**Aircraft Stability and Control**
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**Lecture – 51**
**Longitudinal Modes**

Good morning, I was just going through the recordings and I realize that we need to go deeply into it on the mathematical aspects before we try to go further right. I let us see what we are doing we want to study the dynamic stability of the airplane that means we have to study we have disturbance i see what is its behavior that is how all the perturbed quantities at behaving we get coming back to zero then we say that dynamically stable right.

This is what? To do that what we have done.

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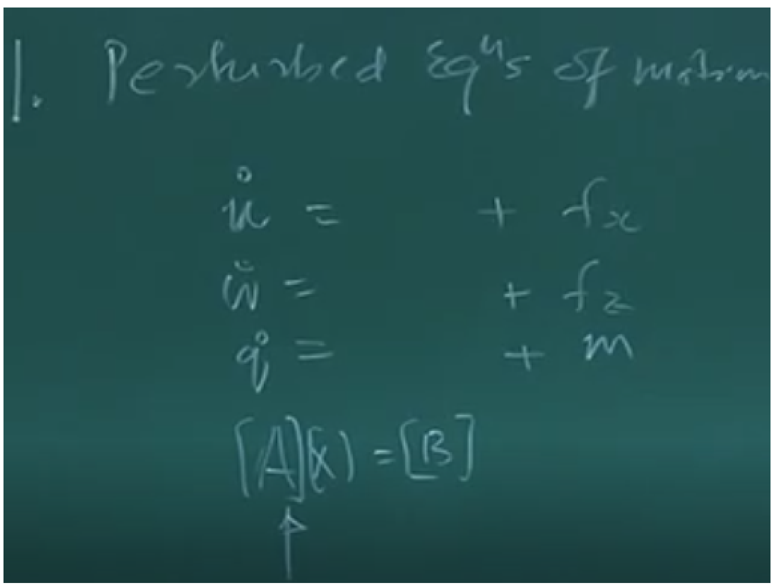


We have developed what we call perturbed equation of motion right. And you know by now that u dot w dot then they are q dot this is the equation where you have f x perturbed atomic force f z perturbed anatomic force z direction and perturbed anatomic moment ok. Because we understand at steady state this m will be zero because there is no such motion at steady state because cruise is our steady state but the moment I disturbance this I will do all this motion execute all this motion so there will be perturbed moment.

Then what we did we know this f x f z a and m will function now alpha, alpha dot, q, delta e extra and we expanded it and then we found though dimensional stability derivatives and plug in this equation to get the matrix? Matrix equation of the form A X is equal to b ok. And then we know the thrust equation is determinant of equal to zero.

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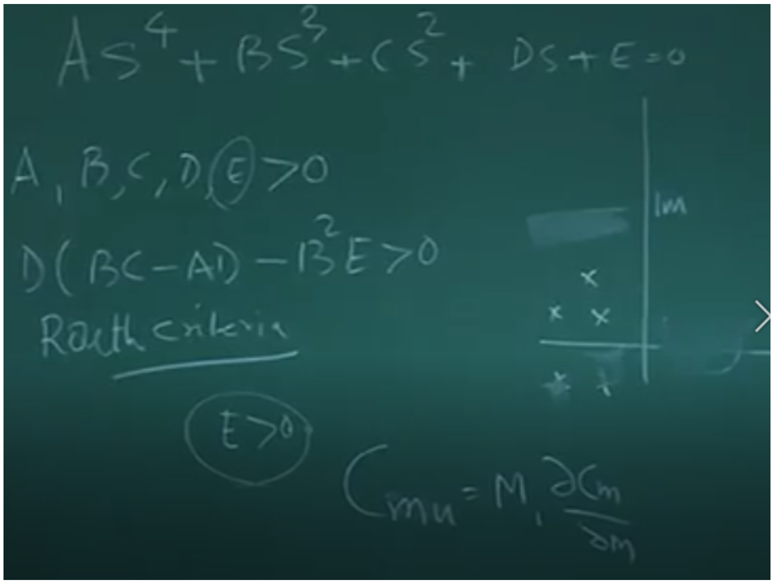


And then we got characteristic equation in linear system as b s cube plus c s square plus d s plus e equal to zero. We also studied a case based on the guidelines that each this system has anatomically stable that means low roots of this equation will lie in the right half plane of you see like this is imaginary this is real no root should be here? If any root is here then it will be dynamically unstable?

So it all it will be there all the root should be on the left hand plane and for that as per criteria the condition that all the roots will be in the left half plane. Is first is A, B, C, D, E are greater than zero and then D into b c minus a d minus b square e is should be greater than zero. This is typically growth criteria. And those who are interested they can read any book we are using this condition to check the stability of the system. We have also studied one thing that we have taken simple case E greater than zero and we have obtained very important relationship that is when c m u that is d c m by d m.

Of c m u this is nothing but m one d c m by d m which is lesser than zero for subsonic case and we have seen what is impact in defining the location of c g in service neutral point. Ok, this we have done.

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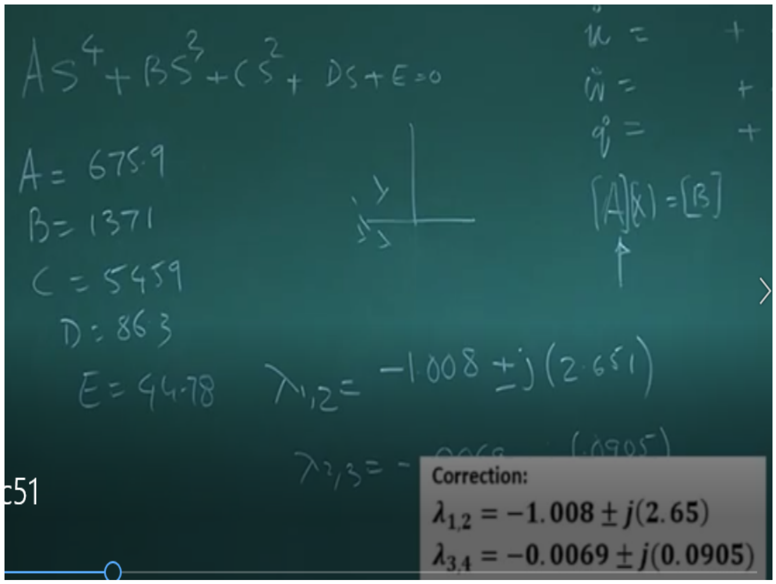


Now we a also trying to see if I agree that yes if this is will be stable. If the roots on the left half plane is some on here, and we have been talking about here roots are here so then how do I find those roots. You can find this root by solving this numerically there were short cut method to solve this equation. But what we have seen through one example were for given airplane the value of a was given 675.9, B was 1371, C was 5459, D was 86.3 and E was 44.78.

You know how to calculate this a, b, c, d, e using the dimensional derivatives or using inertia proper characteristic, mass proper characteristics and aeronomic derivative extra right. This formula I illustrate and you can refer that. What is important when we try to exactly solve this equation we found there are two pairs of root one lamda 1, 2 is minus 1.008 plus minus j into 2.651 in another pair we got this is minus 0.0069 plus minus j 0.0905.

This result tells us something more than this number what does it says. Says this fourth total equation can be product of two second order system right. That is very important and this stands valid for most of the low speed modern speed the aircraft right. So what is the meaning of this, meaning is instead of handling this fourth order equation we can think of it consist of two second order system especially I am talking about longitudinal perturbed equation of motion right, you could see what is this root is it complex conjugate complex sphere these are all complex sphere.

What is this complex sphere means is it must be oscillatory motion. And this oscillatory motion will go on going or go on reducing the amplitude. Who decide that, that is decided by the real part as long as real part is negative you know because complex conjugate which is oscillatory but says this part is negative.

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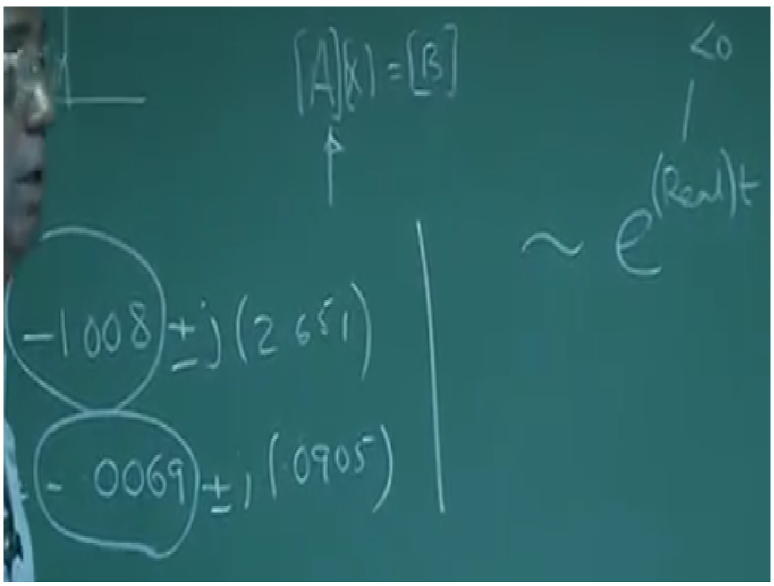


The amplitude go on decaying because the amplitude will go proportional e to the power real into t the real part has to be negative for the amplitude to decay this is the two understanding we have got. So what is it calls for now that let us see what is the second order system and how do you module second order system and how can take advantage of it. And that is exactly why I taught in a pause and go to second order system and try to see little more. Whenever we talk about second order system you understand it comes to our mind mass spring damper system.

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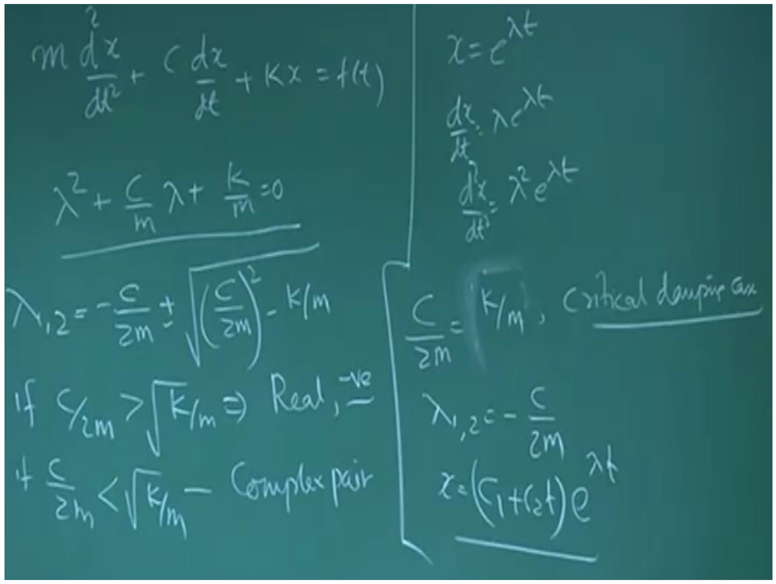

Whenever we will try I think of second order system you all know is mass spring damper system comes your mind and that we know that equation very popular equation d square s by d t square plus c d x by d t plus k x equal to f of t right, this is one dimensional motion of a mass which is having spring type and have a damper this are the popular equation. So know if I want to find the characteristic equation for this homogenous solution.

The characteristic equation is lamda square plus c by m lamda plus k by m equal to zero you are all familiar with this and divide this by m here and then for x equal to lamda t and then put the derivatives and you will get the characteristics equation. Those who are not immediately on touch though we find out by putting it will be lamda t so d x by d t will be lamda e to the power lamda t d square x by d t square will be lamba square e power lamda t so you substitute for d square x by d t square for this one put d x by d t into this one take common and then you will get the correct thrust equation ok.

And then by solving this equation for correct equation we can write lamda t is c by 2 m plus minus under root c by 2 m square minus k by m. I am sure you are familiar with this right you have done it in your some part of the course and you also know that if c by 2 m is greater than under root k by m then this will give root will be real and negative ok this is clear. If this is c by 2 m is greater than under root k by m then this lamda 1 lamda 2 roots will be real and negative and if c by 2 m is less than under root k by m then what happens then you get complex conjugates ok complex conjugate pair, and ofcourse natural question comes to our mind what happens if c by 2 m is equal to k by m.

So lamda 1 and 2 is minus c by 2 m ok and then the solution for this case will be x of t will be equal to c 1 plus c 2 t e to the power lamda t right. I focus in on this case when c by 2 m equal to k by m and the roots are negative and repetitive so roots and in the solution will be like this c 1 plus c 2 t into e into this power lamda t but you know with as the time goes to infinite this product increases but you know the point is e to power lamda t lamda is negative that decays faster.

So you come your amplitude or x goes on decreasing ok. This is typically critical damping case the difference in critical damping and over dump case which is here, this is over dump case right is that critical damping the time required to come down to equilibrium is much smaller compared to over dump case I am not talking about those things those instruction they can study it.

What I am interested here is for our sake is that how do I handle the second order equation find out its natural frequency and damping ratio.

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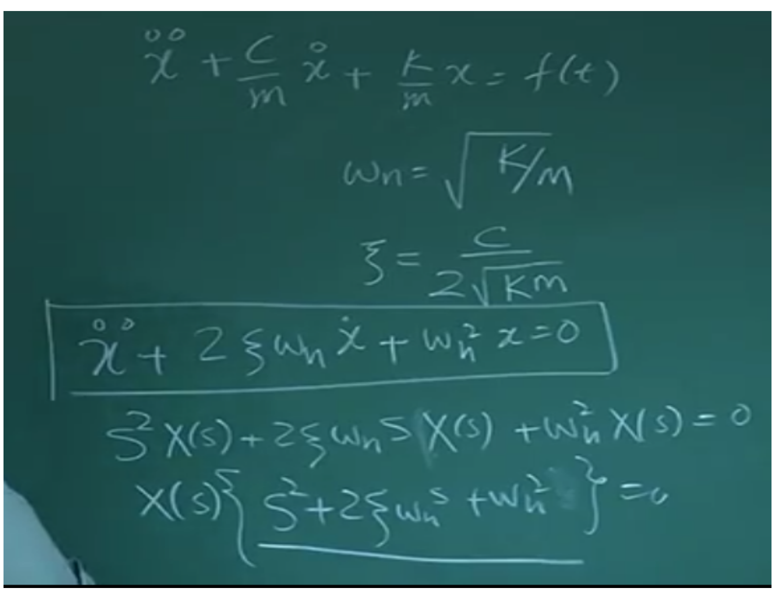


When we say we have equation x double dot which is play d square x square by d t square is c by m x dot plus k by m x equal to f of t right. Let’s say what we do is we know omega n equal to under root k by m which natural frequency which is the frequency with damping zero because in normal case if we see this shot of equation where we have written lamba 1 and lamba 2.

You can write omega here omega equal to these are all the text book material we have already studied in the c by 2 m square right. But if damping is zero, c is zero see is zero then this is becomes omega n which is nothing but under root k by m ok that is exactly what I am writing here and also we are doing one substitution that is zeta equal to c by 2 root k m ok. Zeta is the damping ratio and then when you substitute here you can write this as x double dot plus 2 zeta omega n x dot plus omega n square x is equal to zero.

See for k by m omega n has been written for c by m was written 2 zeta omega n and x dot is here and that is how I can write this equation. Once I agree with this now if I take the Laplace transform what happens it becomes s square x of s plus 2 zeta omega n s into x of s we all putting initial condition is zero plus omega n square x of s is equal to zero. So our equation become s square plus 2 zeta omega n s plus omega n square equal to zero ok.

So this equal to zero this is our characteristic equation which is second order system in Laplace transform using definition of zeta and omega n. So I thought I can be briefly but I thought somewhat in mentioning this those who are interested can read for the so what is the message?

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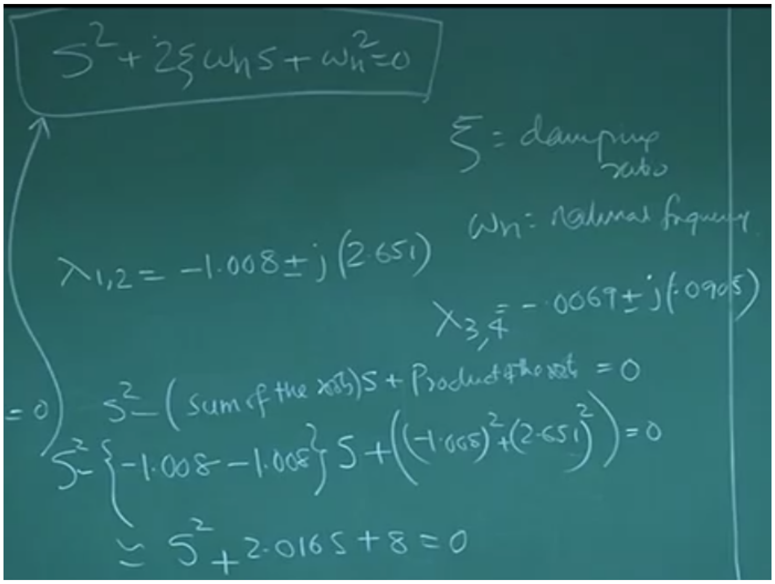


Message is let me write s square plus 2 zeta omega n s plus omega n square equal to zero is the characteristics equation for the second order system using omega n zeta and zeta is the damping ratio. Zeta is defined as damping ratio and omega n is natural frequency ok. Now we come back to our problem where regard roots has a unpaired root lambda 1, 2 is equal to minus 1.008 plus minus j 2.651 got it. The correct another of course was lambda 3, 4 is equal to minus 0.0069 plus minus j into 0.0905. This rules can be also displayed like this.

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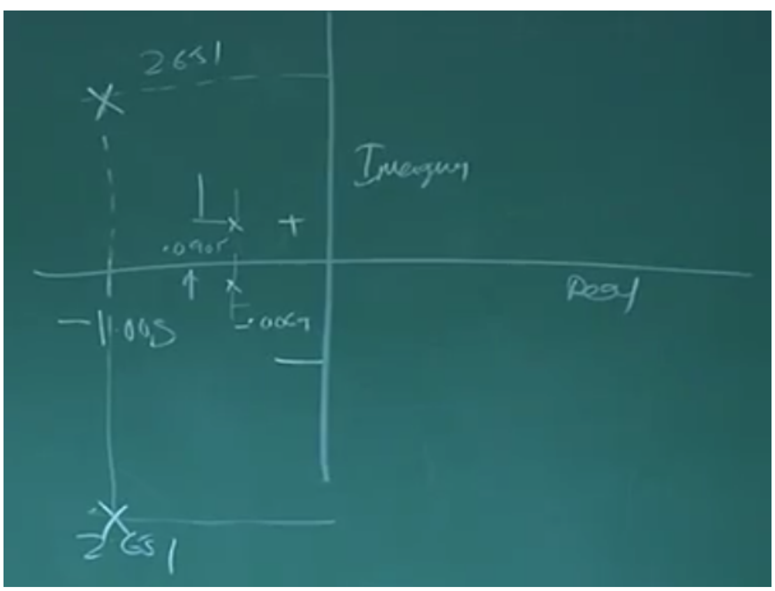

Graphically this is typically you understand that one thing for longitudinal case the solution for that a s four plus b s cube plus c square plus d s plus e is equal to zero will generally give two pair roots complex conjugate the moment we say two pair of root complex conjugate you know this is this corresponds to longitudinal case ok.

And we will find roots are also given like this. This is minus 1.008 so something like this and this is 2.651 one here one here, this is 2.651. this is imaginary this is real axis this is negative this is positive and another pair is given somewhere here where this value is how much .0069 of course minus and this much is this value is corresponding .09050 so immediately you could see this sots of roots are displayed.

You immediately know one complex conjugates one complex conjugate so there are two pairs of complex conjugates immediately you know that this is this corresponds to longitudinal case and one of the pair has larger very large negative fuel value and this is smaller negative value and you know larger negative value means they decay very fast this understanding you should have.

So know I come back here so this is this, whether given that format or this format I know second order system I can immediately form the equation. How can I form I says square minus sum of the roots into s plus product of the roots is equal to zero right. So what is the meaning of that s square minus sum of the roots, sum of the roots means minus 1.008 and this plus minus this will get cancelled so again minus 1.008 so this is into s correct plus product. Product means you will understand this minus 1.008 whole square plus 2.651 whole square this will be equal to zero.

So you will get equation of s square some numbers s some numbers equal to zero. Now you compare this with this to get zeta and omega n. If I do rough calculations I can write this as approximately s square plus 2.016 s plus 8 is equal to zero. The moment I get this equation from these two roots I will compare with this I can write immediately you could see here that I could write.

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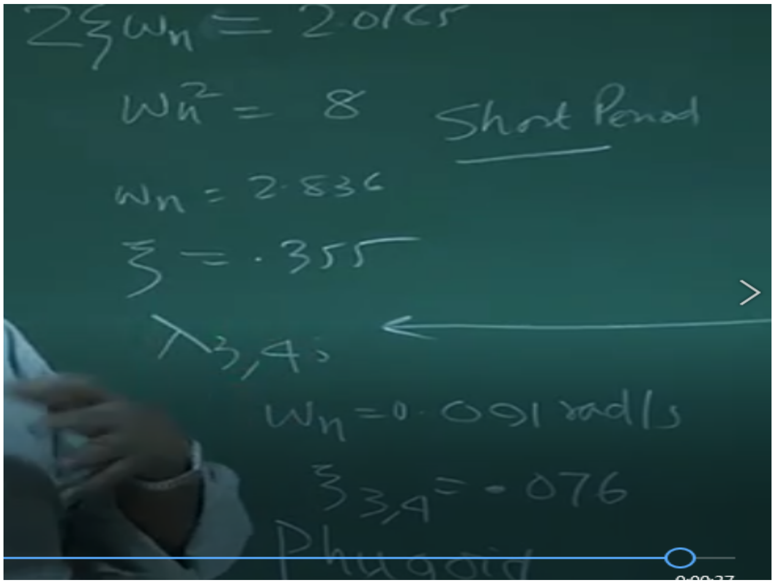


2 zeta omega n s will not be there which is equal to 2.0165 and omega n square will be equal to 8. So omega n square equal to 8 means roughly omega n will be equal to 2.836 per radium radius per second and zeta can find out this equation as .355 so simple.

So what is the message, message is we have an aircraft if you all know the geometry from that equation a s four plus b s cube plus c s square plus d s plus e is equal to zero. A, B, C, D found out from the geometry and another one derivatives then you might be solving it. Find two pair of such equation will come and you find out zeta and omega n on the understanding now you should have from this number is what is this happening in the airplane?

If I give the small disturbance what happens if I give the disturbance the two predominant aircraft can respond one is short period type that is disturbance immediately dumps out. Which real negative roots will be large that will behave like this and the roots were real negative is negative but not that large so that is something like this. So typically for the aircraft with longitudinal characteristic it is dominated by two modes one is short period mode and that mode.

We assume that things happen so fast the changing the u is negligible but in the phugoid mode long period mode it almost goes like this changing characteristic potential energy. Classical books talks about the alpha almost went constant that is the debate but lets see you understand that some shot of kinetic energy and potential energy convention is there and it takes longer time to come back to equilibrium.

Which you could see easily from second equation if again find out the root for this lamda 3, 4 if i solve that equation which is here similar way what we have done here that will get values as omega n is equal to .091 radian per second. If you solve yourself you can expect one question in the exam of this lecture .076.

You could see that zeta is 076, .355 this is highly damped compare to this and this is typically this is closer to the short period titration and this is like phugoid titration that is phugoid and this is short period. We will explain what is phugoid and short period approximations and what I thought we will solve this example and that will give lot of light you will understand what we are doing.

Don't get bogged of with big equations, this equations once derived is known and you did not remember this but you try to understand how to utilize those equation right ok. We talked about short period at phugoid we try see how this is fixed of derived short period equation motion for longitudinal case you can utilize with appropriate approximation get omega n and zeta expression for short period and phugoid mode and that we will be in the next class ok. Thank you.