Aircraft Stability and Control

Prof. A.K.Ghosh
Department Of Aerospace Engineering
Indian Institute of Technology-Kanpur

Lecture -53
Pure Pitch Motion

Good morning dear friends. Summer is almost knocking door of Kanpur you could see we have come back in t shirts of course today is cloudy. We never know after this rain again the temperature may fall and you should not be surprised if we again back with winter cloths. That is the beauty of whether. Climate can change and you were all aware that there is the huge changes in the climatic conditions and this also have the effect of how the aircraft is going to fly.

If you are flying through cloud there will be different type of respond aircraft will have for same type of control. So we need to take care of the air plane, considering what is happening outside one of the way to handling is appropriately designing the stability of the airplane and also appropriately designing the control power ok. So for we have developed six drop equation of motion we have also used Laplace transform to get the equations in frequency domain.

When we have seen for linear system if I have applied Laplace transform in those differential equation I can convert into linear algebraic equation and which are easier to solve but that is the part of computing. What is more important for us is what formation we will get from all those big big equations.

So before we go and start handling those exceptions which are developed in frequency domain where primarily we have developed equation in longitudinal motion and I have seen that it has two mode predominant mode one is short period that is if disturbance is given the short period it will come backs to the equilibrium and this phugoid mode but all most where angle of attack nearly remain constant. It is an approximation right.

But what is the important is, if they long period motion it goes like this prime convention between kinetic energy and potential energy all this we have done. Now at this point I will put hold on it. We will come back to the simple thing before again we will go back there and try to extract maximum information.

So we will take pure pitch case, pitching motion you know it is about y axis right.

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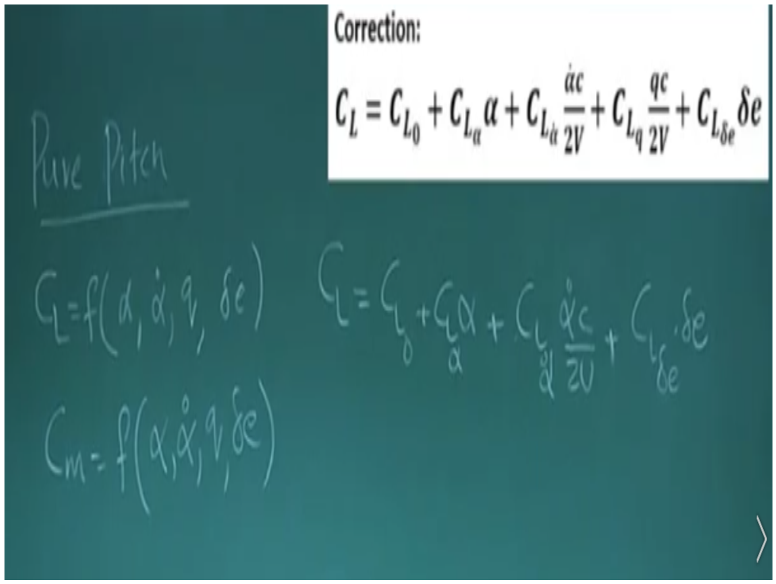

So pure pitch, and pitch you understand it is basically about x and y this is z.

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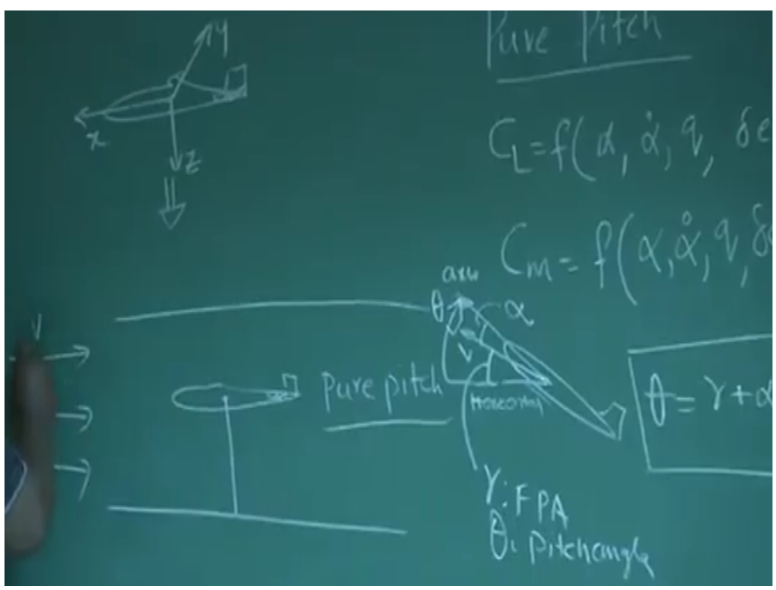

Pure pitch is about y axis and this positive pitch nose is going up and negative pitch going down like this, so it has pitch rate q positive q negative ok. And we know by now that c l is function of alpha, alpha dot, q, delta e right.

For high maneuvering airplane you can see that c l will be function of other state variables also may be their rates with controlled derivatives. Rate of controlled derivatives but we have taken simplified case where where modeling c l function of alpha, alpha dot q and delta e. Similarly c m we express the function of alpha, alpha dot q and delta e and then we have becoming expert we write c l as c l not plus c l alpha into alpha plus c l alpha dot into alpha dot c by 2 u plus c l delta e into delta e.

You know that when I write the great derivative because simply alpha dot or q they are radian per second but you want to use in non- dimensional form. So for q we have written q c by v and for alpha dot we have written alpha dot c by 2 v. Because alpha dot c by 2 v, and q c by 2 v are non dimensional. This part we are clear. Cm also we write as c m not plus c m alpha into alpha plus c m alpha dot into alpha dot, c by 2 u plus c m delta e into delta e right.

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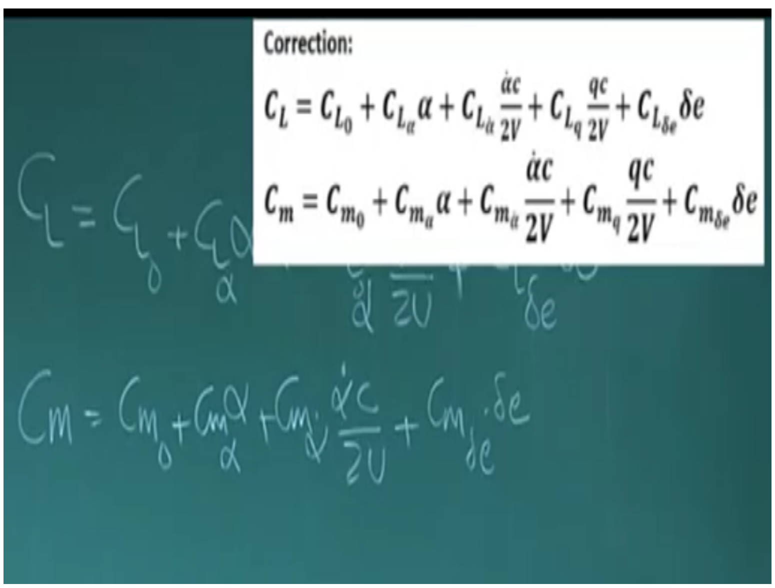

Why need talk about pure pitch. Pure pitching means it only theta is changing there is a q ok and the moment I say theta is changing it has its own effort there is the fall in airplane. If I change the theta angle of attack will change so there will be force change in force in the z direction in the x direction and the airplane center of ultimate starts moving up but here.

When we talk about pure pitch one degree of d dominant is talking about we not allowing the model to go up and down, is ogling allowing the model to do this theta variation ok this is clear. Pure pitch is no such motion no plunging is allowed its only the angular rotation about y axis is allowed but in the actual practice for an airplane if I give some shot of deflection of elevator but only theta will change it will also generate situational were because alpha change and alpha dot change it will have motion in the vertical jet direction.

That is been neglected in pure pitch only angular motion is taken into account. But typically if I try to stimulate this case I can equivalently say that is in the tunnel in the wind tunnel I have put this aircraft model right and this is this model is fixed here and bearing such that it will only allow this motion it will not allow any plunging motion. This is typically I am similarity to pure pitch. Let us see what does this mean practically.

We remember we define if this is the airplane we define this is or axis and let's say this is velocity vector and this is the horizontal right. What is this angle, this angle is velocity of vector is horizontal and that is the flight path so this is gamma the flight path angle what is this angle this is the angle between the axis cord and velocity vector this is alpha angle of attack. So what is this angle?

The angle between the axis and horizontal this is nothing but theta. Where theta is the pitch angle. Here gamma was the angle between the velocity vector and horizontal theta is angle between axis which is cord axis with theta with horizontal so that is theta and alpha is angle between velocity vector and axis ok. So this is the definition and you know from here theta equal to gamma plus alpha.

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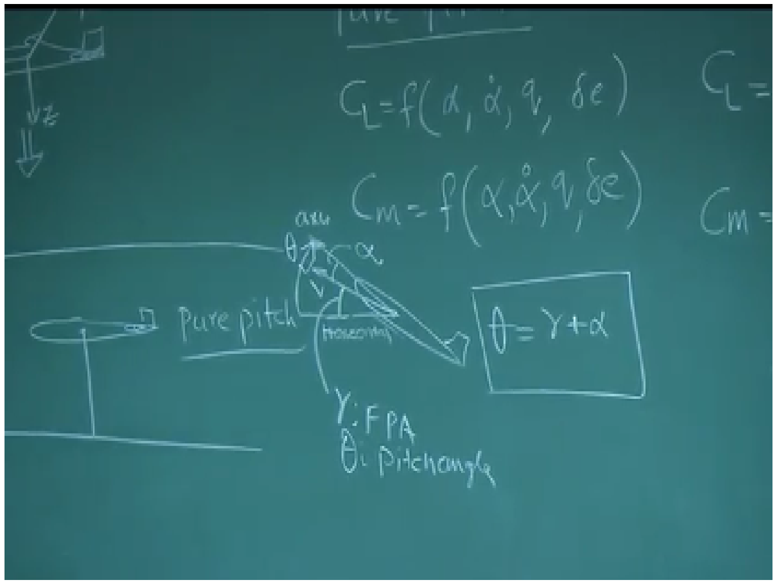

When it is actually free flight whenever there is a controlling input in which changes theta it also changes is alpha as well as gamma engine right. Because if the theta change alpha change. Alpha change moment will come the force will come to c l so that will also change the motion of the c g of the airplane along z axis so gamma will change so all this representation of this actual fight but what happens is tunnel so from the tunnel it is the velocity vector.

So velocity vector never changes direction never changes right, the relative velocity it does not change. So gamma is zero for pure pitch. Pure pitch gamma is zero implies theta is equal to alpha or theta dot roughly equal to q and equal to alpha dot. And you know in a tunnel theta dot is definitely q. Because there are no side are any such motions are there and bang angle nothing this there so I can strictly say in a tunnel theta dot is truly q and q is truly alpha dot right. Ok.

What right, so what is the motion we are trying to study is if I disturb model an air craft model in a tunnel if I give it elevated deflection I am trying to see how theta or alpha is well to vary.

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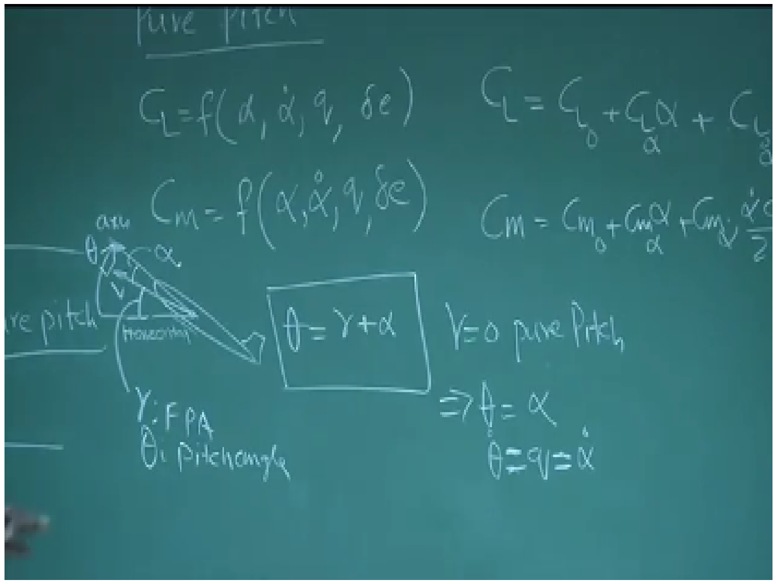

And from the response we will try to find out what is the natural frequency of this motion as well as what dumping the air craft is having right. So what we trying to find out is what is omega n and what zeta is and we are very aware of omega n and zeta. Omega n is natural frequency and zeta is the dumping ratio we have discussed in the previous lectures. So we are trying to simplify. I am trying to see that can you get some feel so that. We need not always work with big big equation as a designer.

As designer is important that we have smaller expressions, handy expressions so that we can get initial estimate ok.

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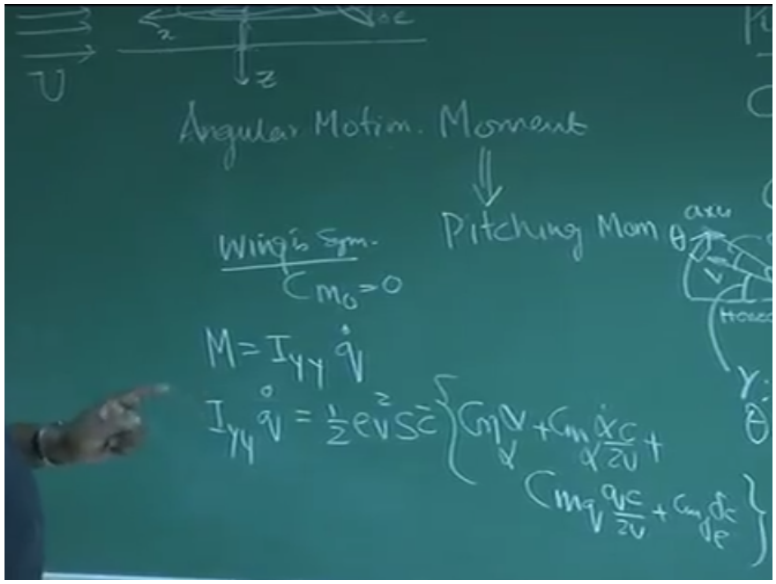

Now think of a situation if this is the tunnels and this is the model this is the elevator. Elevator down is delta e this is the c g and the wing and vertical tail here radar here ok. If I give an elevator deflection and this will close to the cross ten velocity v or it's a u are simplify, then how to write the equation of motion? We are talking about the angular motion. So angular motion so it is the moment that will decide. What is this moment? This is the pitching moment and this pitching moment is about what axis?

This is about the y axis. So this is x this is y and this is z we are talking about pitching motion about y axis we are also putting a condition when we are talking about small disturbance. Why smaller disturbance if I give larger disturbance then it will be very difficult to expand c l as linear combination of all this.

Because that time, the flow and the anatomic become nonlinear. So one condition also your putting small disturbance and that was the basis of small disturbance theory whatever we have developed earlier which we are not variation from that also you know assuming that wing is symmetric and for simplicity we take c m not is equal to zero. It is just for algebraic simplicity.

And now see that we are talking about moment and pitching moment you know this moment is equal to i y y into q dot because we all are talking about this motion there are no other thing coming from u r 8 or roll rate right. Its only pure pitch so I can write m is equal to I y y into q dot and what is m I know I y y q dot equal to moment is half rho v square s c bar into c m alpha into alpha plus c m alpha dot into alpha dot c by 2 u plus c m q into q c by 2 u plus c m delta e into delta e correct note that here c m not a put zero only for algebraic simplicity.

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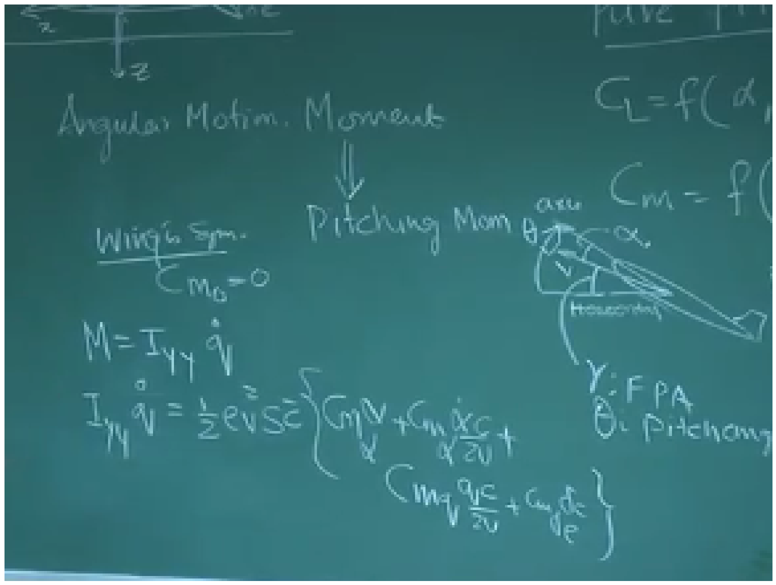

You can push cm not and go on doing any analysis. So let me erase this part.

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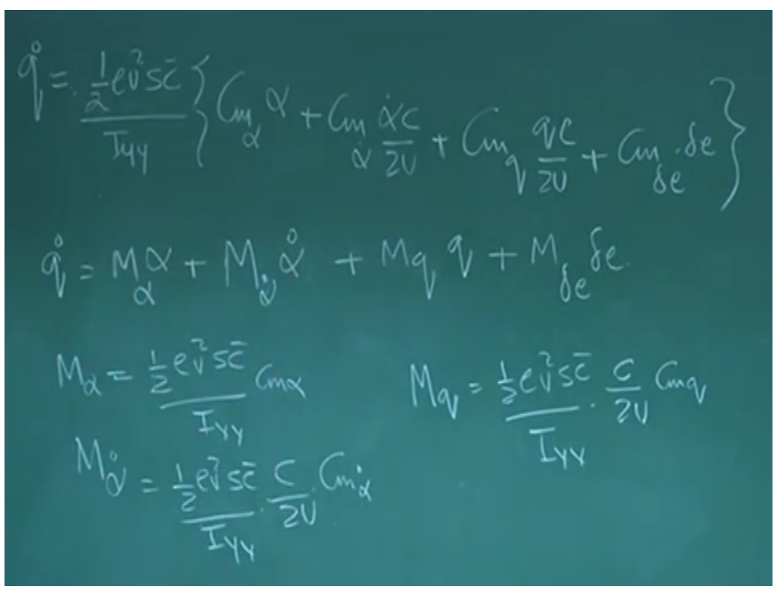

So using this equation I can write q dot equal to half rho v square s c bar by I y y into c m alpha into alpha plus c m alpha dot into alpha dot c by 2 u plus c m q into q c by 2 u plus c m delta e into delta e right. So I can further write simplified, that is q dot equal to m alpha into alpha m alpha dot into alpha dot plus m q into q plus m into delta e into delta e.

What I have done I have clubbed this half rho v square s c bar I y y into c m alpha as m alpha so I have done m alpha is equal to half rho v square s c bar by I y y, c m alpha so what is m alpha dot, m alpha dot is nothing but again half rho v square s c bar by I y y into c by 2 v up to u into c m alpha dot correct. Half rho v squared is c by I y y c by 2 u c m alpha dot that because my m alpha dot. So what is m q then? M q is come here again that is half rho v square s c bar by I y y into c by 2 u into c m q. correct.

You can simplify this by doing some perturbed algebraic equation then v square and v cancel and becomes one v that I leave it to you so that it is now easy for you to know what is m delta e. m delta e is nothing but half rho v square s c by I y y into c m delta e right. So you allow me to write q dot as this where this m alpha, m alpha dot m q and m delta e there refers to as dimensional derivatives.

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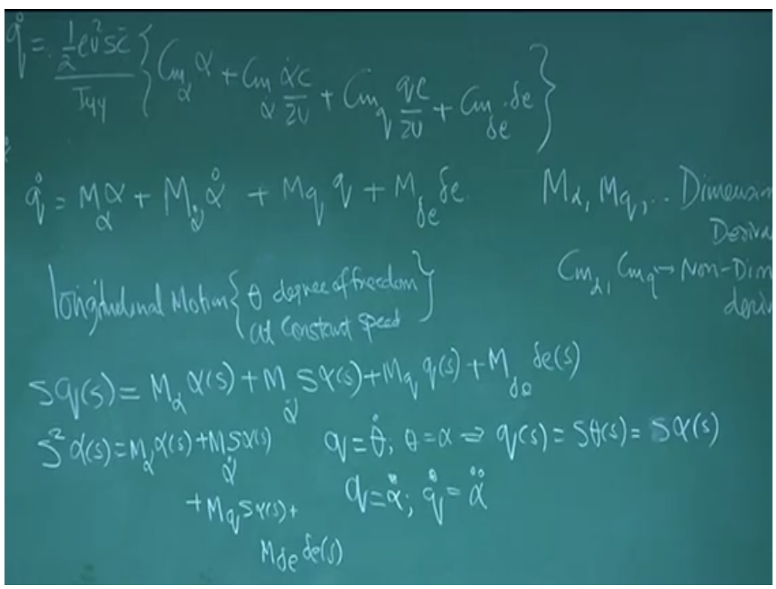

So m alpha m q extra are known as dimensional derivatives in contract if I write c m alpha and c m q they are non dimensional derivatives. It is obvious that if you see the dimension of m alpha we will find yes indeed it is it has the dimension ok, we can check yourself by putting all this dimension. So we are writing pure pitch equation in longitudinal motion so pure pitch means we taking about longitudinal motion restricted only theta degree of freedom so no plunging motion is allowed and also we are simplicity assuming it is at constant speed right from velocity because we are stimulating a case of pure pitch which is easily visible connected in a tunnel.

But pure pitch easily cannot be visible it in a open atmosphere because moment you give some deflection there will be plunging motion as well right. But there is a different meaning for while we are looking for pure pitch. You see that if the plunging motion is not much then lot of information. I get from this equation simple equation in terms of natural frequency of that mode as well as the dumping ratio that is why we are doing this.

So now I take the Laplace transform with the initial condition zero and we know all this think very well when we are talking about the linear system, and it does not depend up on initial condition so I put initial condition is zero it doesn't change anything we have discussed this last lectures so I write this s q s equal to m alpha into alpha s plus m alpha dot into s alpha s plus m q into q of s plus m delta e into delta e of s.

And you know very well that alpha s is the Laplace transform of alpha which is time domain right. You know that we have taken the Laplace transform here this equation was in time domain were converting into frequency domain to the Laplace transform and you could easily see that this was facial equation and this will become algebraic equation there is a ease of no other operation.

Now here we will do a little trick what we will do we know q is equal to theta dot for pure pitch and theta is equal to alpha for pure pitch because if define the angular motion like this whatever theta is that is on the alpha isn't it this is alpha zero theta zero. Theta like this theta this angle between the axis and the horizontal becomes theta but similarly the angle between the velocity vectors in the axis become alpha so it is theta equal to alpha.

So again you can check it if this is the tunnel if I have moved the model like this then this is my axis chord line and velocity vector is like this so this is alpha angle between velocity vector chord line ok and also you know theta is the angle between the axis and the horizontal and incidentally velocity vector horizontal are same the parallel so this becomes theta also wind tunnel in similar case.

Which indirectly means if gamma is zero? Theta is equal to gamma plus alpha it says if gamma is zero that implies theta equal to alpha and if gamma dot is zero this implies theta equal to alpha dot. What is the physical meaning is if the disturbance is so small that it doesn't really create large plunging motion then this approximation will work very well to get some relevant understanding about natural frequency and dumping ratio which is a our target so you are not very far off from here.

We are talking about smaller perturbation and that is where this simple equation has got so much of meaning right for a designer ok. So you come here so now from this relationship I can write q is equal to alpha dot and q dot equal to alpha double dot right I repeat here. Since q theta is equal to alpha and q is theta dot q is equal to alpha dot and q dot is equal to alpha double dot. Why you do want q dot because you could see here q dot was here.

If I take substituted alpha double dot you would have got s square alpha s right. Anyways many ways of doing but you should be clear about this understanding and here I will write the hole equation into alpha s so this will be s for q s I can write from here I can write q of s equal to s theta of s that is equal to theta of s is alpha of s. I can write s alpha of s it is clear. q equal to theta dot so q of s is equal to s theta s and theta s is s alpha s.

So what happens here for q s I write s alpha s so it become s square alpha s equal to m alpha into alpha of s plus m alpha dot it already exist there. So this is alpha s for m q I write plus m q for q of s I can write s alpha of s plus m delta e into delta e of s that's all ok. Let me write it again so you can understand because I could see so many trumps going here and there.

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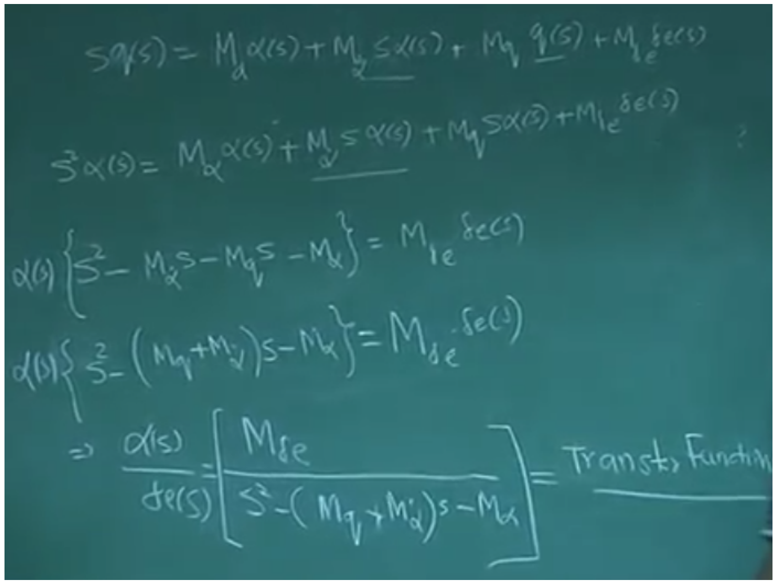

Let us write it again let me write like this s alpha of s is equal to m alpha into alpha of s no problem plus m alpha dot into s alpha of s no problem here plus m q into q of s then m delta e into delta e of s right. But we know that q of s already we have seen q of s is equal to s alpha of s then only it is customized using it as small cap but clarity I am writing like this.

So what you get. We get, this is not alpha s please understand this will be q of s ok. s, q of s come back here s q of s is m alpha into alpha s, m alpha dot into s alpha of s plus m q into q of s plus m delta into delta of s. but what you know q of s is equal to s alpha s right. Simply I put there. So what will happen?

This will become s square alpha of s because q s is s alpha s. s into s becomes s square alpha s is sitting pretty with m alpha s then this m alpha dot s alpha s and then for m q I will write m q and what is q of s. q of s is s alpha s so again I write here s alpha of s plus m delta e into delta e of s that is the expansion.

If I now do one trick I take this term left hand side then what I get I get s square minus m alpha dot s minus m q s. s square minus m alpha dot s this is s square so m alpha dot s alpha s. So I try to take alpha s common ok, so for this s square for m alpha dot I bring it here so this will be s m alpha dot m q I am taking here so it will be m q s ok. Now m alpha I take it here minus m alpha this equal to m delta e into delta e of s right.

Let me write it differently now so I write it as s square minus m q plus m alpha dot s minus m alpha this equal to m delta e into delta e of s let us not forget it again, what are this m alpha dot m q m alpha alpha m delta e they are basically dimensional derivative when I think of m delta e immediately I know the non dimensional derivative that is c m delta e that is what is the increase the c m because of elevator deflection was sign as negative.

And that is this is the dimensional c m delta is non-dimensional right, so you know this expressions so now I can write this as here alpha s has to come ok. So I can write as alpha s by delta e of s is equal to m delta e by s square minus m q plus m alpha dot s minus m alpha ok. We have seen almost similar term in complex manner in our six stuff question this is the special case of six stuff question for motion what you will do in the six stuff stability matrix we will put those simplification so that it results in the similar expression for pure pitch.

So before we go to those exercises we trying to understand what is the pure pitch and how does it help us. Now what is this alpha s by delta e s this is conventionally known as transfer function ok. Now if you want to find out natural frequency and dumping ratio for pure pitch longitudinal motion because this is very very straight forward you know that.

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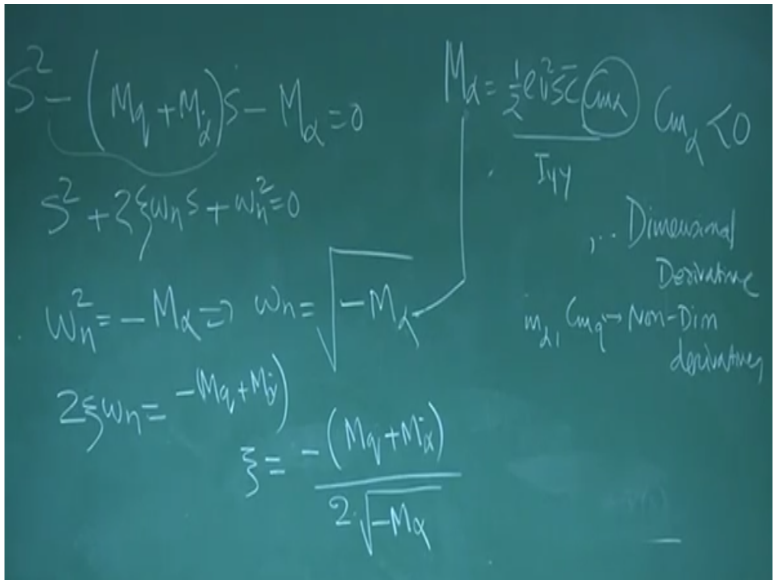

The Correctest equation is s square minus m q plus m alpha dot, s minus m alpha equal to zero right. This is the greatest equation s square minus m q plus m alpha dot s minus m alpha equal to zero and I compare it to it second order system s square plus 2 zeta omega n s plus omega n square equal to zero if I compare when I get omega n square equal minus m alpha and 2 zeta omega n equal to minus m q plus m alpha dot because this is 2 zeta omega s ok.

So what is the natural frequency, natural frequency is on the root of minus m alpha and you could always see zeta will be minus m q plus m alpha dot right divided by 2 into omega n which is nothing but minus m alpha what a neat expression right. You may wonder this omega will become complex is true. No why because if c m alpha what is the special of m alpha nothing but half rho v square s c bar c m alpha by I y y.

That is m alpha now for statically stable airplane c m alpha is negative sign is o, m alpha sign is negative so this negative when you push negative will be positive so omega will get a positive number so no confusion or no conflict ok. This is the very simple way of getting handling a pure pitch motion through modeling and we check when we solving six stuff equation motion and we will be getting solution for some motion closure to pure pitch how much they come close and that we can take a decision especially at the design stage don't want to solve big big equation.

Because you don't have complete information so this shot of understanding makes you very very comfortable because you know that geta I want may be .6 or .5 immediately we can see what the m q is required what is the m alpha required if I want natural frequency of 13 hertz or 10 hertz 2 hertz.

I know what shot of configuration required for me and quickly I can find out those derivatives, but understand this whole is for motion longitudinal motion angular motion right. In aircraft we find dumping and natural frequency of different modes required ok it only address this which you will see fairly close to the short period excitation.

Today we have just made an attempt to understand pure pitch motion and how to compute natural frequency and dumping ratio and you will see that how handy is this when you will be getting information relating natural frequency and dumping ratio you would have six stuff question motion in precaution of remain. And you will also appreciate how wonderful these approximations are for a designer and designer life becomes very comfortable otherwise we want to handle with big matrix it is not like that.

To start with when I try to configure it I was simple things I should be have to feel I should be smell the number not the way for the computer to give me some numbers in decimals. No that's not designer look for. Designers look for pure understanding good approximations relevant approximations and get conctrete number ok. So this I am just ending we will come back to this again right you see how beautifully this can be used for designing a stability argumentation system which is life line for all control systems. Thank you very much.