Aircraft Stability and Control
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Lecture-47
Perturbed Force: fz continued

Good morning friends, it must be tried of feeling be expression like the an it is again ask yourself what are you doing what are you doing we are try to developed mathematical model which will be using characteristic to at aeroplane it come of instable are not as fresh will as static stabilit.

As constant we have seen what conditions are but now you are do more generically we try to Develop equation of the motion and try to see that can I use this truth coming done stability the airplane I know very will dynamic stability in to the stability for the more cases and for an airplane.

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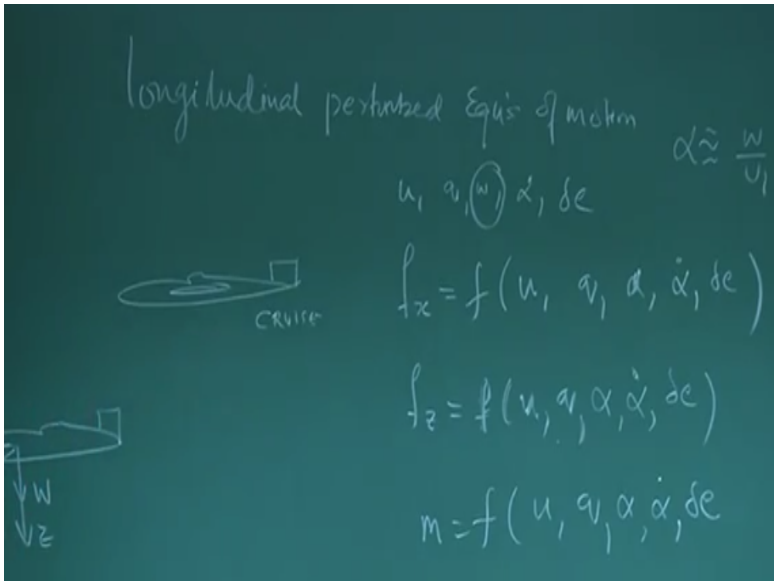

It is very true and what was a approach the approach was we develop longitudinal to terminal perturbed equations of motion at what exactly will mean by that set ok. This is my airplane this is cruise by equilibrium right and about cruise I will giving the dispense small dispense and you try to see what happens we please understand the airplane is going like this and I cruise dispense

then but ok longitudinal case that has been this motion could have a this motion and the have this motion and there be a perturbation in pitching motion.

And it is derived at this motion and derived at a small q there is perturbation in the speed so In the derived at the small u the perturbation of the vertical velocity we denoted that by small w ok so u, q small w these are the particular of quantities and also we know that at this airplane is pitching like this and this like so pitching this there will be changing alpha we can't rate of change of alpha become is important z and also note down that alpha dot that playing important roll.

Important roll where if I try to find out f x and f z and the pitching movement is it because of the small disturbance this u there will there the perturbed u fracture of u fracture of q is will be there and perturbed then w component of the aircraft are be there I have will be there and also alpha dot will be there because you understand the body will just moving like this is introducing the alpha dot right.

The alpha will be jointing with in between the velocity vector in vertical component in the right so the alpha that also begin is doing like this as it doing like this is q alpha dot that what weight it is moving the alpha it is so it is how is that use it include alpha dot and then when said f(u) ok and then q then of Couse there is delta e also and alpha and alpha dot and delta e move down here will be w but here it is alpha because you know alpha is nothing.

But w/u1 so is there is w there will be alpha that is take this body like this if there is alpha perturbed alpha and there will be compound of this velocity it is w along z direction right during perturbation. Because we have which use stability axis which has steady states .that direction is fixed and the condition of steady states because whatever velocity vector of there.

The x axis there on the perturbed then there will be perturbation alpha what about the perturb alpha this perturb w that is compound along the z direction ok similarly here if I write f(u) , q , alpha , alpha dot , delta e and u , q , alpha , alpha dot , delta e and one more thing you realize please you note that all your lecture put not alpha dot will explain you will so approximation but

coming back the you understand that will want to ensure these arguments are non-dimensional that write we said inset of u.

We will write on u/u1 is set of q , we will write qu/ 2 u1 the It set of alpha dot will write the alpha dot c/ z u look at their non-dimensional ok all on the non-dimensional is in similarly here again u/ u1 qu/2 u1 alpha dot was alpha dot c/ 2 u1 similarly to get u / u1 this is q c /2 u1 alpha dot c /2 v one simply q alpha dot they have dimensional radians per second right ok fine once we agree to this and we made this apposition that was a normal airplane the small dispense I may eliminate alpha dot z u one effect on effects for most of their airplane**.**

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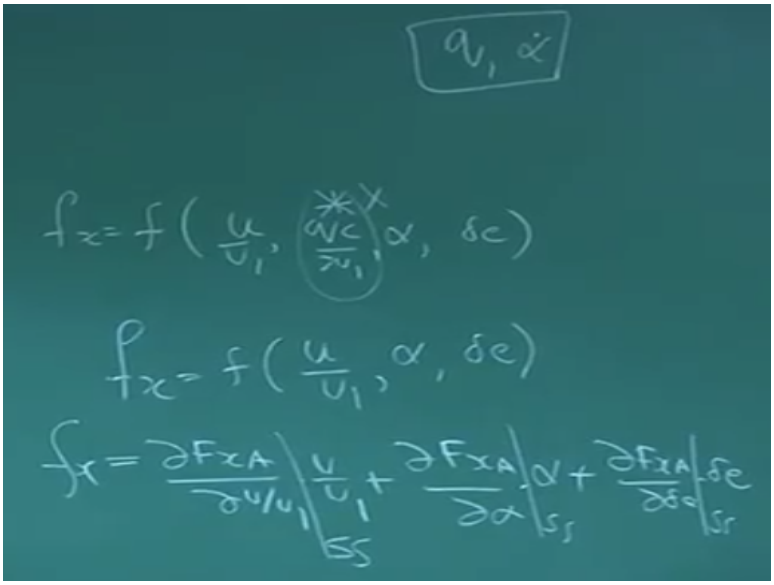

So we wrote f x is equal to function of u/ u1 then q c/ 2 u1 alpha and delta e right again file there are normal condition airplane will see by experience and this effective also not that much but I know understand that is effect is because of q and what is he effect of alpha dot we will discussing this thing today also right but this point of time the assuming that these effect also cut terminate f x wrote as f function of u /u1 alpha and delta e ok and then we wrote the expand it delta d f x a/ d u/ u1 and u/ u1 plus d f x a/d alpha .

Alpha plus d f x a / d delta e by delta d and right this is the f x. What is f x this the perturbed aerodynamically force and then we know how to valuated this it. You are very clear was at the evaluate and steady state and we have derive the expression and then no explicitly what is the

from of f x right that will be done ok I note down here this q and alpha dot was a we need a attention we will need the distraction we will go for that similar think we need for f x ok here although the first lecture we need the lecture liked as I have eliminated the alpha dot like to u1 let us skip it so all so get someone fell.

what is happening remember alpha dot will has no effect on f z it will serve another then or right although for all airplane most the airplane we made still in this but just see you that handling with handing alpha dot derivate I will keeping with assume movement with consent u/ u1 q very important because you know that c m q it will very very important diversity from the dynamical stability point of you alpha and there is alpha dot c that is will be very important delta e.

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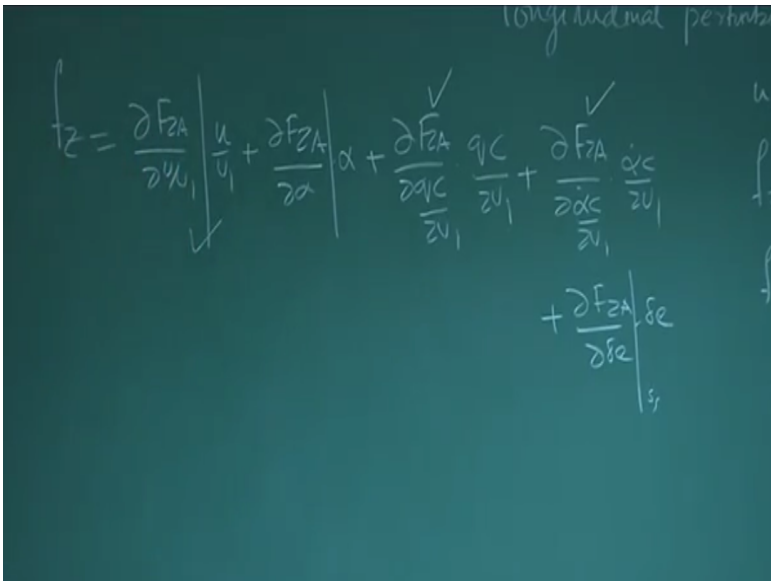

So it will be developing expression for although derivative ability now still yesterday when we find use the f z what is the dame is not mistaken every time fx is equal to d f z a/d u/u1 and u /u1 plus d f z a/ plus d alpha and alpha plus d f z a / d q c / 2 u1 and q c /2 u1 plus now we adding d f z a/ d alpha dot c as / a u on alpha c / 2 u1 plus d f z a / d delta e int delta e so we had assuming that f x the function of u and qc and alpha and alpha dot and delta e right.

I would to talk the alpha dot in the last lecture like to last exit now you are relaxing is expanding now we are become comfortably handle this we find to know fx it know. So what is the expression of d f z / d u by u1 which have been done what is the d f z The with by d e u by alpha

we have done we have just check for this one this one this also already done we have the last lecture the today first.

Will do the d f z a by the d qu by u1 and d f z a by d alpha dot 2 u1 ok let us focus on that it I have all on already mention.

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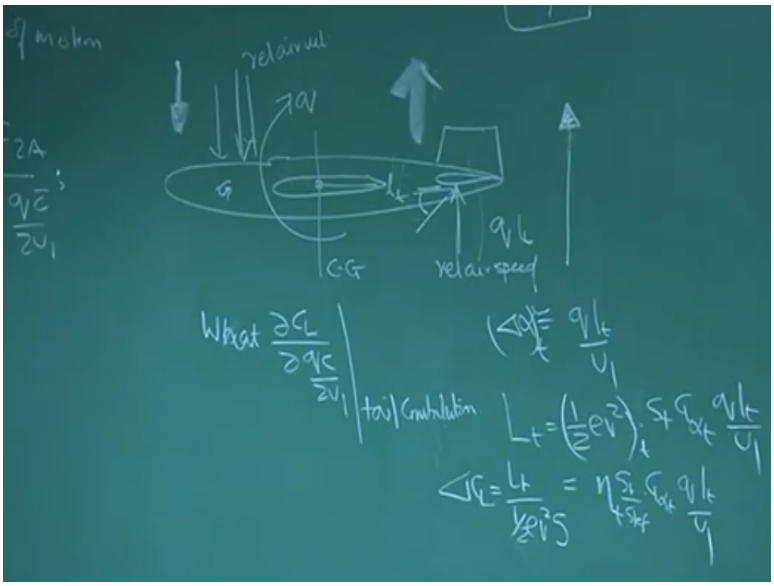

On all the d f z a by d q c by q u1 but let see the revisit again let us check this is airplane this is airplane ok and this is tail and here the wing and let see this the cg of the airplane right what we are try to find out. We try to find out what is the parceled derivative d f z a and d q and e of course q c by 2 u1 and non-dimensional the base on.

The sitting is if I give the positive q why that there will be change In the force in the z direction ok for a small perturbed frozen now I see if I give a perturbation q is a small q what to happen this will be go down so there q into l t in the relative speed right similarly as this is the cg so this position is going down so there will be also and some set of a and relative the velocity this position also this position also and of cg and could see one thing as it goes down there is force cl and additional cl is come the vertical direction

Which will purpose in to q and as it is going to up there will be sound force is coming down like this right but for airplane this position is predominant the force is consider this and this ok what

will do now because you understand and the another thing this l t is find larger the ok compare to any center of pressure if I take for the what every force pressure distribution is because of q this length is smaller compare to his and this is not that lifting as compare to the tail so tail will be more the tail alpha so natural that force will be more and if I want to just check.

What is d cl by q u1 just take the is tail contribution because we are assuming that the tail contribution is more significant same cent for the contribution ok how to I find out so very simple because of q what is the delta alpha at tail additional it is q l t by u1 approximately righty what is force and the tail and the l tail will be ½ roe v square at tail into S tail and cl alpha tail into alpha the alpha is q l t by u1 ok.

So then we are expect now what is delta cl. the delta cl is will be l t by ½ roe v square into f reference so this will become Neeta tail l is still now s reference into cl alpha tail into q l t by u1 and what will be signed we could see that is going down like this it is like to alpha in to is positive so delta c will be the lift direction so as for the conversion that is the leave direction is opposite to the z direction right for the small alpha so whatever you get here right it remember cl is consider in positive.

Why will be transfer in it in to z direction there is become negative we will watch out for that ok

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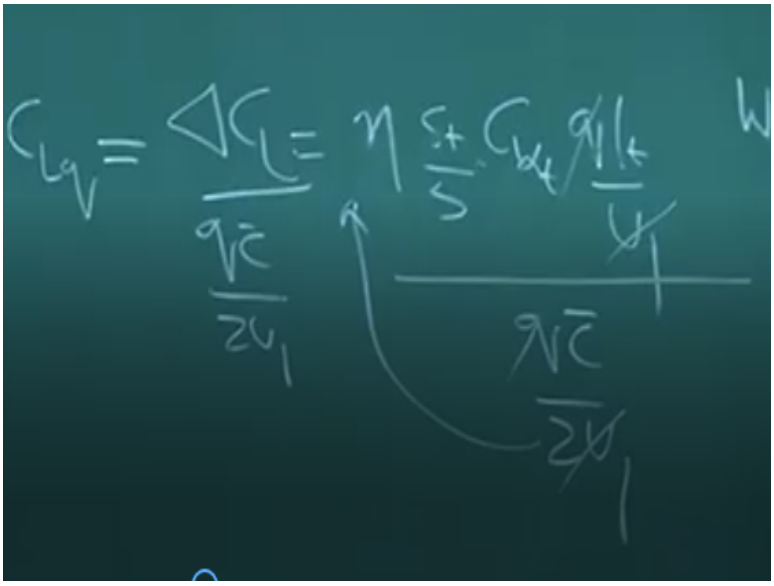

Now for this is cl so we have a delta cl is equal to Neeta s t by s reference into cl alpha tail and q l t by u1 and you know d cl by d q c by 2 u1. We find out see it a linear everything is linear so I simply can write cl q this divide by the q c by 2 u1 and that will be q c by 2 u1 for U1, U1 is cancel qc by qc cancel so I get d cl by d q and cu by 2 u1 is equal to two Neeta tail s tail by s reference with wing cl alpha tail then l t by c ok .

Could it check here q and q will be canceled so Neeta exceeded s cl alpha tail l t by the c bar and two both upwards this is the expression for cl q ok.

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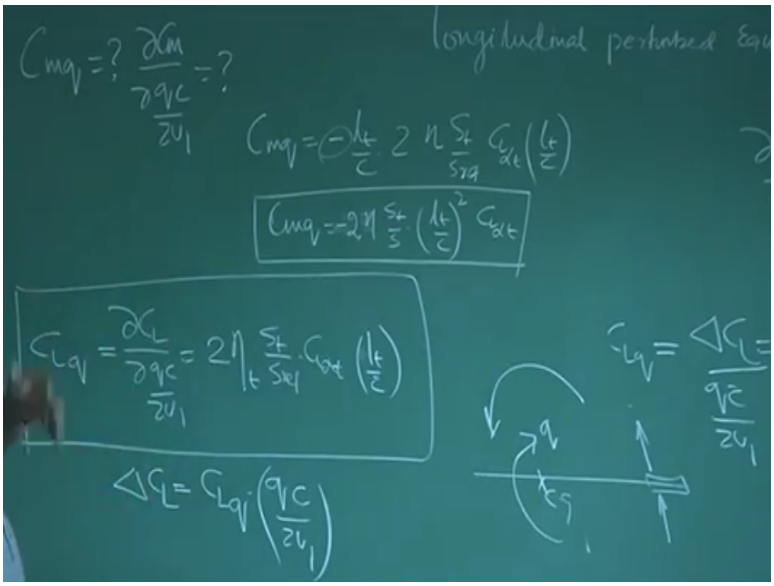

This is the expression for the cl q if are and they find how much delta c will be coming because of q what should be answer it is the cl q and q cl by q u1 that is we understand it is not cl q into q because the cl q will defend the as d cl by d q c by c two that is the important ok this is the understood cl q now we not leave it here we also at for completion what will do at we it at one we did we also try to see what is see in what is c m q this will come ok.

This is the pitching movement changing the pitching movement because of q and like to change the c because of q this is also changing of the cm by d q c by 2 u1 ok. So let us derive the expression we all come to close to force movement is simple so what will be c m q .C m q will be nothing but minus l t into c into cl q that is two Neeta at still by s reference and cl alpha tail into l t by c right this is cl q simple we find to do example this is the delta l into l t.

In the movement we tail and sure and that is should be minus sin. Why put the minus sin because the positive tail will be negative pitching movement and sin conversion are from as right so you when get expressions of the d m q as minus two , two Neeta s t by s it is l t by c whole square right to the cl alpha t and of course the minus sin write correctly minus two theta s t by s and l t by c also so we know the expression of d m q you know the expression of cl q .

The cl q right is negative that is very well now it is could be c g and this is the tail if I am giving the q like this so it is going down the negative value will be there the delta cl will be there the delta cl will be give movement about cg notes down that is the this the minus and this here and we have known what is expression cl q and cl q will be try to know the expressio0n for the cl alpha dot and cm alpha dot and I have done come back to the perturbed equation.

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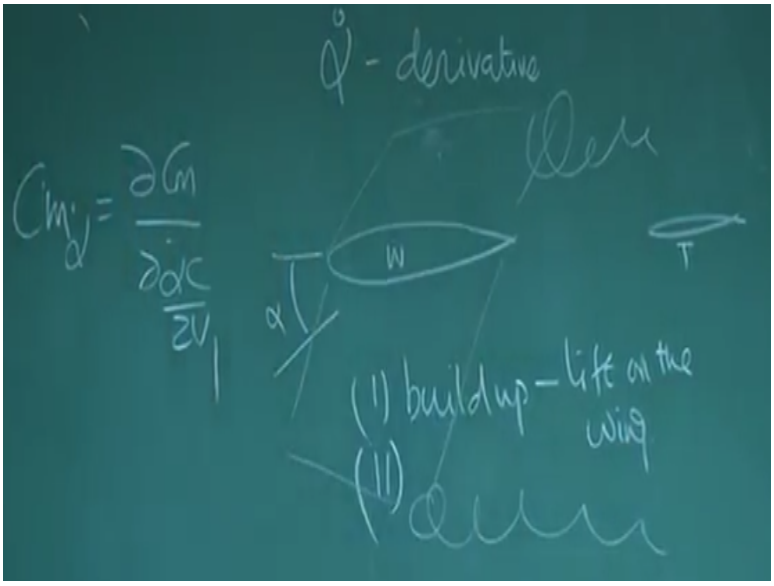

We have understood c m q and cl q it is done. now we are focus of c m alpha dot cl alpha dot and you know the cl alpha by definition d cl by d alpha dot by 2 u1 and c m alpha dot is equal to d cm by d alpha dot c by 2 u1 and you know why this c u 2 u1 because the alpha dot the dimensional to converter to the non-convertible the dimensional quality and these are the text book are the covered and the alpha dot derivative right.

Let understand one thing very fundamental if this the wing ok and this is the tail. Wing and tail as the airplane seen the alpha in this the perturbed alpha please understand it doesn't lift develop list it take time to build the lift distribution understood this why you understood this if is alpha lift cl alpha into alpha but it is text time.

Text time to buildup lift so the time component and so the second time so I right buildup of lift on the wing and text book they also take about in diesel lift right the second is very important you remember the because of the final of the right they will be traveling like this they introduce downers at the tail they introduce the tail so what is this for text this is wing from wing introduces.

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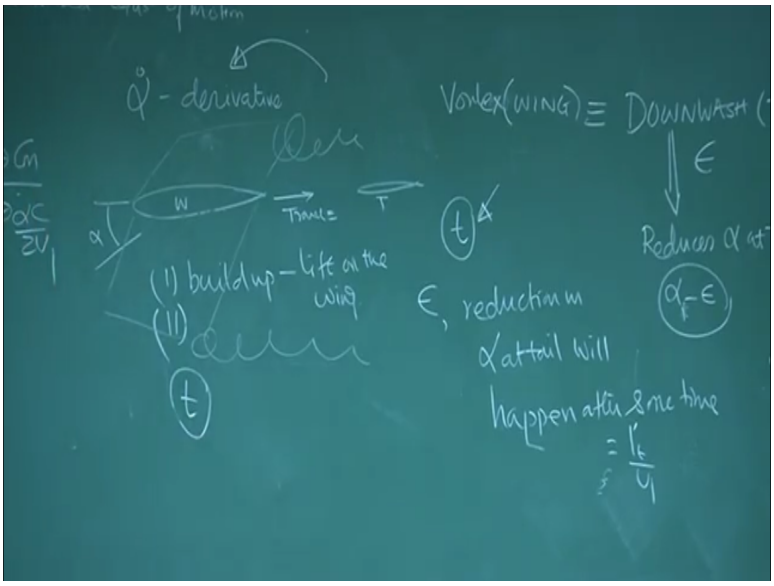

They introduce downwash at the deal vertices or vortex which is introduces downwash at (tail) right and what are this to this downwash epsilon it reduces alpha at the tail right so something alpha t -or alpha1-epsilon so get reduced no good see that if there is time component in epsilon right it is not that something fix them word it is changing with time then the angle at the tag the tail also becomes function of time ok

We will at steady state but you have be careful and give this statement what I meaning is please understand this vertices will travel right approximately to the speed of the air craft there are device there are this 50%speed but I assuming that travels same speed of the air plane so that means suppose analyzing the time t right what is happening at time t. the epsilon reduction will not sink of the plane because epsilon reduction in alpha in alpha will at tail will happen when will happen after some time .

Because what is this will travel of the sometime at the tail then all the downwash angle will be seen at the tail at the time t that means what is happening how much time will take roughly lt/u1 it is the dc from the at the tail dc of the wing to dc of the tail. I call you l t frank it is the distance between the cg of the airplane and ac of the tail and ac of the wings to ac of the tail to be the both size. So what is happening what is the understanding if I am looking at the time to then the downwash seen by the tail will happen later at time.

L t/u1 if that the time the downwash does not wing starts if I start thinking at time to the what is the generator from the wing at the time to this effect have not been by fell by the tail correct that means and I am analyzing for a time t. this is actually sing the more angle it is not see the epsilon that more angle Give me a downward movement pitching movement and that is an alpha dot the force is a cl alpha dot it is clear please understand hear I am focusing the phenomena at the time t at time t angle of the what is angle of the wing at what is the angle m of the tail.

At the time t is what is it be generated but it will take around lt prime u1 time to each tail to downwash correct that we have time to that always effect t is not seen by the tail. The tail has more angle effected correct reduced angle only after time l t/u1 so at a time t is analyzing it is actually practically have a more angle effected then what a thinking minus epsilon that more angle effected all pitching movement down on force upward this is will be upset cl alpha dot and this is movement will be associate to cm alpha dot . It is the physics.

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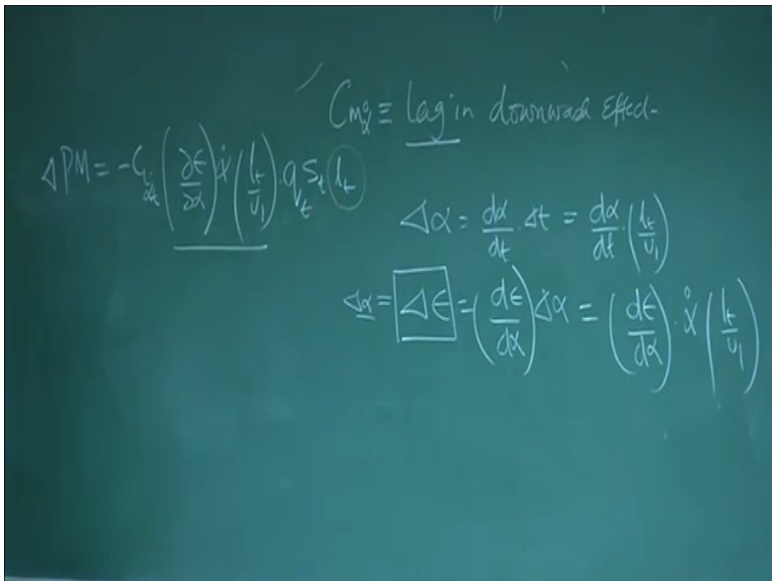

Let us how to develop the model. Ok we discuss this second monkey path I know there will be confusion doesn't matter we again discuss this things monkey path .

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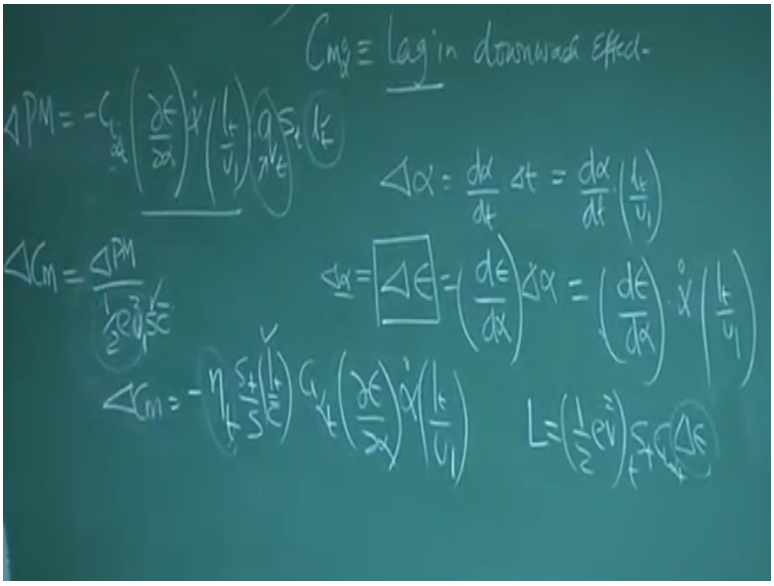

So Cm alpha dot is genuinely term us take =lag in downwash effect you understand what is why this lag is they because at the time t when the we focusing at the wing then its text time it will by you be downwards want to reach the take the downwash.

So there is the lag and downwash is the lag. ok you know delta alpha =d alpha/dt delta t=d alpha/dt lt/u1 right and what is delta epsilon =d epsilon/dx delta alpha=d epsilon/d alpha alpha dots lt/u1 right so this delta epsilon the angel of tag this more by this actually because this actual this happen you see delta epsilon of the angel which will be realize the till only after the time lt/u1 and at the time t this is the extra angle you got so I will write pitching moment and delta pitching moment.

And as minus cl alpha tail d epsilon/d alpha alpha dot Lt/u1 cl alpha into alpha this is because there is the angle it a more at the tail because that downwash has not reach what is the reach the downwash .angle vector is more than this much and this into q tail and v square tail s tail and lt will we the pitching moment is in it half row v square till s till cl alpha tail into alpha into momentum there is the pitching movement.

No problem I will explain delta c m what delta cm will be delta pitching movement divide by half row v square s c bar .the delta c will be delta pitching movement into half row v square if

you do that what will happen. V is nothing but u. delta c m =-n t s t/s s c bar is (lt/c bar) and of course you have cl alpha tail.

D epsilon/d alpha alpha dot l t/u1 right let us check .this is pitching moment divided by half eu1 square sc bar so half row eu1 square q till and n till the ratio s t by s and lt by c bar. Taken here lt/c1 taken here now what is happening the left will d alpha and alpha is here right what is cm alpha dot.

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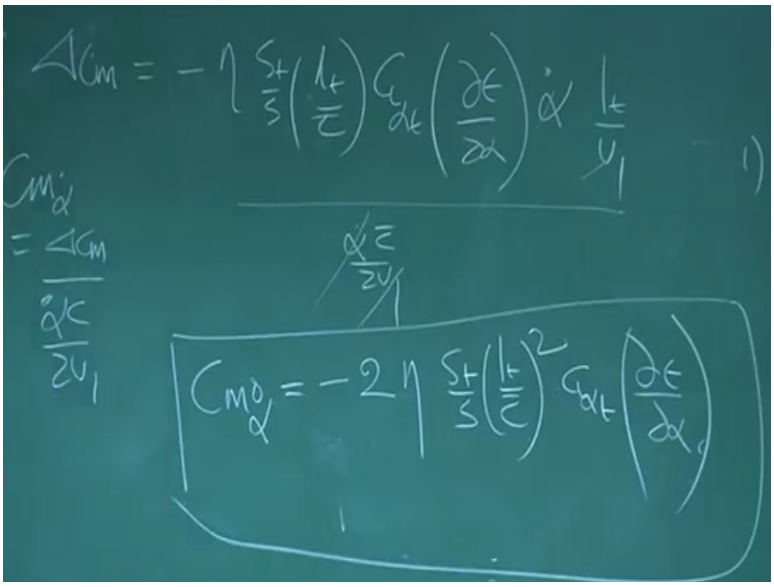

delta cm is equal to minus Neeta s t/s lt/c right so what is cl alpha till d epsilon/d alpha alpha dot (lt/u1) then cm alpha dot is nothing but = delta cm/alpha dot c/zu1 so there is slope so if I divided by alpha c/2 u1 what will get will get cm alpha dot = u1 will go. First of all alpha dot t and alpha dot is go and 2 Neeta s t/s lt/c whole square lt here c bar is here cl alpha t d epsilon /d alpha ok .this is expression for c alpha dot how to alpha dot t you know what you mean d epsilon/d alpha dot those we simplify that the expression also get impartial charge ok.

You will understand that cm alpha dot you can easily say. If you just for the force generated .d axel an and d alpha low speed simplify the expression also be the empirical charge for that ok so we understand what is cl alpha dot so you can easily to see cl alpha dot just you how much is the force generator ok that that matter. I will leave with you .you try to find out expression for cl alpha dot ok clear.

Very simple delta epsilon is the left equal to find lift. Lift is equal to half row v square as till s till cl alpha till epsilon delta epsilon this is expression with given by this then follow the definition of cl alpha dot and find it ok .i expect you will do this, thanks.