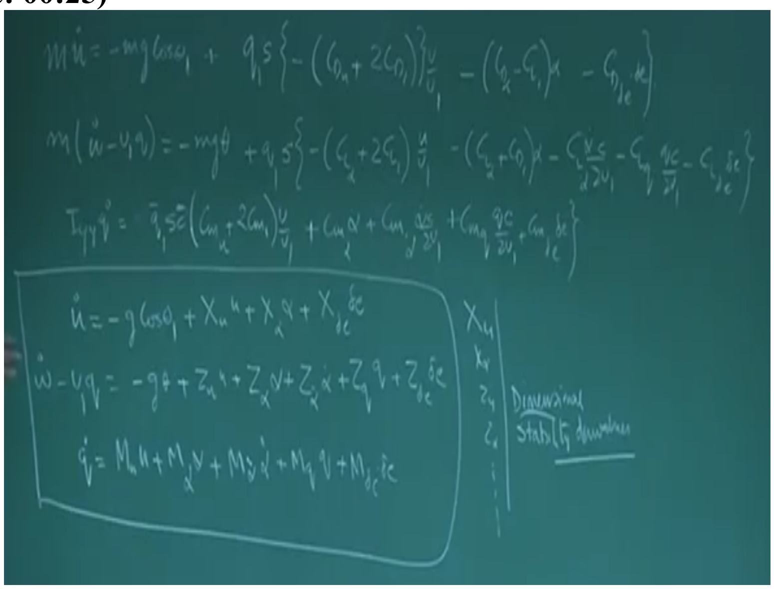
Aircraft Stability and Control

Prof. A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture-49
Longitudinal dimensional stability and derivatives

Good morning, everybody. We are continuing small perturbation, equations of motion for longitudinal case.

(Refer Slide Time: 00:25)



And if you recall, we knew that mu dot equal to -mg cos theta 1 plus fx along the x direction. Similarly mw dot -u1 q equal to -mg theta sin theta 1 plus fz for the second equation and the pitching moment equation was I y y q dot equal to m. And what are these f x and f z m? They are the perturbed aero dynamic forces f x, f z and perturbed aero dynamic moment m.

How do they generate? Suppose the airplane is at steady state, the cruise is our case. Then, if I disturb it, small disturbance if I give then naturally it will have pitching motion, there is change in the velocity. So accordingly moment as well as forces along X Z directions also will change, moment will change and these are perturbed moment. What is R A? We want to see as the disturbance vanishes whether this perturbed quantity u w and q become q zero or not.

So that in a way the airplane has come back to its own equilibrium right. So its standard will be stable. What was the approach? Approach was we wrote f x functional u by u1 alpha then alpha q and alpha dot etc. And once we do dig this for fx says that m we got some dimensional derivatives and then we wrote this equation as to be more precise, what we did was we for fx using those stability derivatives, we wrote this as q1 s - cdu plus 2 cd1 u by u1 then - cd alpha - cl 1 into alpha then - cd delta e into delta e.

And for fz we wrote like this, we derive these expressions. q1 s and then - cl alpha plus 2 cl 1 into u by u1 - this should be here, it should have this sort of a curly bracket, because q1 is common here. So this is u by u1 - cl alpha plus cd 1 into alpha - cl alpha dot into alpha dot c by 2 u1 then - clq into qc by 2u1 - cl delta e into delta e and for this equal to moment and the pitching moment is expanded as q1 sc bar it is the moment.

So c bar will be there cmu you realize cm is very important derivative especially for supersonic case and then plus cm alpha into alpha plus cm alpha dot into alpha dot c by 2u 1 plus cm q then q c by 2 u1 plus cm delta e into delta e, okay. And then we did little bit of algebraic adjustment. We divided each term by m. So I have u dot here, I divided each term by m. w dot m here each term by Iyy.

And then we wrote the equation as u dot equal to - g cos theta one plus xu into u plus x alpha into alpha plus x delta e into delta e and xu x alpha x delta e we define the dimensional derivative. So w dot - u1q is equal to - g theta plus zu into u plus z alpha into alpha plus z alpha dot into alpha dot plus zq into q plus z delta e into delta e.

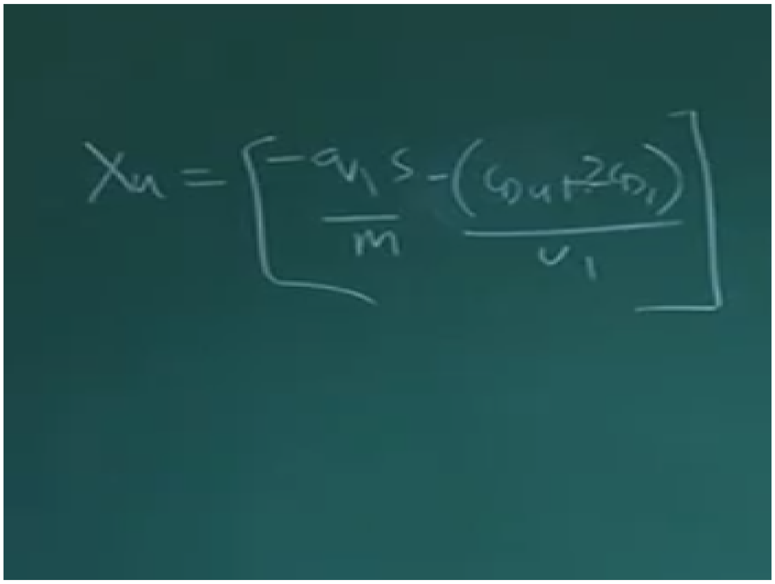
Note down this xu, x alpha, x delta e or zu, z alpha, z delta e extra are dimensional derivative. That is why I don't mind that z alpha into alpha. Then I have that q dot equal to mu into u, m alpha into alpha plus m alpha dot into alpha dot plus m q into q plus m delta e into delta e ok. All these things we have done. Now the question comes what to do next, right. Before we go for next step we realize that all this xu, x alpha, zu, z alpha etcetera, all these derivatives are dimensional derivatives or dimensional stability derivatives.

This is important, for example if I take m alpha the corresponding non-dimensional derivative is cm alpha okay. If it is alpha dot then the corresponding non-dimensional derivative is cm alpha dot okay. So once we have this, this is the equation of motion where I have got u and perturbed u, w and the perturbed w and q is perturbed q okay.

Now for completion what I will do, I will also post the exact expression of xu, x alpha, x delta etcetera for your ready reference, which I will not write here but, you know very well. What is xu? You need not require anybody's help. xu will be simply divide this equation by u by m sorry.

So x will be q1s by m into - cd u into u by u1. So u1 will come x will be very simply you can see which we have shown last time.

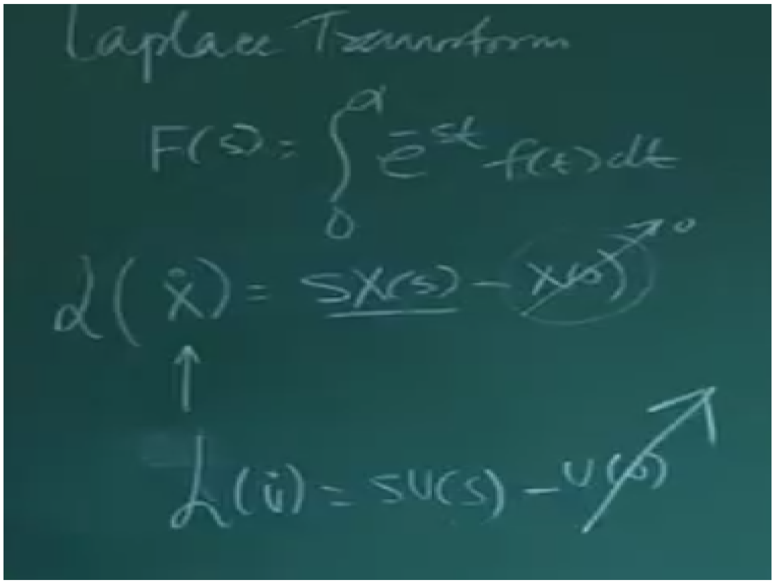
(Refer Slide Time: 07:36)



Also x will be - q1 s by m into - 1 into cd u or let me write like this q1 s - cdu plus 2 cd1 divided by u1 that will be xu. Because if I divide by m here I get q1s by m - this bracket term into u by u1. So u1 have to get denominator. So this became xu, so this term xu into u is this complete term divide by m ok. So please do it yourself and you can see nothing big mathematics or mathematical expedition as simple as dividing by m ok.

Let us come back here these, we see perturbation question equations of the motion. So what to do next, what is our aim? Our aim is to solve this equation and try to find out the stability characteristics of the system in longitudinal mode ok. So we will do that. Now we will do another trick because we know this is linear system because we are talking about small perturbation system.

(Refer Slide Time: 08:55)



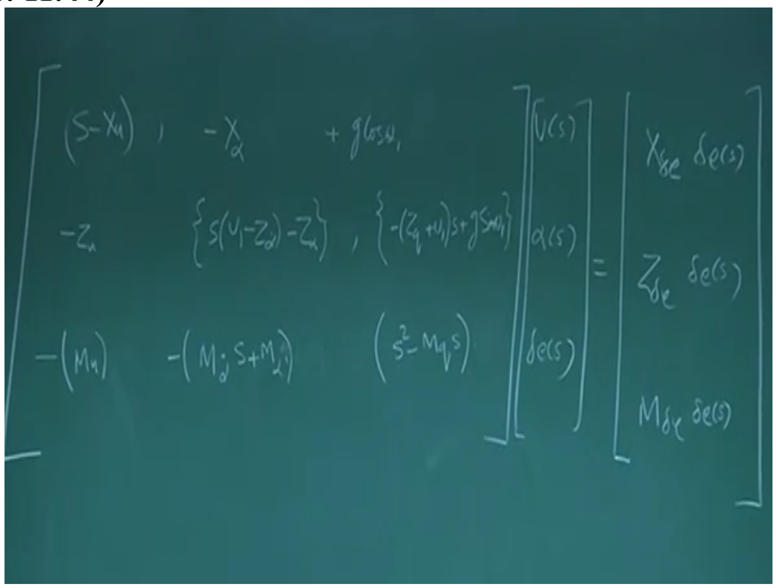
So we will use Laplace transform and that is defined as f of x is zero to infinity e to equal – s t, f t into d t this is Laplace transform. We transfer time domain into frequency domain ok. What is the advantage? See if you recall Laplace this the operative Laplace of x dot is nothing but s x of s - x of zero which is the initial condition for a linear system we can neglect the initial condition for stability point of view for linear system.

It does not depend upon the initial condition. So we don't really care. If I see it here, what is happening? This is the first derivation form x dot. Now this is the x of s. So now this is become almost linear form it is the algebraic form it will come. If I take Laplace transform here. What you happened for u dot it will be s u of s equal to - g cos theta 1 into theta of s right.

Now we do Laplace transform on this equation and then you see we have done some mistake here you have the perturbed theta plus xu into u of s plus x alpha into alpha of s plus x delta e into delta e of x. If you see here. If i take for understand Laplace of x dot is s of x of s then Laplace of u dot will be s. But u Laplace of u dot will be s u of s that is u in Laplace transform of u, u t - u of zero. We said we don't care about initial condition because we are studying the stability aspect of the linear system. The lesion of the initial conditions.

So u dot became Laplace of u dot become s u of s. So here u dot is s u of s - g cos theta and theta is here theta of s then x into u of s. x alpha into alpha plus x delta into delta. Surely you can do it here take this Laplace transform second equation, third equation and then you can write finally meter form of equation in matrix form this will become once you take the Laplace transform all that equation.

(Refer Slide Time: 11:44)



Let me write what will you get you will get S - Xu into v of s then - x alpha into alpha of s plus g cost theta 1 into theta of s right equal to x delta e into delta e of s right do not write directly write matrix form now. Now we write the equation in standard form then you convert it into matrix from. The second line will be - z u into u of s then s u1- z alpha dot - z alpha into alpha of s.

The third term will be the big term will come – z q+ u1, s+ g sin theta 1theta into theta of s equal to z delta e into delta e of s. And the third equation will become –mu into u of s - m alpha dot s + m alpha into alpha of s then s square - mq s into theta of s equal to m delta e into delta e of s you could see that we have not used q of s we are not getting anywhere this term q of s why because we know q is equal to theta dot right.

So of q of s will be equal to s theta of s under small disturbance these things are fine ok. Now we will make a trick and try to make it in a matrix form. So what is the best way to do is your simply put here matrix sign like this take out this U S from here take out this alpha s, alpha s from here theta s, theta s from here and then you write this and this also out ok.

You write this as u of s alpha of s delta e of s and this is equal to x delta e into delta e of s z delta e into delta e of s and m delta e to delta e of s right. I will send you texting you all these small

small I have one manipulation or adjustments would you understand the blackboard to become too heavy to handle this.

(Refer Slide Time: 15:38)

Longitudinal Dimensional Stability Derivatives

Note: Variation due to thrust are not modeled in equations of motion.
It can be included as these terms appear with drag.

$$X_u = \frac{-\bar{q}_1 S\left(C_{D_u} + 2C_{D_1}\right)}{mU_1}\,(\text{sec}^{-1}) \qquad X_{T_u} = \frac{\bar{q}_1 S\left(C_{T_{X_u}} + 2C_{T_{X_1}}\right)}{mU_1}\,(\text{sec}^{-1})$$

$$X_\alpha = \frac{-\bar{q}_1 S\left(C_{D_\alpha} - C_{L_1}\right)}{m}\,(\text{ft sec}^{-2}) \qquad X_{\delta_E} = \frac{-\bar{q}_1 S C_{D_{\delta_E}}}{m}\begin{pmatrix}\text{ft sec}^{-2}\\ \text{or}\\ \text{ft sec}^{-2}\text{deg}^{-1}\end{pmatrix}$$

$$Z_u = -\frac{\bar{q}_1 S\left(C_{L_u} + 2C_{L_1}\right)}{mU_1}\,(\text{sec}^{-1}) \qquad Z_\alpha = -\frac{\bar{q}_1 S\left(C_{L_\alpha} + C_{D_1}\right)}{m}\,(\text{ft sec}^{-2})$$

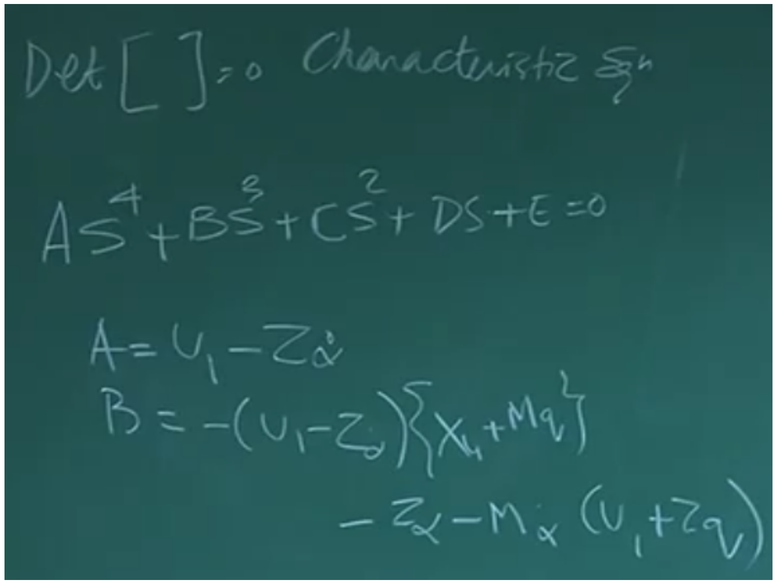
$$Z_{\dot{\alpha}} = -\frac{\bar{q}_1 S C_{L_{\dot{\alpha}}}\bar{c}}{2mU_1}\,(\text{ft sec}^{-1}) \qquad Z_q = -\frac{\bar{q}_1 S C_{L_q}\bar{c}}{2mU_1}\,(\text{ft sec}^{-1})$$

$$Z_{\delta_E} = -\frac{\bar{q}_1 S C_{L_{\delta_E}}}{m}\begin{pmatrix}\text{ft sec}^{-2}\\ \text{or}\\ \text{ft sec}^{-2}\text{deg}^{-1}\end{pmatrix}$$

So what is this equation form a is equal to b and this is linear system you know and you know for the linear system whether the airplane is stable or not. That can be easily decided by studying its characteristic equation by the bunch statement. Number one is Linear system and for linear system if we want to find out whether it is stable or not. Best way you do is find its characteristic equation and, find the roots of characteristic equation and then you can command with the system is stable or not dynamically stable or not. So we will do that and if I now see.

By you have use my half hour matrix understanding so I can find out what is the determinant.

(Refer Slide Time: 16:37)

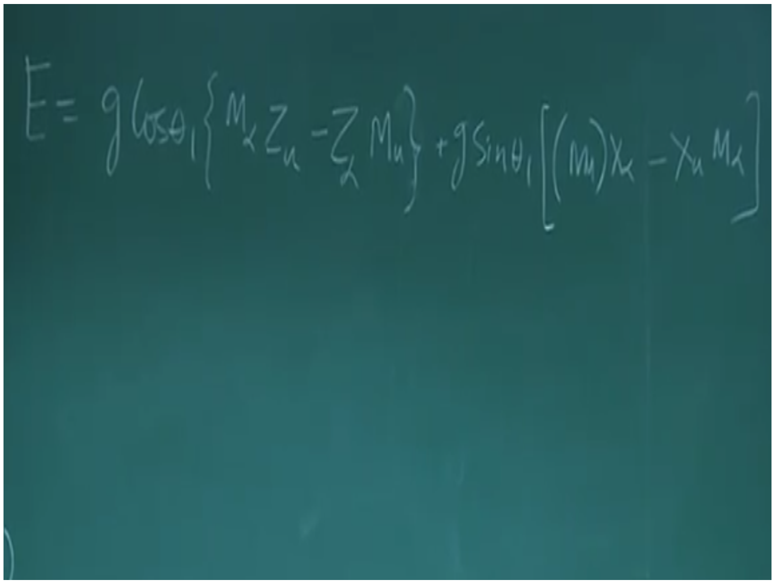


We determinant of this matrix equal to zero. We will give correct characteristic equation right. So what is the determinant if I put the determinant I will get equation is form As4+ b s cube + c s square + ds+ e=0. Do you see what I mean what is the determinant of this first term will be s - xu right into this - this so when second term will be this.

This I eliminate this - this like that I can find the determinant and I can club them in the form of a, b, c, d, e as constants and I can write equation in this form. For the completion I will just give you one value of this co- efficient. Let us say if I do the determinant part of we can put it to zero. What is the A I will get you see soon you will get and you can do yourself we not do on the blackboard here. A will be a - u1 - z alpha dot.

Similarly B will find that will be - u1 - z alpha dot into xu+ m q-z alpha-m alpha dot into u1+zq like this you know similarly c, d, e, f and all. And I will write the expression of e also here t will be here. I will erase this part as I told you this thing you have to do once we have to understand what is the physical interpretation of it.

(Refer Slide Time: 18:43)



If I see the correction you will come like this. You will come as g cos theta 1 into malpha zu - z alpha mu + all this is like this + g sin theta 1 mu x alpha - xu m alpha. If will come expand you will get it ok.

(Refer Slide Time: 19:29)

$$\boxed{D_1 = As^4 + Bs^3 + Cs^2 + Ds + E}$$

$$A = U_1 - Z_{\dot{\alpha}}$$

$$B = -(U_1 - Z_{\dot{\alpha}})\{X_u + X_{T_u} + M_q\} - Z_\alpha - M_{\dot{\alpha}}(U_1 + Z_q)$$

$$C = (X_u + X_{T_u})[M_q(U_1 - Z_{\dot{\alpha}}) + Z_\alpha + M_{\dot{\alpha}}(U_1 + Z_q)] + M_q Z_\alpha - Z_u X_\alpha + M_{\dot{\alpha}} g \sin\theta_1 - (M_\alpha + M_{T_\alpha})(U_1 + Z_q)$$

$$D = g\sin\theta_1[M_\alpha + M_{T_\alpha} - M_{\dot{\alpha}}(X_u + X_{T_u})] + g\cos\theta_1[Z_u M_{\dot{\alpha}} + (M_u + M_{T_u})(U_1 - Z_{\dot{\alpha}})]$$
$$+ (M_u + M_{T_u})[-X_\alpha(U_1 + Z_q)] + Z_u X_\alpha M_q + (X_u + X_{T_u})[(M_\alpha + M_{T_\alpha})(U_1 + Z_q) \pm M_q Z_\alpha]$$

$$E = g\cos\theta_1[(M_\alpha + M_{T_\alpha})Z_u - Z_\alpha(M_u + M_{T_u})] + g\sin\theta_1[(M_u + M_{T_u})X_\alpha \pm (X_u + X_{T_u})(M_\alpha + M_{T_\alpha})]$$

Let us focus here once you have got such a characteristic equation see of 4th, cas, see has 4 cube 2, 1 right. Then numerical ways of finding this root you solve this equation numerically and find what are the values of this root. And depending upon this you can comment on this is stable or not right.