advantage of everything will be linear ok. That is why we are restricting ourselves small perturbation equation.

Once I do that, then what is fx a? fx a would be half you may what will be minus drag this is the drag and drag you know as minus half rho v square v will be u 1 plus u whole square s cd as simple as that. Okay. What we are in? To find d fx by du b y u1 so we will do little mathematical juglary here. We will write this as minus half rho u 1 square into 1 plus u by u1 whole square s cd. no problem. Ok.

Now we want to find out d fx a you should be able to do faster than me this will be minus half rho u 1 square into first I will take the derivative of this. This will be 2 1 plus u by u1into s cd in the next term I will take minus half rho u 1 square s into 1 plus u by u 1 square into d cd by du by u1 right, this is clear. This is minus half rho u 1 square s is there this is same so this d by d will give u1. Now what is d fx by du by u1 what is the another condition?

I need to evaluate this at steady state that is very critical ok if I want to evaluate at steady state I know at steady state U is zero. So what will happen if I substitute that I will get an expression.

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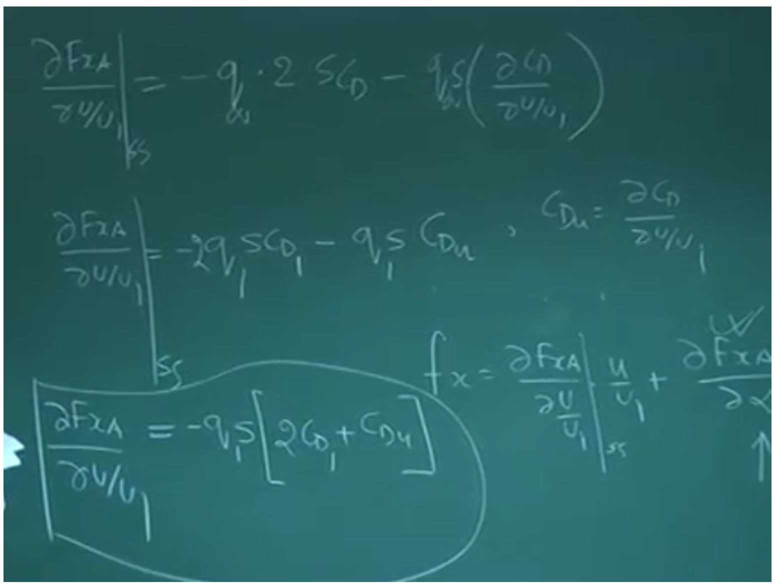

I will get D fx A by du by U1 and is equal to I put U is equal to zero because this is at steady state so this difference is minus q so at steady state ok. So I write Q infinite then there will be then two, U is zero is gone so S cd then minus again Q infinite and then this is 1 this is half rho V square is U is zero so this goes beautifully I write s and then I have D cd by du by U1 ok can I check this we want to see Q infinite. S this is zero because at steady state no perturbed value so this becomes 1 so this debate is very good.

Now I can write this as D fx A by du by U1 as minus at steady state we are evaluating so I agreed we will not infinite at steady state we will put the substitute 1 to understand this is the quantity evaluated at steady state. So 2 is a 2 I write here S cd, cd1 and again S D evaluated at steady state minus q1 S then cdu. where cdu is nothing but D cd by du by u1 will understand what is cdu when we complete this. This is basically minus q1 s right. And so this will be 2 cd1 plus cdu that is what is d fx a by du by u1.

Let me check am I getting the same. D 1 very good. Let me check am I getting the same. D 1 very good. Now let us see as I told you have to do all this things once and try to understand and get physical interpretation of this derivative. So now we talk little bit about this derivative. Ok let us do that.

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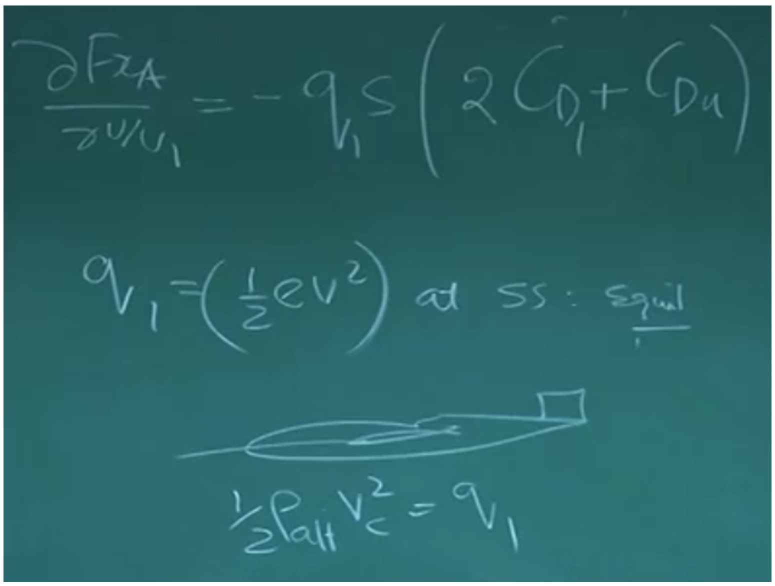

So let me write this d fx a by du by u1 is equal to minus q1 s into 2 cd1 plus cdu. You could see that we have developed this expression and we are try to give an interpretation write inside q1. What is q1? q1 is half rho v square with dynamic pressure at steady state that is at the equilibrium right.

For our case let say it is cruising and the cruise we have taken as equilibrium so this will be half whatever the altitude will be there into v cruise square that is our q1 ok. Now no issues. cd1 what is cd1? cd1 means cd at steady state. How do you know? What is cd? cd we know through drag puller that is cd not plus k c l square. You are all familiar about this, so what will be cd1? cd1 will be cd not plus k cl1 square. What is cl1 square?

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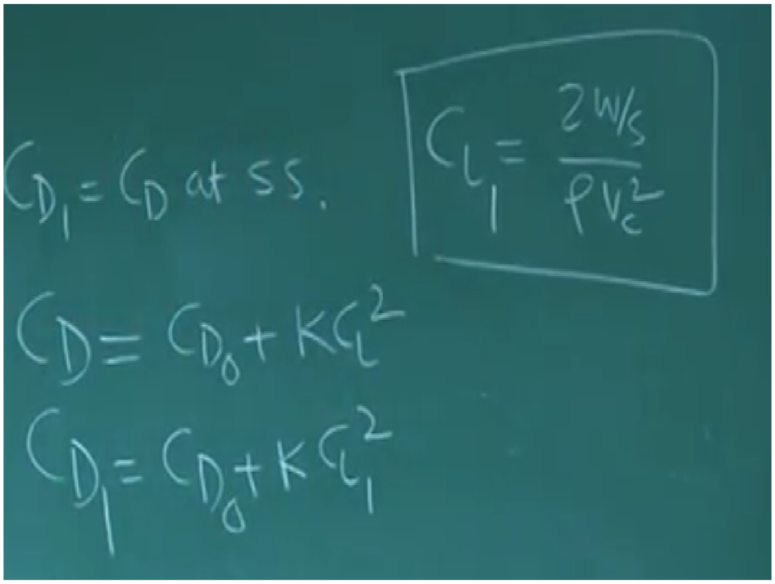

cl1 is c l at steady state for the case at cruise so what is c l at cruise? C l at cruise is 2 w by s by rho v cruise square. So c l1 will be this. We allow lift equal to weight. So all the things we are knowing. Now the question will come cdu. What is cdu? Let us understand what is cdu ? This is very important please understand this cdu concept ok.

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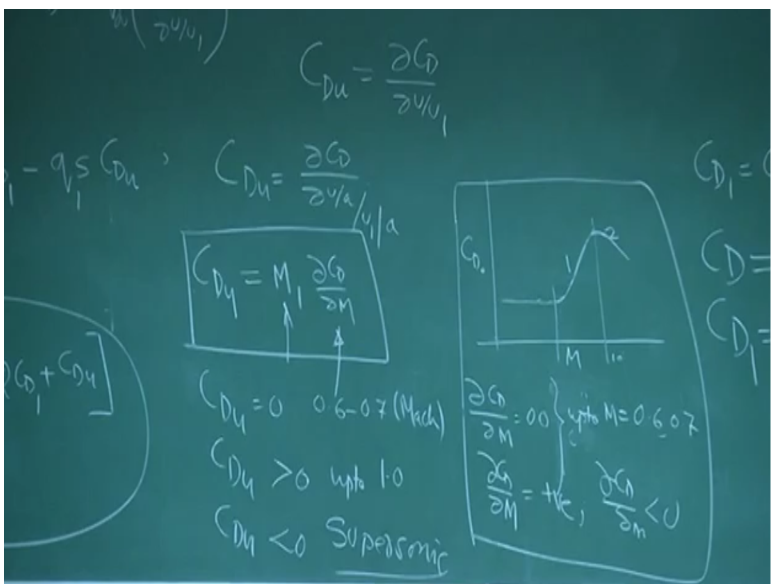

This is the first time we are seeing it and they are very important for high speed airplane ok. cdu you know by definition. It is d cd by du by u1 right. Physically what does it mean? It means the changes in u will there be change in cd is there or not. See please let us understand so cd versus mac number plot right.

Generating something like this right. Up to 0.6 or 0.7. This is fairly constant cd. This is cd not. We are talking about drag parasite drag, because of shape, because of mac number, because of flight regime. We are talking about cd not. And you know that from some point there is the formation of shock waves and here around 1 we will get the pick. This is the cd versus mac number

And if I ask you what is d cd by d m which I mean d cd not by d m is what you could see upto 0.6 or 0.7 which is almost zero up to 0.6. Zero this is up to mac number equal to 0.6 or 0.7 ok. It varies different different configuration and d cd by d m is positive in the first part here. This slope is positive and you could see d cd by d m is less than zero for second part at high supersonic speed.

You could see that when I am accelerating towards supersonic right. That time the d cd by d m is actually positive. At here d cd by d m at supersonic speed is negative. So once you understand this let us use this understanding, to understand what is d cd by du by u1 is. This I can easily write as cdu as D cd by du by A U1 by A. I am dividing u 1 by u 1 by a which cancels each other but it gives very important information.

This tells us this U 1 by A will be M 1 and this is D cd by D M so I will look for cdu through this relationship. Now you see cdu, M1 is the mac number at the steady state now important thing is D cd by DM. So now since you understand D cd by D M. How it varied for subsonic transonic to supersonic? So you can easily say cdu is zero let say between mac numbers 0.6 to 0.7 is the mac number.

And d cdu or cdu is positive at this part may be upto 1.0 mac and cdu is negative for supersonic. Extremely important derivative and you see that once you understood this what happens during the moment. ok. So what is our success till? Now we have derived d fx a by du by u1 and we have understood what have in the right hand side.

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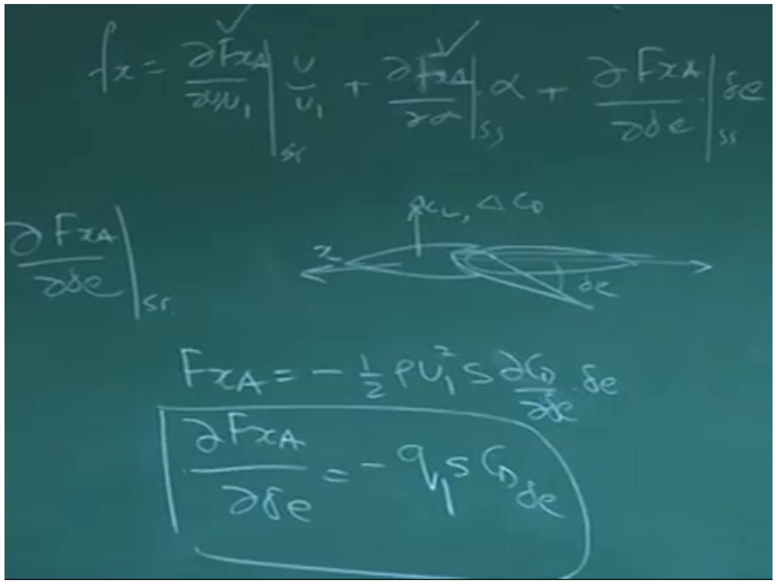

So let us again come back that what we have fx equal to f d fx a by du by u1 u by u1 plus d f d fx a by d alpha into alpha plus d fx a by d delta e into delta e we have already evaluated this evaluated this we know this have to be evaluated at steady state we have known left with evaluated at steady state.

So let us see d fx a by d delta e at steady state is very straight forward thing but physically you should understand. If this is the elevator and this is the tail. When I am deflecting by delta e, and if changing c l will come corresponding delta cd will also come. So that will contribute to fax this is for small perturbation I can write this as fx a is equal to of course drag will be in this direction x in this direction.

So I write minus half rho u 1 square s cd delta e or let me write for your classification d cd by d delta e into delta e. Remember this is fx a because of elevator deflection helding other constant so that why we are talking about the partial derivative and here you could see clearly that d cd by d delta e will not be very strong number but then for completion we will complete this d fx a by d delta e will be equal to minus q1 s cd delta e that's all.

Please understand for small small aircrafts serial time may not be very large significant but then any amount can drag can correctly model because this are directly link with the fuel convention.

Of course this is out of the steady state so let us see what we are doing and why we are doing I again take you back to this fx minus m g theta cos theta 1 is equal to mudot and fx.

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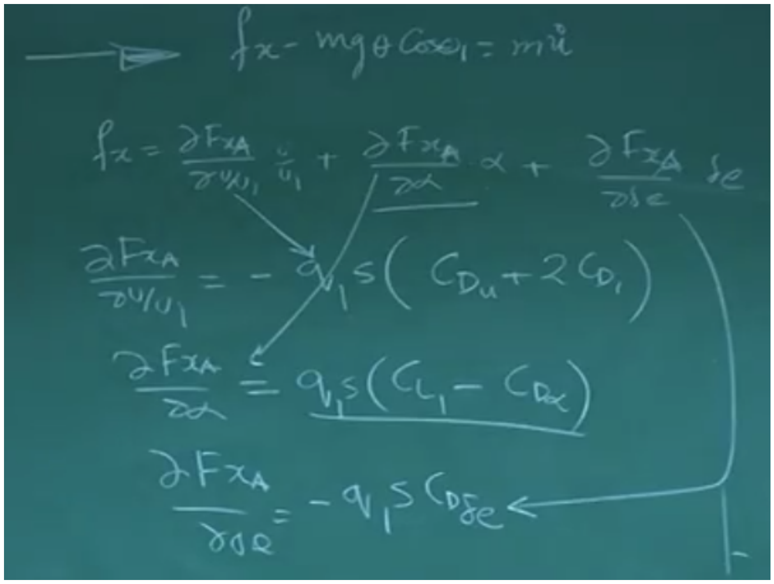

We have written as d fx a by du b y u1 into u by u1 plus d fx a by d alpha into alpha plus d fx a by d delta e into delta e of course this term is aerodynamic force that I have been telling you repeatedly similar expression you can do for thrust expression also. At once you know how to do for aerodynamic and you know how to encounter thrust effect also.

So now what is the expression for d fx a. d fx a by d by u by u1 is minus q1 s into cdu plus 2 cd1. Then D fx a by d alpha we have seen in the last lecture that was equal to q1 s into cl1 plus cd or cl1 minus cd alpha. You understand what is cd1? You understand what cd alpha is. Then we have seen d fx a by d delta e is equal to minus q1 s cd delta e right.

So what is fx then? Very simple fx will be this one for this, this one into u by u1 this term will be this, this term will be into alpha and third is will be this term into delta e. clear I repeat in the fx d fx a by du 1 which is given by this expression I want to substitute multiply by u by u1 for this I have to put this expression into alpha, third want to put this expression. And I will put all this is here in this equation ok. So what we will get first purely mechanical so I can write this as.

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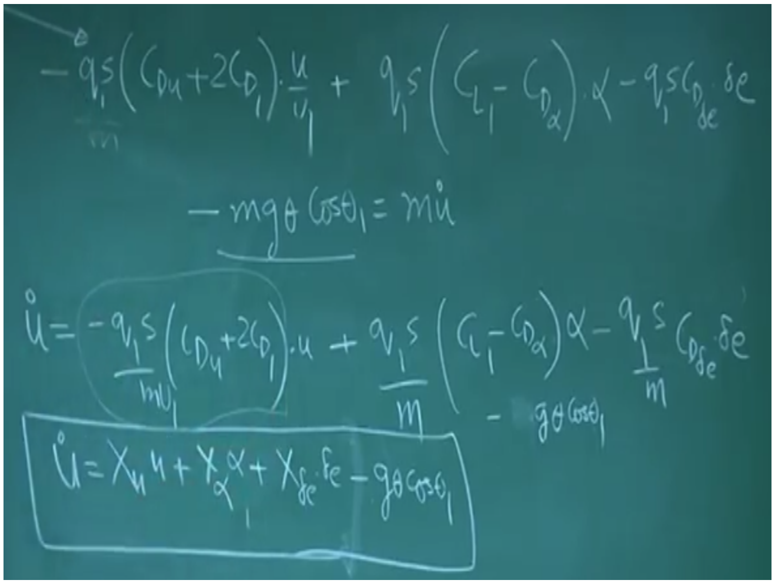

Let me write it d fx a by d alpha is q1 s minus cdu plus 2 cd1 into alpha. No sorry. I have written wrong here. That is the problem. Let me erase this we have to be careful. What I have to write here. I have to write here the expression for d fx a by d alpha so what is that q1 s cl1 minus cd alpha into alpha let me check.

q1 s right cl1 very good then of course for third d fx a by d delta which is q1 s cd delta e so I will write q1 s cd delta e into delta e. So this is completely the first term fx. whole fx I will take it here clear. Now what is there in the next it is minus m g theta cos theta 1 equal to mudot no issues simply mechanically I am doing as if with few expressions minus m g theta cos theta mudot.

There I do some sort of jugulars then I can write which I leave it to you understand immediately what is this u dot write is because I write in acceleration form so m I divide everything by m. so what I will get I will get minus q1 s by m minus cdu plus 2 cd1 right and I put it here q1 and put it small u. you understand u1 I have to give under this m u1. M has come because we are dividing by m so this is u dot clear.

This m I am taking denominator here so I take u dot in left hand side and this u by u1 this u1 have taken it here so what we are doing is? We are writing u dot equal to so this m has come here so that m is here and u by u1, that u 1 I have taken it here so left with u second term is q1 s again divided by m right. And then, this is cl1 minus cd alpha into alpha, and of course this is cd delta e into delta e, no issues. So now I write u dot in neat form u dot is equal to xu into u plus x alpha into alpha plus x delta e into delta e no problem.

We are missing something know. This, this fine u dot is equal to this of course have here minus m g theta so it will be g theta cos theta 1 please understand I have missed this term. What we are doing u dot we writing divided by m so we are this part is taken here divided by m this part is taken here but we forgot to take care of this term. This will be m will be cancelled so g theta cos theta 1. So I will write this here minus g theta cos theta 1.

Now let me ask you a question what is xu? Let us see with me or not as I told you please sit with pen and pencil when listening to this lecture. So what is xu ?

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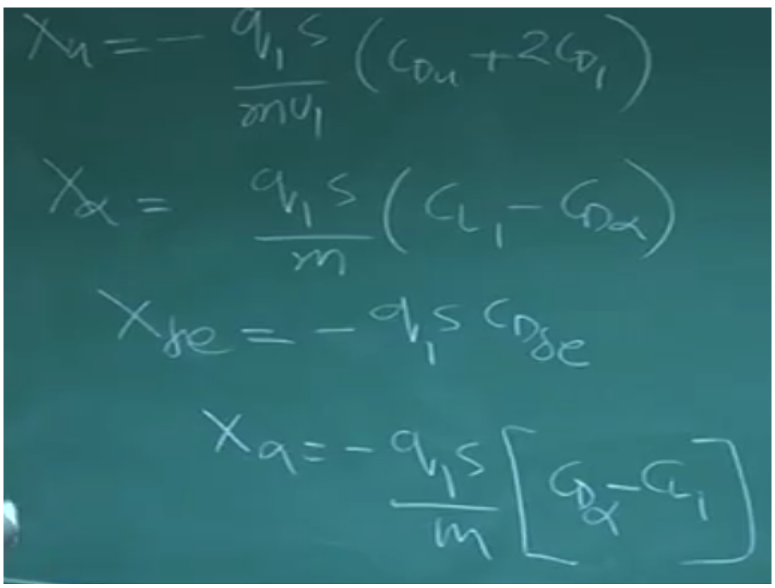

Very simple see from here xu into u that means this whole team is xu that is xu is minus q1 s by m u 1 into cdu plus 2 cd1 right. And you know u 1 is the steady state velocity at cruise because all the equilibrium in steady state is at cruise. So nearly you are smart enough to tell me what is x alpha? X alpha you can check yourself. X alpha will be q1 s by m c l minus cd alpha as simple as that okay. So X delta will be what? X delta will be minus q1 s cd delta e ok.

Many books you see there to ensure every time this minus sign will be there so they write as minus q1 s by m, and they write cd alpha minus cl1. So everyone will find minus q1 s is coming.

Okay. So this equation now we have been able to derive again we will come back to this why we have derived.

This equation because you know that, this u dot is a perturbed u how the u dot is change that will be decided by the change in the aerodynamic because of motion and those are decided by x alpha x delta e and u alpha perturbed quantity. Where I want to see whether it really decays or increases in the dynamic stability, but as you understand here this is u perturbed quantity alpha is there so this one equation is not sufficient.

So we have to include equation because of plunging as well as pitching which completes the longitudinal perturbed equation of motion.