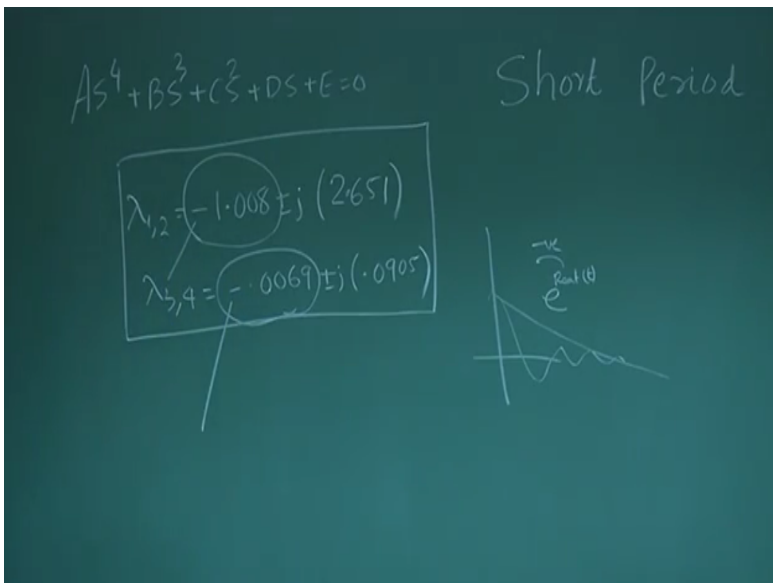
Aircraft Stability and Control
Prof. A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture-52
Short Period and Phugoid approximations

Good morning friends. We were try to understand that dynamic stability aspects of an aircraft if can see that when we will analyzing longitudinal mode and we put get the equations of the form as as4 + b s cube + c s square + d s + e = 0. And you know a, b, c, d, e all this can be computed using atomic derivative and national property extra.

(Refer Slide Time: 00:38)



And when I solve this numerically we get suggestion for typical aircraft lambda 1, lambda2= - 1.008 + minus j 2.651 as similarly – 0.00069 + minus j 0.0905 what is important here it is note is that. These rules are complex conjugate right so it tells you oscillatory motion. Whether its oscillation = decay or not. What it tells us that is decided by the real part of the root. And you can see both appear the real part is negative. That is the oscillation will dump out we all know for the second order system.

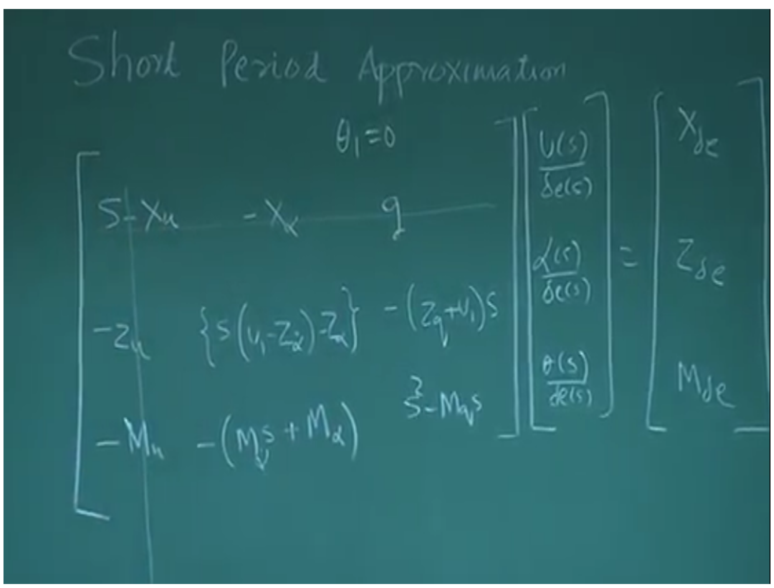
Now with another different we could see that the minus one this is 0.0069 so which one will the decay faster. This will decay faster. So this excitation all most like the second order excitation this will decay faster because you know that for a second order system. If this decay like this,

this edible is governed by like this e to the power real part into t and the real part is negative the naturally this will decay faster as compare to this right.

So What will have seen for most of this type of aircraft you find solution of this equation will result in two pairs of complex conjugate this one lambda one and lambda two lambda three and lambda four the one prior the real part is highly largely negative larger than other roots that is the real part. And physically what we understand when the aircraft is disturbed with the primarily the two modes excited one is short period that when we disturb it immediately comes back.

And it will be short period mode you can understand there no much can the velocity u okay. However this is long period is the longer time and it is at the phugoid mode and it is accelerates like this and then downs the equilibrium. So we know do a short period approximation of with understanding this all of it in background and will do the short period approximation and let me write the equation first.

(Refer Slide Time: 03:35)

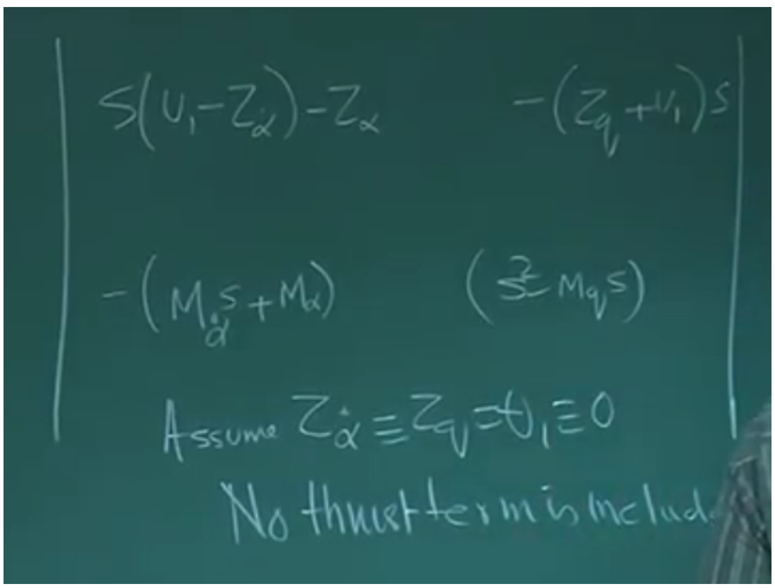


This is our standard equation s minus x u minus x alpha g and the minus zu then s and then u1g minus z alpha dot minus z alpha then minus z q + u1 into s and minus m u minus m alpha dot s + m alpha then s square minuses mqs this of course into u of s alpha of s theta of s equal to x delta e and z delta e m delta e you write it like this theta e by delta e. We should note down that we are note writing q here.

We have here understanding q is equal theta dot for small perturbation. So q alpha is nothing but s theta of s that was absolutely done. So that is fine is the part of algebra now we are almost going towards applying short period approximation, and what is the short period of the approximation that during this disturbance or during excitation you remain constant there is no change in the u perturbation right.

Perturbation u will be 0. Because you could understand this is very short period that come the very short time it comes to equilibrium so that's not a bad approximation. So why to handle all those three by three matrices, since u and u matrix is not consult for us. So you neglect this to this term first this one here we neglect this.

(Refer Slide Time: 06:02)

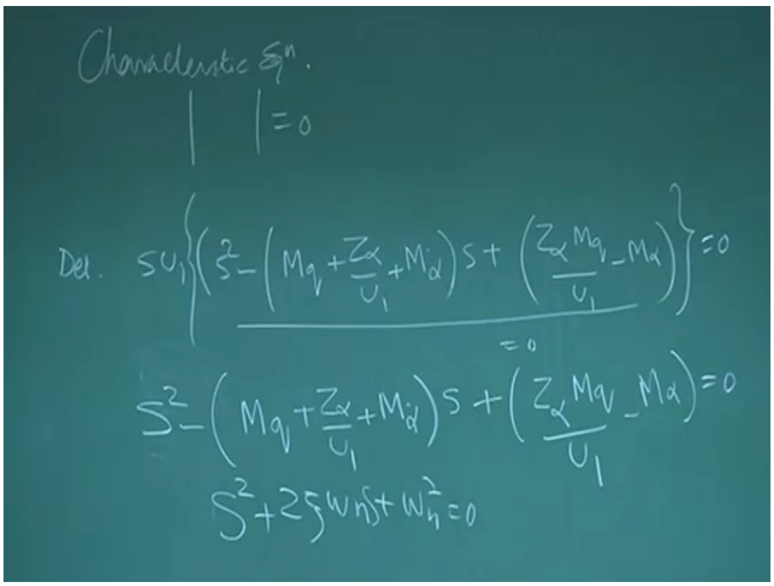


And then we get the equation from the determinant su1 minus z alpha dot minus z alpha minus z q + u 1 s minus m alpha dot s + m alpha and s square minus mqs okay this determinant matrix form get reduced to this and I am directly writing this determinant. Because I want to find the characteristic equations and I know the determinant = the 0 is the characteristic equation right. If I write that, that I get equations then which will be very you will see that very handy for designer to design the airplane.

Before I go to that we give this is g means I update theta one equal to 0 which is fine if I am doing it in steady state. And also when I do it here when I assume z alpha dot = z q and also theta one all other identical is 0 and please understand here. We have not incorporated the thrust term right so no thrust term is included because you know that if you understand this how to derive this I think it will establish no way handling the drag time okay.

So this whole structure is included please note down this. So when I write the determinant = 0 I get an equation.

(Refer Slide Time: 08:02)

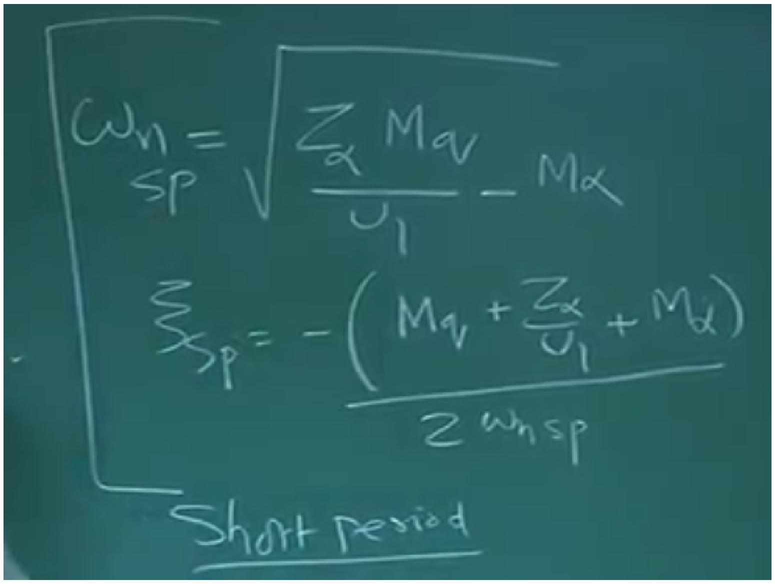


Now we are looking for the characteristic equations and that will be determinant of this = 0 and that will be give me the result something. Like this determinant is su1 s square minus m q + z alpha by u1g + m alpha dot s + z alpha m q / u1g minus m alpha that will be equal to this is on this one is here and here and this equal to 0 right. This equal to 0 means this one is 0 or this one is 0 or both are 0 from this you will get s = 0 one this whole term = 0.

s = 0 long term this one is shout of the neutral stability because the airplane whether it goes strength goes like this and like that it is that this atomic concern this is same thing for it right. So we are not take this suggestion at all and we will use this equation and then I get final characteristic equation for this short period as s square mq + z alpha / u1g + m alpha dot into s + z alpha mq / u1g minus m alpha = 0 as simple as that okay.

Could you see that when I put short period approximation we have got equations which is second order equation and it very much handy in handing this equation because you know find out damping ratios and natural frequency and that is only comparing it with this form which I already did.

(Refer Slide Time: 10:25)

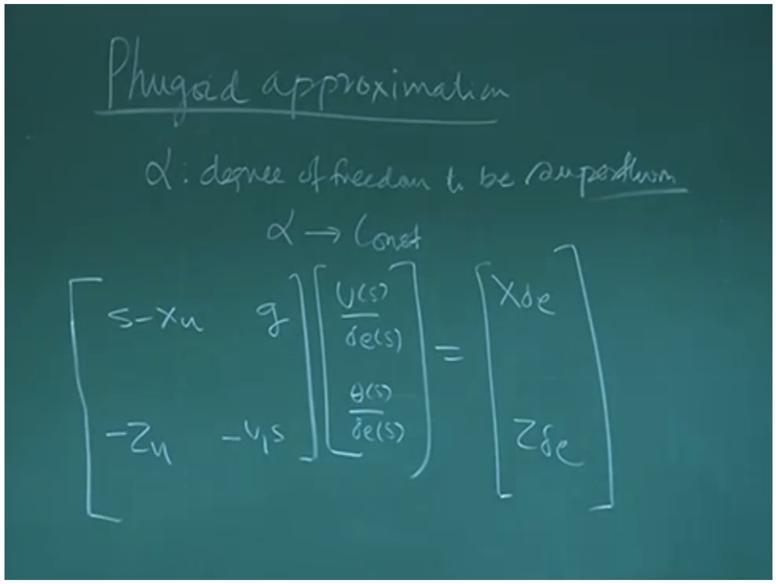


So now if I compare it what do I get. If I compare I get omega n short period equal to root z alpha m q / u1g minus m alpha and zeta short period = minus mq + z alpha / u1g + m alpha dot / 2 omega n short period. So this is basically we are getting for the short period. So I repeat it again which airplane is fly if you there is an excitation from the external disturbances. The airplanes will have a tendency should excite primarily in the short period or phugoid mode.

What is short period disturbance and it is comes like this and it downs like equilibrium through conversion of potential kinetic energy that is long period this is only about short period which is this. What is the exceptional in short period mode and how you are simplifying the equations. We assume that the short period is you will not to going change. So that the three by three matrix is primarily two by two matrix neglecting those equations right.

And we have got natural frequency for short period and type z for short period mode okay. This is very important relationship short period mode we will see I learning of SAS when operating at the initial relationship for developing the final solution. Now we come back to the phugoid approximation.

(Refer Slide Time: 12:14)

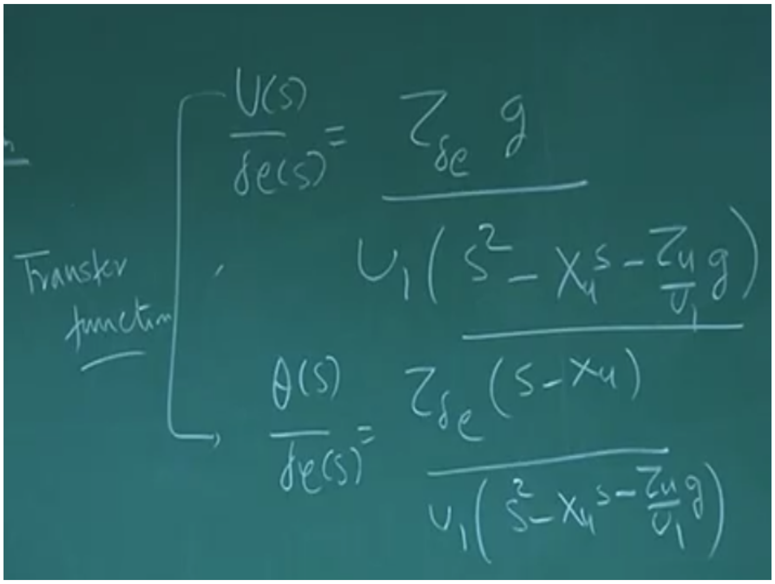


As the Phugoid approximation is concerned which is not really very great approximation but how about it gives the good result. And Phugoid approximation is the alpha degree of freedom should be superfluous but many are superfluous that. The Phugoid mode approximation which is not good approximation but it is not that bad I also and it is not worse approximation and not a very good approximation.

It gives you something in the P phugoid approximation the angle of attack perturbation angular perturbation is angle of attack is neglected okay. In the short period u was neglected right. With that alpha remaining constant will be the approximation. If you do that we will get the equation for you that is important to note down this. We can easily find out which one will be omitted. u of s / delta e(s) as zeta of s / delta of s you see no alpha is here two by two matrix = x delta e and z delta e now the characteristics equation will becomes very simplified let us see in phugoid what are the additional thing we are going to get.

Please remember this point of the dynamic stability of next to trick to as different course together we will get lambda of course for dynamic stability that's the huge. But we are giving the some favor and that ready for next course don't get disturb because of so many equation and so many interruptions of coming try make the understanding and make yourself ready.

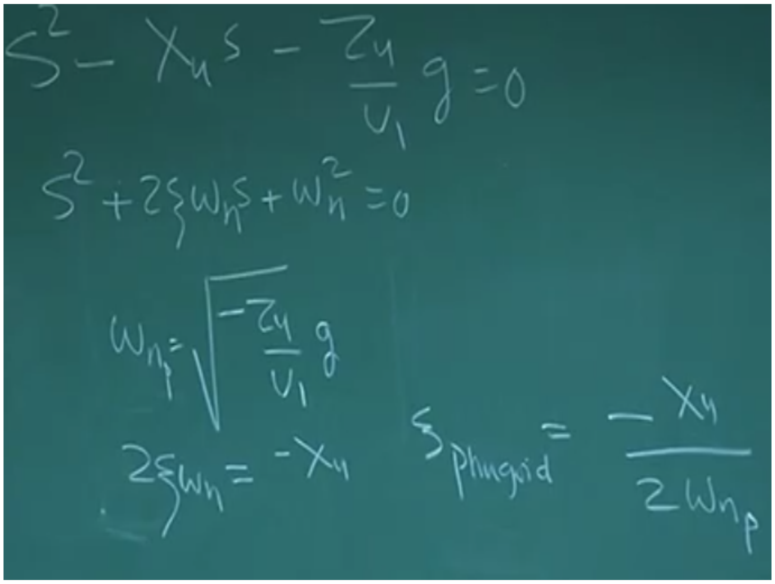
(Refer Slide Time: 14:40)



If I do this then I find from this u of s /delta e(s)= z delta e g / u1g of s square – x u s – zu/ u1g and the theta of s/ delta e of s = z delta e int s – x u / u1g int s square – x u s – zu/ u what is u of delta x and delta a is a transfer function from the short period of you know that is alpha e and delta e delta was a transfer function it was a transfer function okay.

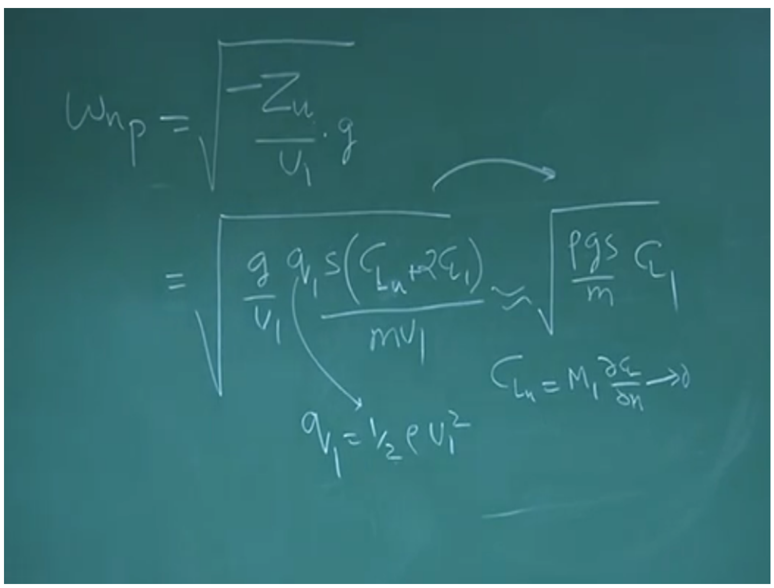
Now if I am to find the roots then what I have to do and ensure that in settlement to 0 characteristic equation.

(Refer Slide Time: 15:58)



If i to the 0 here that s square - x u s – zu/ u1g = 0 here you see the form S square + 2 delta w n s+ wn2 =0.

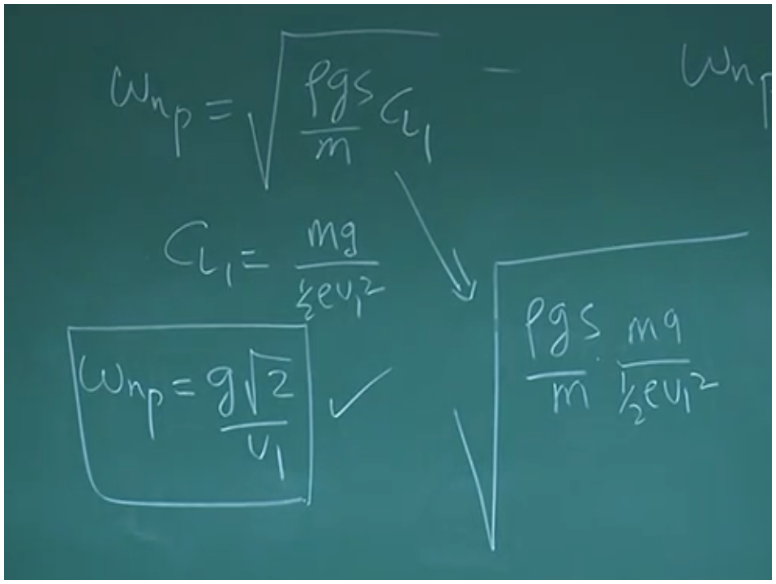
(Refer Slide Time: 17:05)



The omega n is equal is to root of – zu / v1 into g, and two zeta omega as n = – x u that is all okay. That zeta phugoid = – x u/2 omega np and this so Phugoid you know by now what is z and x u almost fresher's at it all now we will keep at up this and see that what happen let as 2 zeta omega n take and try the exact meaning of omega np = and root – zu/ u1g dot g right I for zu I got a expressions that = g / u1g q1 int c one alpha + 2 cl1 / mv one okay which I can approximately to and a root of p g s/m int c l one what has been done.

To see here c then is not a c alpha this is a cl u okay. What is approximation here let us related as cm the cln = m1 delta cl/ delta n = 0 that I am typically talking about so up it here up mark point 6 what is q = half e v1 square . q l is use this the moment are use this smart enough this get this expression okay. This is the expression how do I get from here to here your smart of q1 = half e v square is here the u1g square and u1g square is cancel the cv1 and v1 is cancel.

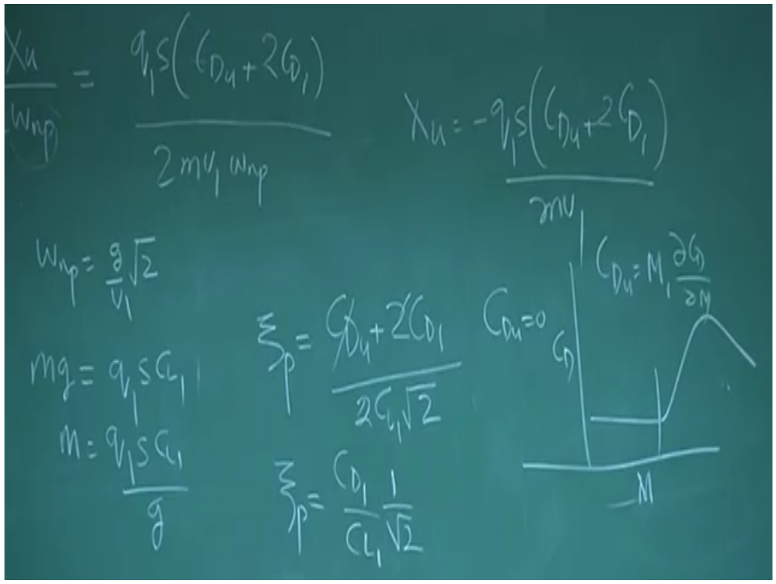
(Refer Slide Time: 18:51)



And omega np = root of pgs/m cl1 what is the cl1 the cl1 is nothing but mg / half e v1 square. To this expression omega n phugoid I s equal t as g root 2 / v1 will you see this is cl1 here under root of rho g s /m and m g / half rho v1 square**.** Will you see that 1/2 was here rho and rho will be cancel so what is the best edge here? For a phugoid mode you see that natural frequency in phugoid mode version provided to the forward velocity u1 so more u1g is less okay right.

It is very very important of the version rather you could see zeta phugoid we watch .

(Refer Slide Time: 20:18)

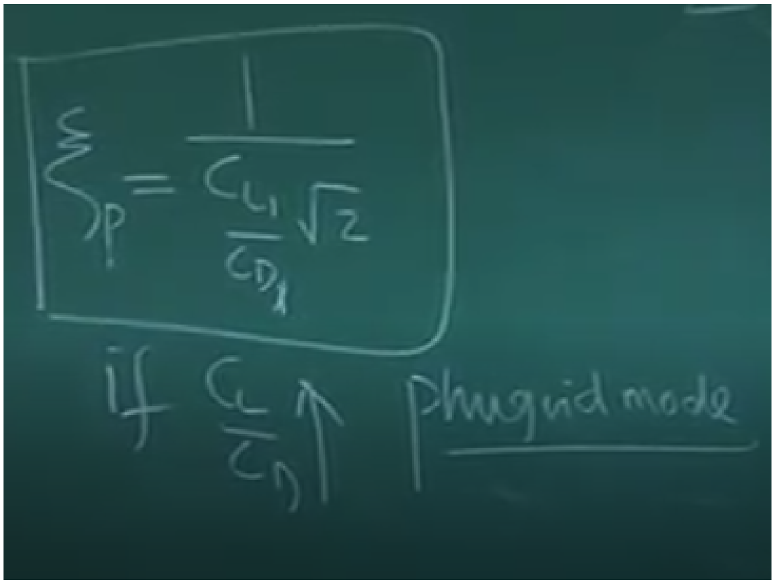


We watch the zeta phugoid is got as –xu/z wn p = q1 cdu + 2 cd1/ 2 mu1 omega np what have we done we substituted the equation for xu and what are the xu what is expression of the x u is – q1 s cdu + 2 cd1 / mu1 nothing the big deal will be done the q1 is derived as right look something big at as simple omega n = g / v1 root 2 understand one thing for beginning we understanding.

We assuming no transfer is right is all like a fall of case and I have been always telling a thrust and drag was similar nature. What is doing here similar fiction expend doing as so now omega v is substitute here and also write mg I s equal to q 1 s c l one and m = q1 s c l one / g we substitute this the omega one is here the alpha a is here this is expression I get.

The zeta p is a very important in this lesson cdu + 2 cd1 / 2 cl1 root 2 and if cu will be 0 this will be remember cd versus match number0.6/0.7 and goes like this and cdu = m1 delta c d / delta but this point is will be a 0 so assuming I can neglected is zeta p = cd1/ one and the q c1/ root two okay. These rows are canceled in this expression.

(Refer Slide Time: 23:01)



What is important of the is not a good approximation in let us do that 1/ cl1 / cd1 root two it tell you if cl and c v is large try to see that care full by doing that damping ratio for phugoid mode became weaker and weaker that is very important right okay the slide is traction time to phugoid mode and this is very very important okay. so we have finish long tutorial part is concerned it what will doing immediately go for stability augmentation system and then give only make lecture.

Lateral relational case which and send of the if for I am sure by now we will get tired and then wined it up please try derive the explanation. okay thank you very much.