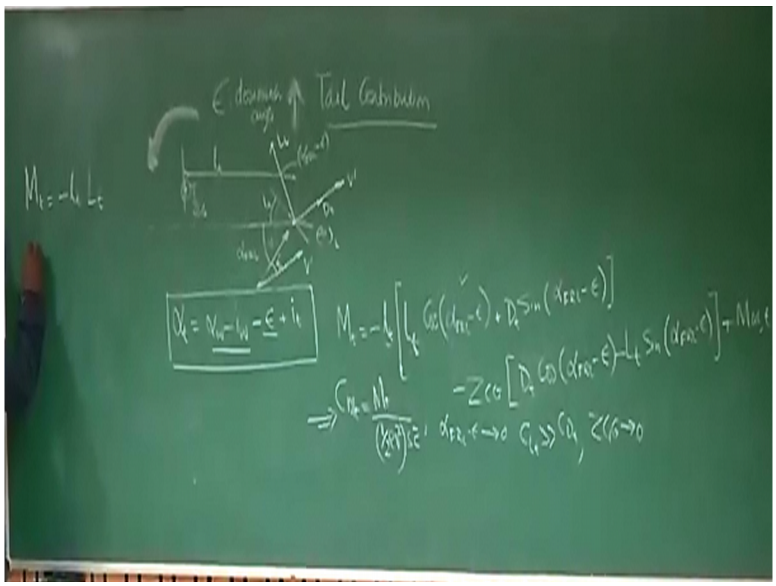
Aircraft stability and control

Prof: A. K. Ghosh
Dept of Aerospace Engineering
Indian Institute of Technology Kanpur

Lecture- 05
Stability: Tail Contribution and Static Margin

(Refer Slide Time: 00:15)



Now we will be talking about tail contribution. If I draw this diagram, you know the wing was here and this is Alpha FRL. We are talking about tail contribution, let me draw the diagram, this is the wing part, we have already done CG somewhere here. This is ZCG okay and this tail somewhere here and we are having tail also having some setting angle we called IT right? And then, please understand one thing if this is the V direction the velocity direction free stream.

When tail sees this dynamic pressure the velocity vector is no more the same as free stream velocity vector. You know very well as there is the lift under wing, this is the higher pressure and the lower pressure. There are vertices, and their vertices travel like this and induces a downwash at the tail to the velocity vector, which was free stream like this okay? But, because of downwash it gets tilted and I call it V prime and this is V. So, if I draw here

I have to draw 1 line parallel to this, which is not the local velocity direction. Local velocity direction is V prime direction and the angle between these two is Epsilon, which is the downwash, not clear okay? Let me explain, you know that if this is the wing and here there is a tail right and because of lower and higher pressure there are vertices and that induces a downward component at the tail. That is if this is the tail and wing is somewhere there because of vertices the downward component will be induced at the tail.

Let see the velocity vector free stream was like this because there is another downward component because of downwash so velocity vector will be tilted downward that is exactly as happened. This is the V free stream direction because of downwash this velocity vector is tilted. Tilted by how much by epsilon which is called downwash angle right. You know that lift and drag are perpendicular and along the velocity vector respectively.

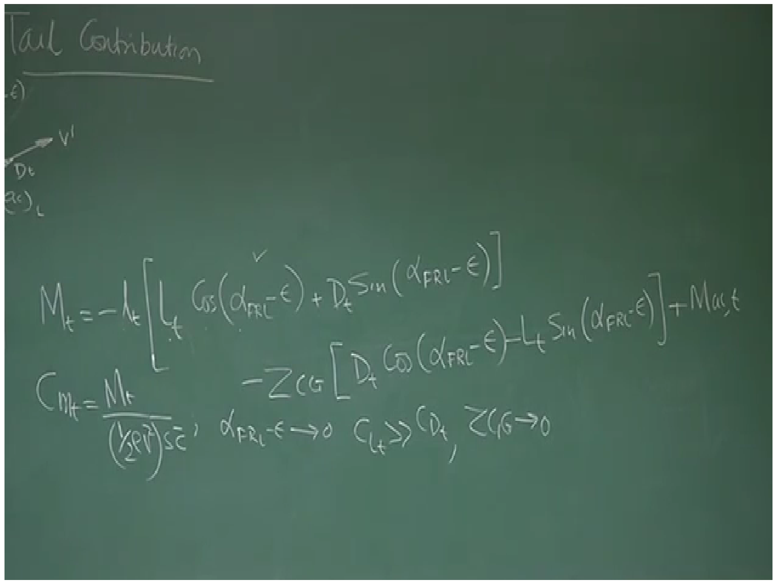
That is lift on the tail will be perpendicular to V Prime lift tail and it will be drag tail will be like this okay clear? Please note that it will not be perpendicular to the free stream direction V, it will be perpendicular to the local V because that is causing the dynamic pressure that is definition of left. So, if I do that and now I again use this diagram. So, I understand two things here. What is one is what is a Alpha T? What is the tail angle of attack?

So, from the diagram you could see Alpha T will be what is Alpha T you see here this is the tail okay. Alpha T will be let me write then you understand better Alpha W- IW- Epsilon + IT. Is it clear or not? See Alpha W - IW is nothing but this angle which is Alpha FRL right from Alpha FRL your angle is reduced by Epsilon downwash also we have to be sure that I have given a IT angle setting angle here so, your Alpha tail is simply this.

Is it okay? This is the velocity vector this is Alpha FRL So the angle seen by the tail is if Epsilon was not there Alpha FRL + IT there would have been the angle similar tail. But, now what was happened because of Epsilon the reduction in the angle so that is being put here okay. Once you know that now you can see also see from this diagram that this angle is nothing but Alpha FRL – Epsilon. So if that is true nothing stop us from writing this expression.

Let me write the expression MT because of tail is LT into COS of Alpha FRL - Epsilon + D of tail SIN of Alpha FRL - Epsilon okay. And then we have another term minus just I will explain you that wait for a minute D of tail COS of Alpha FRL - Epsilon - LT SIN of Alpha FRL Epsilon + MAC tail let us see what it is.

(Refer Slide Time 06:35)



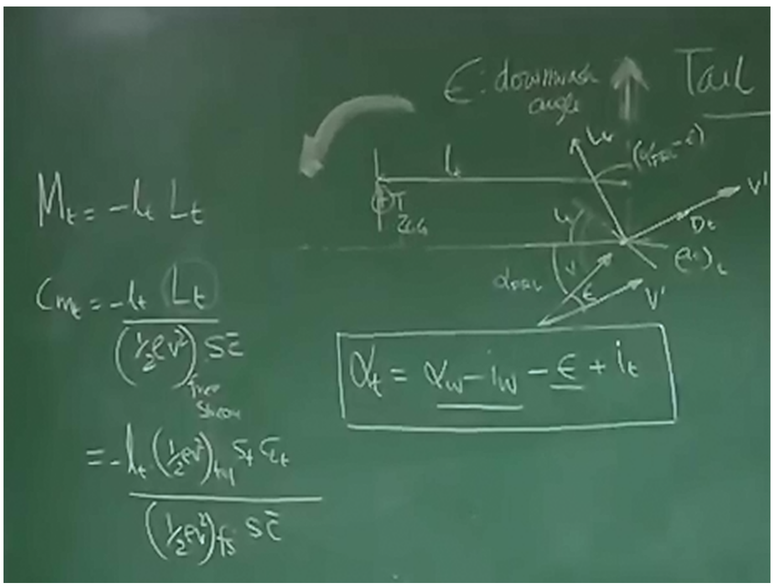
What is LT first of all LT is this is AC of the tail and LT is this is the distance from XCG to AC of the tail that is called to tail momentum okay. Now you could see this is LT so this component LT COS Alpha FRL - Epsilon put this into LT so LT COS of Alpha FRL - Epsilon is this component so this into LT will give a nose down movement so I have - SIN and LT is here LT COS Alpha this part is done. Similarly, you could see for other term as well okay. Next is what we have already learned what is next from MT.

Next from MT I will come to CMT okay. How do I come to CMT? CMT means pitching moment because of tail that is you divide MT by half row V square S into C bar okay. If I do that please remember here, I am dividing by half row of the V square free stream not half row V prime square that is why this needs draws an attention okay. Please understand this is the lift stage perpendicular to the V prime and one component of the lift which is a along V is indeed induce a drag right.

Because we are non-dimensionalizing everything every coefficient with respect to free stream speed or velocity or dynamic pressure. That is the point to you should clearly understand. If CMT I define as MT by this and also now I put Alpha FRL - Epsilon all these small quantities and again CL tail greater than CD tail and ZCG is to zero. If I do this simplification then I get the neat expression.

Which is MT = - LT into LT from because when I put this one this is put to zero because drag I'm lifting ZCG0, so I already have pitching moment as a lift because of tail into LT tail momentum - because it will given nose down moment right?

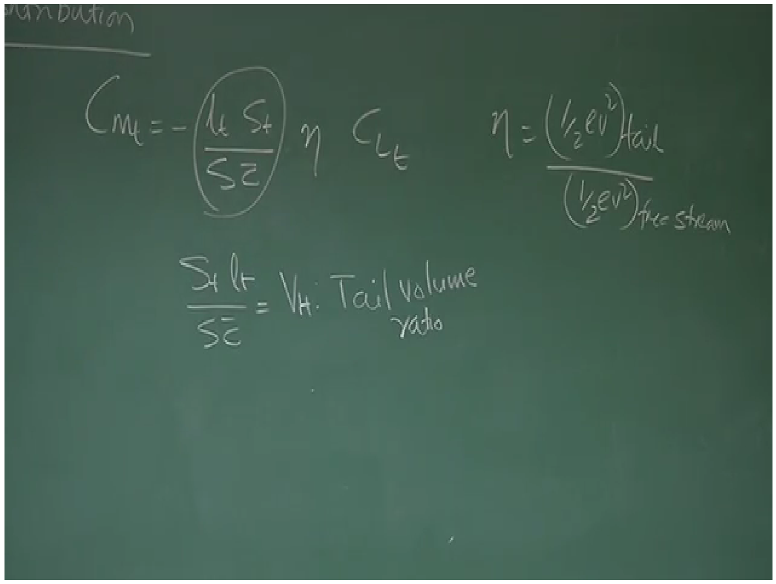
(Refer Slide Time 09:25)



So, from here I know how to get CMT how to do this trick divide by half row V square SC, so I will get CMT as mimus LT into LT by half row V square free stream free into SC bar watch here carefully what will be LT, LT will be say - LT is

The tail momentum lift from the tail it will be half row V square at tail into ST into CLT right? This divided by half row V square free stream into SC bar so what is this CMT now stands for? What is the modification and expression of CMT when you take care of a local dynamic pressure at the tail which is different from free stream dynamic pressure.

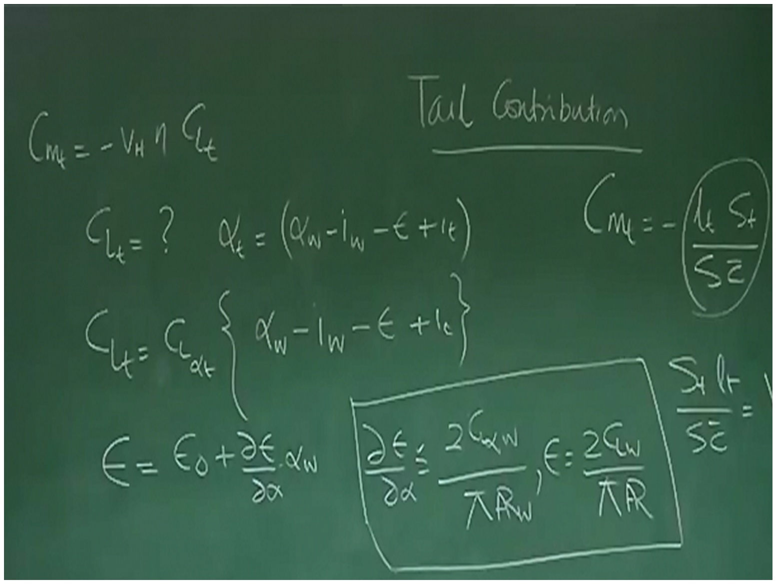
(Refer Slide Time 10:29)



So this CMT finally takes the shape as minus LT, ST by SC bar into Neeta into CLT. What is Neeta? Neeta is the ratio of dynamic pressure of the tail and free steram dynamic pressure. So, the Neeta is half row V square at the tail by half row V square free stream right. And you could see ST LT SC bar has been separated out this is Neeta into CLT and what is this ST LT by ST LT by SC bar this is V popular diameter called tail volume ratio tail volume ratio we soon discuss what is the importance of this?

But one thing at this point you should be able to appreciate that pitching moment because of tail will largely depend upon this ratio tail volume ratio for a given CLT. That means and what is this? This tail to a designer if you want to increase VH you can increase this contribution and increasing the VH means either to increase the tail area or tail momentum or both right or in a combination. So that gives a very good design of flexibility.

(Refer Slide Time 12:16)



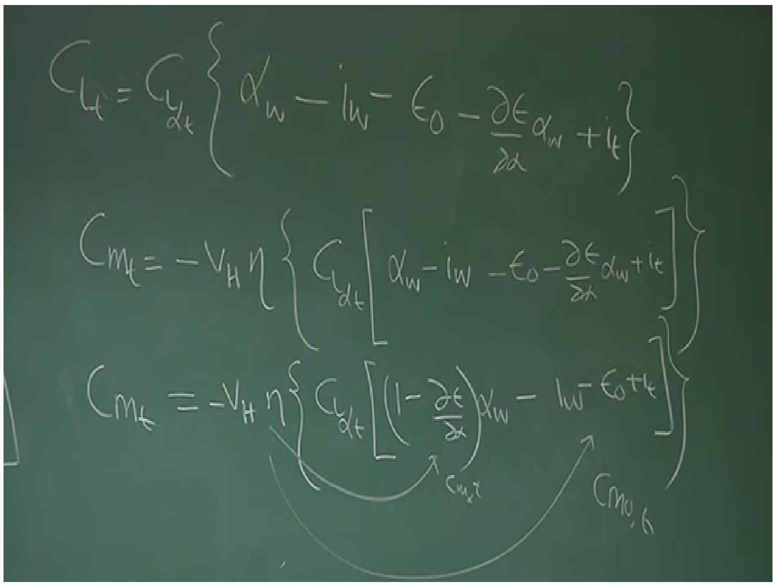
So, we are continuing with tail contribution. We have seen CM tail as - VH Neeta into CLT. What is CLT? CLT will be = what CLT the lift coefficient at tail what is the angle of attack at tail we already know Alpha T = Alpha W - IW - Epsilon + IT right where the IT was the tail setting angle. So CLT will be what very it’s very straight forward this is CL Alpha tail into Alpha W - IW - Epsilon + IT. You may ask me a question, why not this is = CL0 + CL Alpha T into all those thing why CL0 is not here.

Answer is very simple tail is generally symmetrical and is advisable to have tail symmetrical that is why here there is no CL0. Is this clear? So, once I have this then, what is my next step? Let us revisit again what is Alpha W it is a wing angle of attack what is IW the wing setting angle what is the Epsilon it's a downwash because of wing and IT is the tail setting angle correct now how the downwash is going to be change can be approximated by.

This model of wing where D Epsilon by the D Alpha written as 2CL Alpha wing by PI aspect ratio of wing with the approximating but it works and assuming that E is one and Epsilon we can write as 2CL wing by PI expectation okay. Please take this there we can read some books, you can take to read my first lecture on introduction to airplane performance or any good aerodynamic books, flight mechanics book will tell you this gives expression.

We are now coming back. How do use these things that we will be seeing. So, once I have written this let me go step forward do not forget, what are you going to have what we are looking for we are looking for the contribution of tail in terms of giving pitching moment about CG of airplane and if you can still write that contribution also as CM0 tail + CM Alpha tail into Alpha tail or Alpha wing or Alpha then I will get a general expression okay. That’s exactly I am trying to do.

(Refer Slide Time: 15:10)

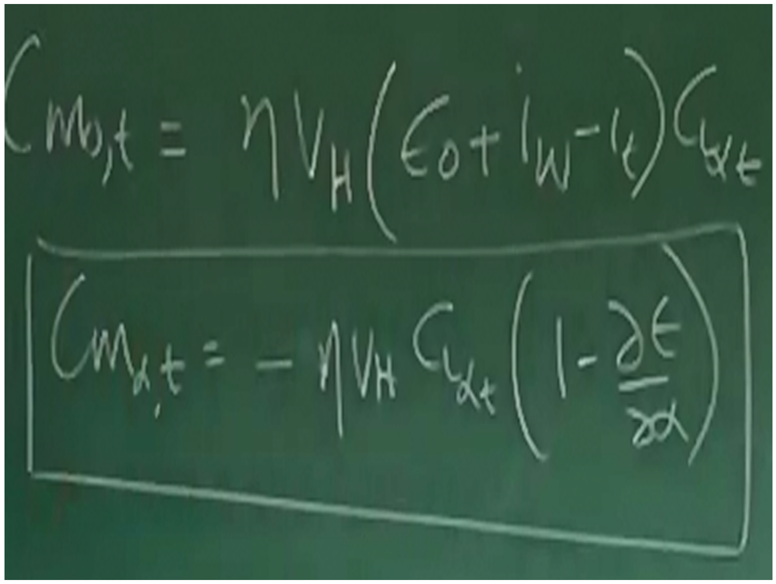


So from this I can write CL Tail as CL Alpha tail to Alpha W - IW for epsilon I am writing Epsilon 0 - D Epsilon by D Alpha into Alpha wing + IT. If that is true then what is CMT? CMT already you know expression of CMT has minus VH so minus VH Neeta see here Neeta CL tail so for CL tail. I will write this expression. So, I will write CL Alpha tail into Alpha W - IW - Epsilon 0 - D Epsilon by D Alpha into Alpha wing + IT. This further I can simplify as - VH Neeta CL Alpha tail into 1 - D Epsilon by D Alpha into Alpha W - IW - Epsilon 0 + IT.

What I have done I have just taken this common Alpha W and D Epsilon by D Alpha W so I have written like this the rest of I have preserve the way it is and this is CMT. Now have a closer look here also we have seen in the CMT, that is contribution of tail to pitching moment about CG here also there is a one term first term that depends upon Alpha W change it with Alpha W and other is the constant.

So, immediately your mind should think. “Oh” this is constant with this part will give me CM0 tail and this part will give me CM Alpha tail. So, I can write CM tail = CM 0 + CM Alpha tail into Alpha wing. So if I do that then what expression I get is very simple and you can handle it.

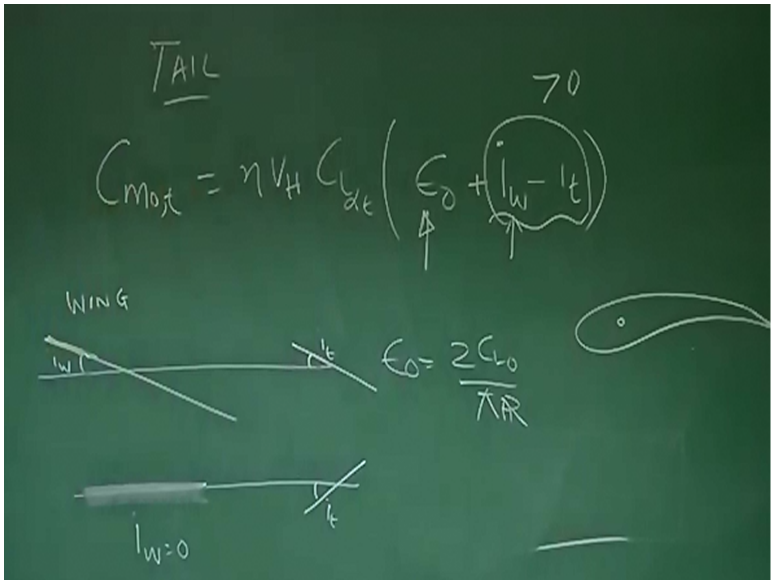
(Refer Slide Time 17:26)



So I am getting CM0 tail = Neeta VH you could see here Neeta VH and minus observe and change the signs. So, I will have Neeta VH into Epsilon 0 + IW - IT into CL Alpha tail that will be CM0 tail and CM Alpha tail will be minus V Neeta into VH. This term this into this is the CM Alpha term minus VH into Neeta into CL Alpha tail 1 - D Epsilon by D Alpha this will be CM Alpha tail.

Let us also now go back to the expression what we develop up for CM0 tail. I have written already here Neeta VH is tail volume ratio CL Alpha tail Epsilon 0. What was the Epsilon 0? Epsilon 0 is because of cambered wing right even at Alpha = 0 there will be a pressure difference that will give you lift and that will give Epsilon 0 and if you further understand Remember this is CL VS Alpha at Alpha = 0 there is a CL not and CL 0 is there means is the pressure difference so because of CL 0 there will be or because of pressure.

(Refer Slide Time 19:01)



Difference, there will be vertices and their vertices will give a downwash at tail even at Alpha = 0 is the key point here that is why Epsilon 0 okay and you know how to calculate that okay. What is IW, IW was wing setting angle and IT was tail setting angle. Now from here you could see if there is cambered aerofoil wing some value of Epsilon 0 will automatically come which 2CL0 by PI aspect ratio I am assuming it to be one roughly

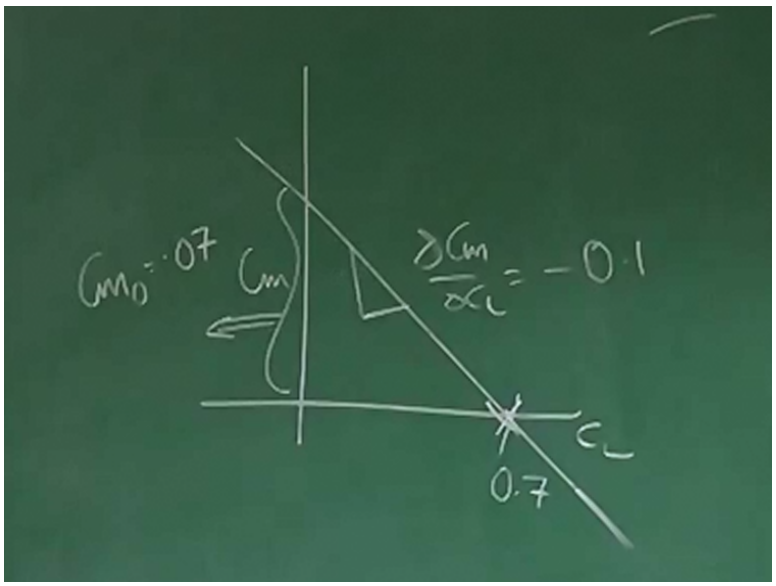
So if there is a cambered aerofoil wing that will give some value of positive Epsilon 0 right. What about IW. See this is just the wing setting angle IW. This is the wing right if you see this expression here IW wing setting angle IT is the tail setting angle right, if all only we are thinking in terms of tail setting angle that is that is no wing setting angle wing is like this then we need to be a negative IT to generate a positive CM0 from here also you see see all are positive.

For timing even if I do not include that this is a positive number, a small number. But if IW is 0 if IT is negative that is like this the then this gives positive CM0 tail this is the case when IW is zero. But, the question comes you can still get CM0 positive if you put IT positive. How it is possible as long as this difference is greater than zero you will get CM0 because of tail is positive that is suppose if this is IW is 3 degree and IT is 2 degree although 0 - 2 degree not negative angle.

But If we cleverly 3 degree here and 2 degrees here you see the differences 1 degree positive which will give you CM0 tail as a positive. So, you can get CM0 positive by giving tail setting angle negative one CM0 positive by appropriately calibrating IW and IT in a fashion that IW - IT is a positive number. So these all the aspects are optimize to get a appropriate value of CM0 that could be generated through configuration and design.

But, please remember what through the value of CM0 who decides that the designer decides that depending upon the stability margin or degree of stability they want.

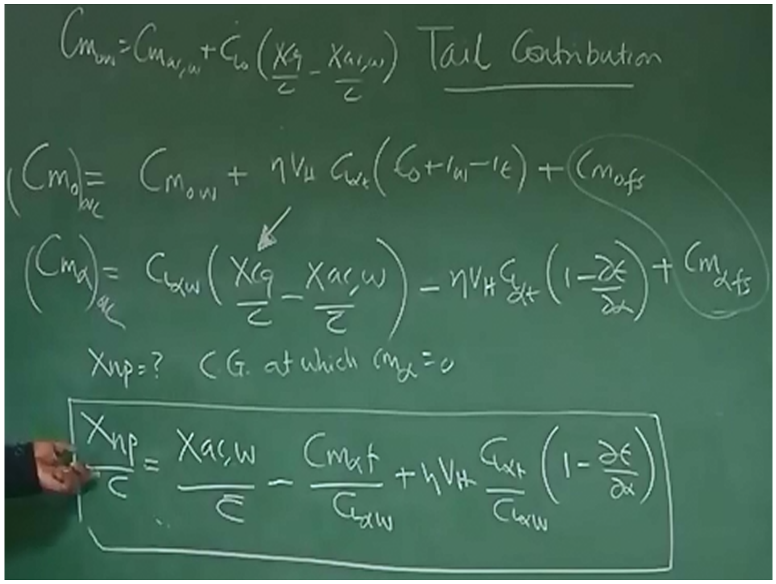
(Refer Slide Time: 21:57)



And what is the trim here this is CL and CM with designer will say DCM by DCL will be minus point one that is 10% static margin and CL, I say around point seven I want to trim the airplane at point 7. So, these I have to tell you have to configure the airplane such that.CM0 is point zero seven could define from here okay.

And the moment designer stage you know that yes, I have to generate point zero seven CM0 and DCM by DCL as minus point one you now align your wing fuslage tail everything. So that finally, you can get these values clear. So, this is the tail contribution okay. Now if you recall we had similar expression for wing contribution also. So, if I now add wing +

(Refer Slide Time 23:04)

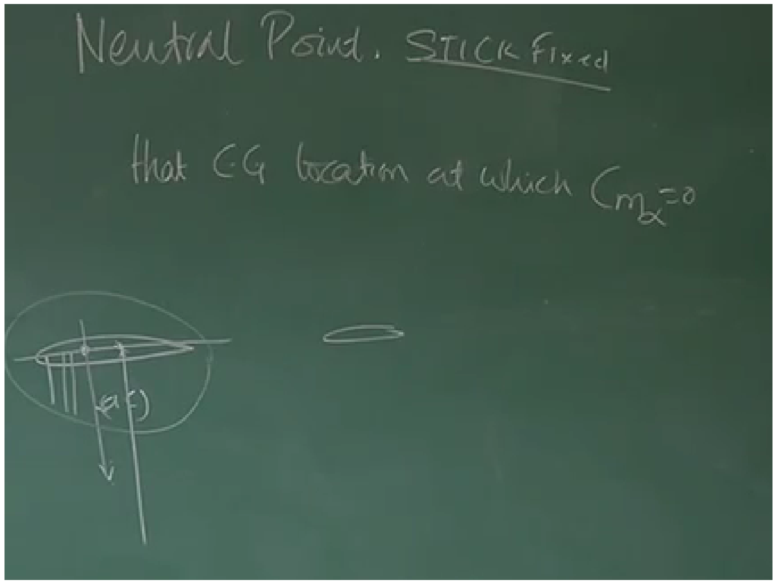


Tail, what is the final expression looks like. So, I will get CM0 will be because of wing and tail I will add there things together so I write CM0 wing + Neeta VH CL Alpha tail into Epsilon 0 + IW - IT and let me add CM not fuselage which we have not done but we are adding get we will do in some example exercise right. So CM Alpha will be CL Alpha wing into XCG by C bar - XAC wing by C bar is to all we have derived okay.

Plus let us say this is because of tail is - Neeta VH just now we have completed CL Alpha tail into 1 – D Epsilon by D Alpha and let's add + CM Alpha fuselage this two things we haven't done but, we are putting this number for contributing let us revisit this, this CM0 of the whole airplane, this CM Alpha of the whole airplane, and this CM0 is CM0 wing for CM0 wing.

You know that the expression derive was CM0 wing was CM AC wing + TL0 into XCG by C - XAC wing by C already we have developed that. And all these these expressions now we know. So if I want to really ensure that I have particular CM0 of aircraft required I can manipulate this VH can manipulate location of CG and AC of the wing and you see how we do it by through example. Now next question comes if this is expression, how do I find a neutral point? okay so that is next I'm doing neutral point of the airplane using this expression.

(Refer Slide Time 25:14)



We generally call it stick fixed. So, when you realize that stick fixed means we are not allowing the elevator to float what is neutral point? Neutral point is stick fixed is that CG location at which CM Alpha is zero or aircraft neutrally stable.

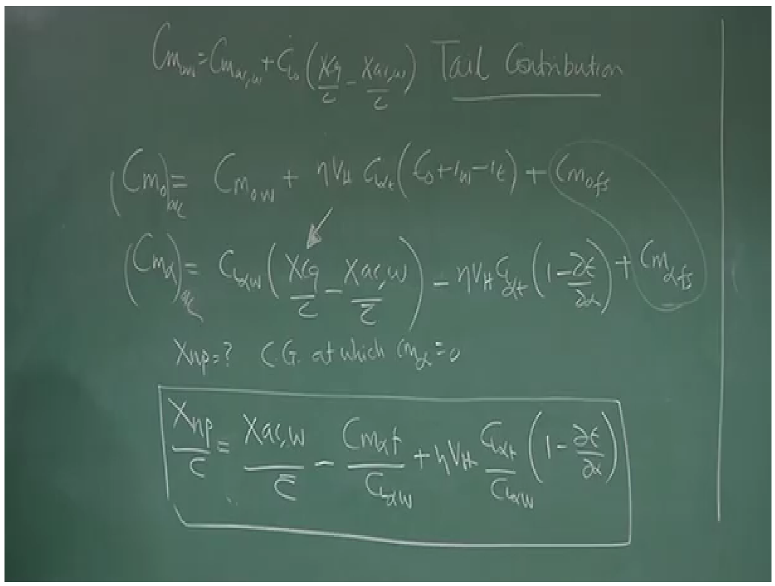
Suppose if I am talking about simply a wing then if this is the AC then suppose CG was here it is statically stable. But, as the CG one side of the AC of the wing. So, this wing along configuration becomes neutrally stable for a airplane. There will be tail so this point not the aerodynamic center of the wing somewhere here if I bring the CG we will find the aircraft will become neutrally stable so what is CG location. I am trying to find out can I do I write the complete expression of CM Alpha for the whole the airplane.

We have seen CM Alpha is here here although whole airplane is this one. Now, if I want to find out neutral point I call it XNP then what is the condition it is that CG location at which CM Alpha is zero. So it is that CG location at which this gentleman become zero. If I do that then I get XNP by C = XAC wing by C - CM Alpha fuselage by CM Alpha wing + Neeta VH CL Alpha tail by CL Alpha Wing into 1 - D Epsilon by D Alpha.

You see here I am putting this zero. So, I want to the find what is the XCG. So, this term will come on the left inside is become a positive then divided by CL Alpha wing which is here CM Alpha from CM changes its sign minus divided by CL Alpha because of this and of course CL Alpha divide by this to XAC wing. So, this is the neutral point stick fixed for an airplane. So, if you want to to calculate neutral point what do you require is you need to know,

What is the XAC location or wing aerodynamic center of location. What is the value of CM Alpha fuse large what is the value of CL Alpha wing what is the tail volume ratio CL Alpha tail and D Epsilon by D Alpha to immediately you will know what is the point which is the neutral point.

(Refer Slide Time 28:15)

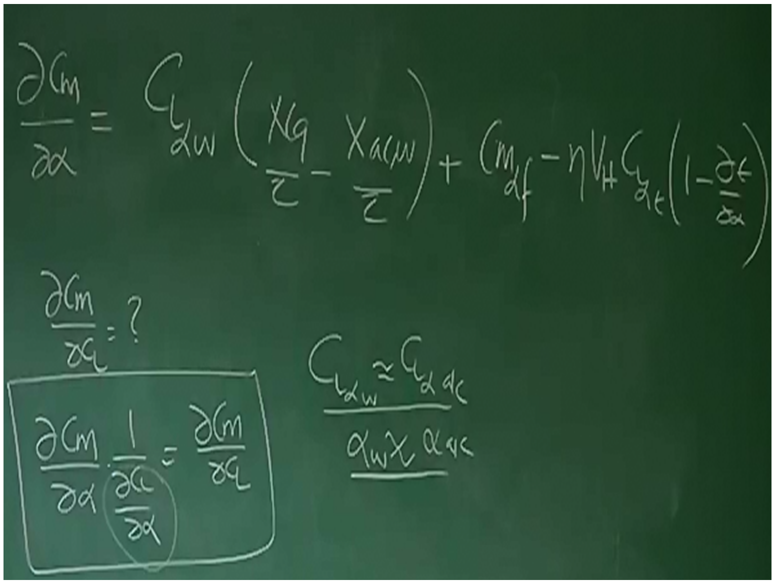


So we should be careful enough when you're layout your aircraft your CG should not go beyond neutral point in fact it should be little ahead of neutral point. So, that we have got static stability margin. That exactly we are now going to talk so this is the neutral point expression what is the meaning of neutral point it is that CG location at which the aircraft will become neutrally stable right? So, it depends upon what it doesn't depend upon CG please understand guys.

It depends upon what is the AC of the wing, it depends upon what is the CM Alpha fuselage CL Alpha of the wing what is the tail volume ratio what is CL Alpha tail what is the DX Epsilon by D Alpha right? So, it is on the characteristics of wing fuselage aerodynamic characteristics right. So manipulating this you can always adjust you are neutral point okay of the Airplane right. It is very important because in more case CG should go beyond the neutral point okay.

But, If you want to make it statistically stable so you need to have your CG of the airplane little ahead of neutral point how much ahead of that going to plan out. Let us do some approximation and try to see can you we get some better field in terms of what is that separation should be between neutral point and center of gravity of the airplane.

(Refer Slide Time 29:41)

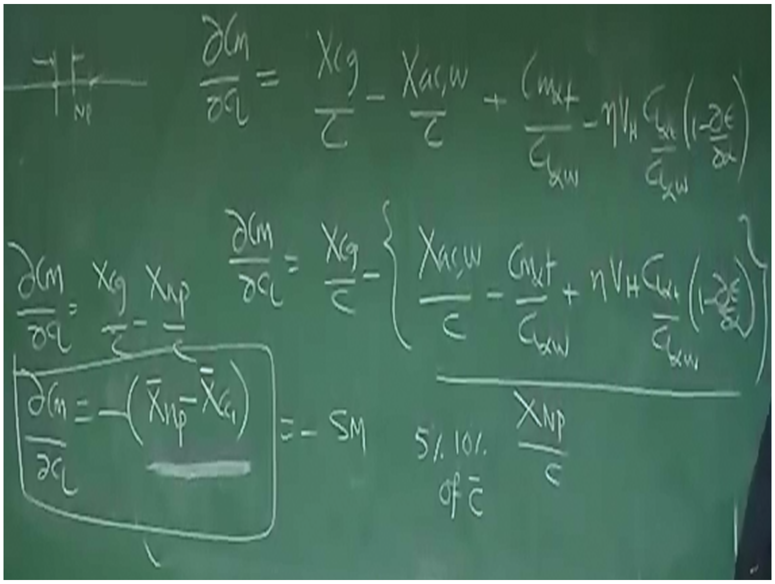


We have seen DCM by D Alpha can be written as let me write this CL Alpha wing. Please be careful I'll be doing some approximation fuselage – Neeta VH CL Alpha tail into 1 – D - Epsilon by D Alpha this is the CM Alpha expression what I will do I will try to see DCM by DCL = ? So now I am doing that approximation what I am doing is I am writing this DCM by D Alpha into one by DCL by D Alpha = DCM by DCL right? Okay.

What are the assumptions we have made? I have made one assumption that CL Alpha of the wing and CL Alpha of the aircraft are same almost same right is this clear one is CL Alpha of the wing is basically close to CL Alpha of the aircraft and also we have assume the Alpha the wing is Alpha of the aircraft. This approximation we are making to get some useful understanding about the static marginal we defined, just see that so what I have to do.

I will divide these by CL Alpha of the wing and there are some we add there these assumptions could be made for this specific study okay. That should not be forgotten. We are doing these approximation for a particular case.

(Refer Slide Time: 31:44)

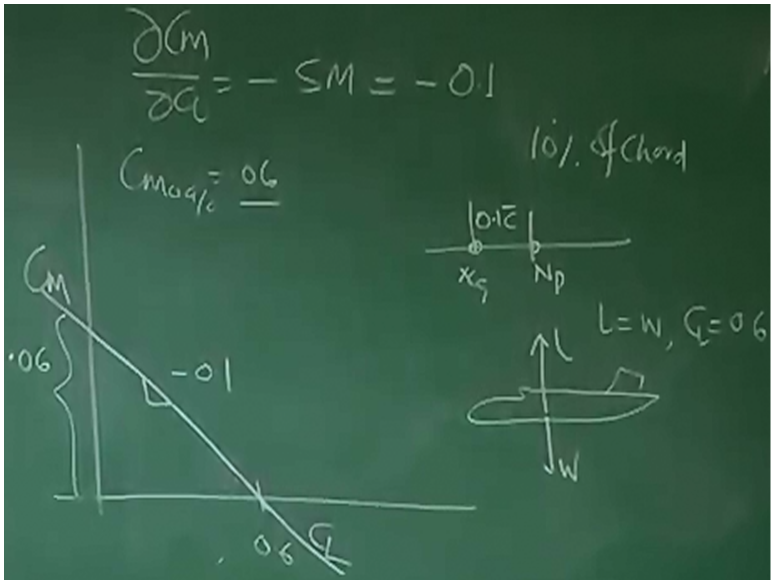


So if I divide by CL Alpha the wing so left hand side will become DCM by DCL and this will become XCG by C - XAC wing by C right then plus CM Alpha fuselage by CL Alpha wing - Neeta VH CL Alpha tail by CL Alpha wing into one -D Epsilon by D Alpha. But, see here carefully I can write this DCM by DCL also have XCG by C - XAC wing by C - CM Alpha fuselage by CL Alpha wing pulse Neeta VH CL Alpha tail by CL wing into 1 - D Epsilon by D Alpha.

What is this term this is nothing but X what neutral point okay this is X neutral point by C. So, now we have DCM by DCL as XCG by C - X neutral point by C all this is equal to I can write it like this minus of XNP bar means divide by C bar - XCG bar and dear friend these separation I will asking you how up to what point I should take the central of gravity. So, that in case when central of gravity beyond neutral point it will because statically unstable. But, I was telling you if this a neutral point if CG comes here it is unstable but CG has to be ahead of neutral point but the portion was how much ahead right.

So, this is the answered partially by this what do you say that your DCM by DCL is minus of static margin static margin is what XNP bar XCG and typically for transport airplane static margin it could be 5% to 10% of mean aerodynamic chord. If suppose it is 10% let's understand this. How beautiful this expression is how wonderful this expression is for a designer. What does it tell you let us have a closer look.

(Refer Slide Time 34:32)



DCM by DCL = minus static margin and let say we are talking about - point one or say 10% of chord. What is 10 % of chard means if this is the neutral point and this is a CG, this distance is the point one C bar. So, when you plot CM versus CL. if we know I need CL of point 6 to at a particular speed to ensure lift = weight so you could always have suppose that improves and we have a lift with the eight for left = weight suppose CL required for that condition is point six so mark here point six and you know.

That aircraft starting margin will be around 10% or DCM by DCL is minus point one so draw line slope straight line slope is minus point one. So, this will tell you CM not required will be point zero six. Will you see that CL point is come from my trim, 10 % static margin from here is guideline. So, if you draw slope of minus point one it cuts CM axis somewhat and from here and we will find this distance will be point zero six so for a designer CM not aircraft should be = 0.6 and where will get 0.6?

We will now come back to wing contribution come back to tail contribution and ensure tail, wing, etcetera are in such a way that CM0 of the aircraft is point six then the aircraft once flying will be automatically trimmed as CL equal to point six so this is the beauty. We will be solving some example on that okay. Thank you