Aircraft Stability And Control
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Lecture – 55
Lateral-Directional Motion

Good afternoon friends, we have come to the last lecture and that is brief introduction to lateral-Directional perturbed equation of motion and stability analysis in particular for dynamic stability analysis you are now aware how we derive perturbed equation of motion for longitudinal case what was done we pick up the equations of motion one new direction double direction and the pitching movement equations.

Then we introduce small perturbation and then give simplification assuming that small perturbation and steady state some condition are true. The automatically satisfy and also product up to perturbed quantity are negligible enough and we neglected that and you got equation solution, and then we have different stage by how do I module perturbed aerodynamic force fz for that we derive at machine at the mechanism don't you finally you got a matrix and when you took the determinant =0 which was like characteristic equation.

We found equation in form a x 4 plus b and plus a square dx plus equal to 0 and then we find out the roots and from there we try to interpret the short period mode and the phugoid mode and try to find out the natural frequencies and time deviation. We will be doing similar thing let us directional case but the different please understand one thing. By the way first of all please understand that when I say lateral motion is.

Motion about x axis and right wing goes down is a positive back ok and the directional the right wing going back this way ok so laterally right wing going down and the right wing is going like this flying this direction so this are two modes of the coupled for simple reason to understand if a given a role disturbance like this like this then you could see that lift will be more compared to this.

That is because this is thinking a bank as soon as lift here is more the drag here will be more it not only bank like this. You also like this the similar think is more that the opposite case so it is lateral case and directional case are coupled and that is why we will taken this equation is the motion along the y direction.

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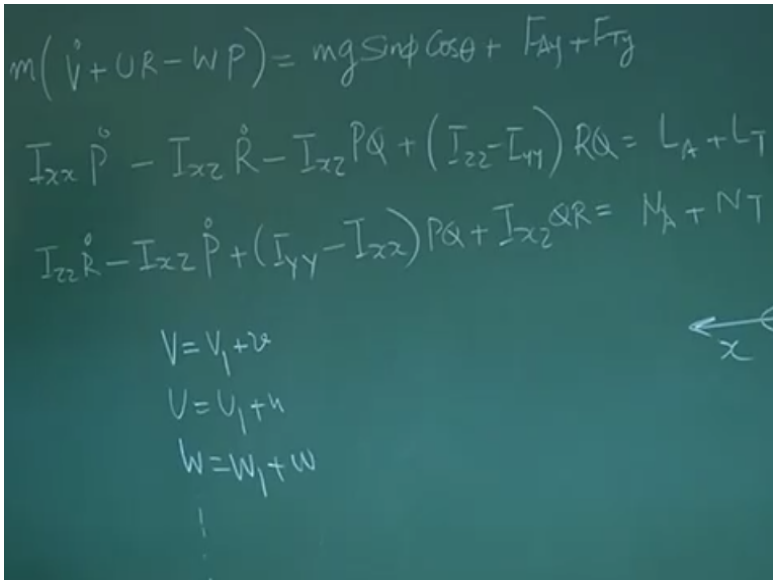

What was y I recall if I recall the y direction was .this is the x direction this is y and this is z. And this is motion along y direction the ixp.

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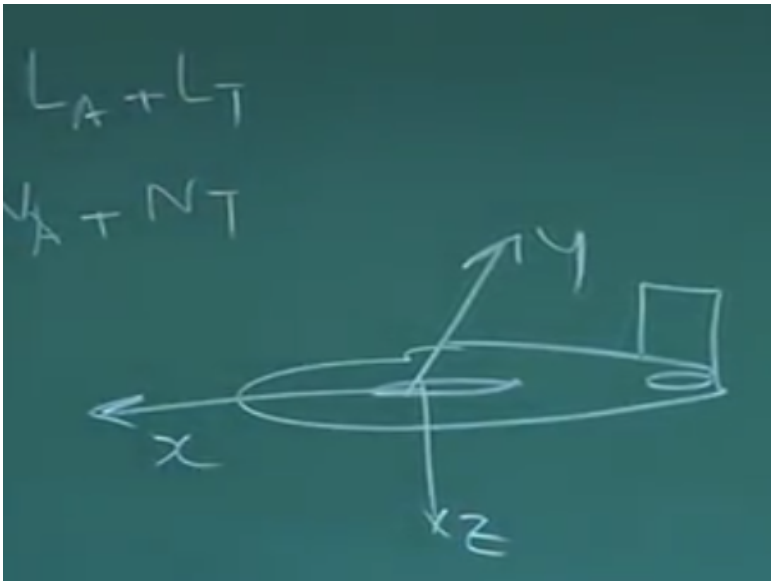

This is rolling movement about the x arises. And this is about the z arises ok these are three equations will be picking up and try introduces perturbation at get it last time for longitudinal case it is very simple v is will be written as v1 and at the step as a equal to v1+v. u is equal to the u will be written as u1.and the small u. U1+ n and here will assume that there may be not much change in the u ok.

But it is doesn't matter we will put this u we find that all the taken care all the perturbed equation w=w1+w like this. We are subsisting here and then by doing this substitution and exactly following this what are follow would longitudinal case will be get perturbed equation motion.

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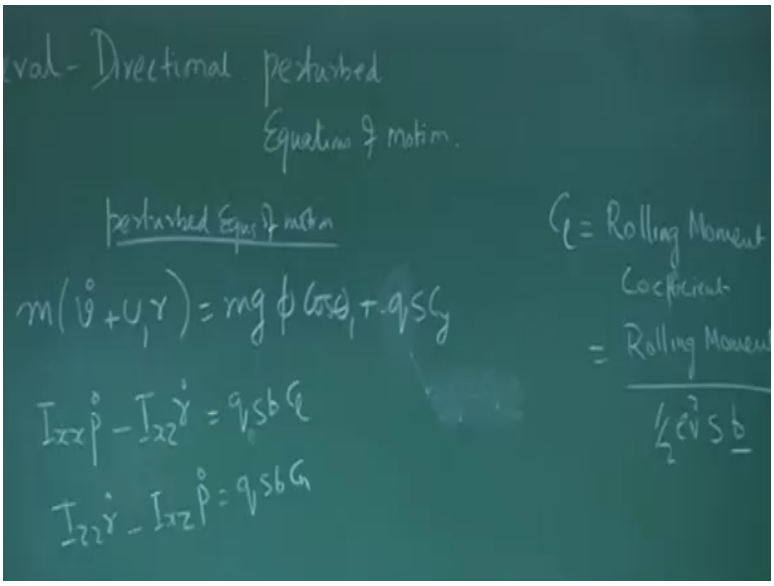

As m (v dot +u1 r) = mg phi zero not phi cos theta +fy similarly I will got I x x p dot minus ixzr dot is equal to q1 sb cl and the I Z Z r dot-Ix Z dot p dot=q s b into cn .Please come back here if we see the right hand side if I write fay this is the aerodynamic force along the y direction and fty is the propulsive force which for the our case is related this.

Because we understand if I put do this if only a fay and the fall and fan and the exchange the additional term delta on right we have taken to this is zero are not considered second thing in this perturbed equation of motion and this is the perturbed equation of motion in lateral motion case what is v the perturbed r is a ya rate right about z direction the p is perturb quantity and what is fy here fy will be simply qs int c y right ok.

Half rho square is most efficient only here it is rolling movement but perturbed equation q sb and cl is by the lecture movement is equal to cl know by now that cl means rolling movement coefficient. It is defined as rolling movement divided by 1/2 rho v square s b please note that directional and lateral case and no dimensional conventionally used as span and not the chord by using longitudinal case.

So this is the perturbed equation of motion before I got detail in to it each understand this just give your glance do not forget it ok. We need to have a separate dynamic stability is exact as I thought I complete this the net shade you get enough interest and you find that the equipment no

why different longitudinal case ok. Now here see fy will I have cy as valued here which is ideal it cl ideal it cn .

If I write it here no more write fy for your understanding cy and your understanding force is perturbed force and the perturbed rolling moving movement no problem.

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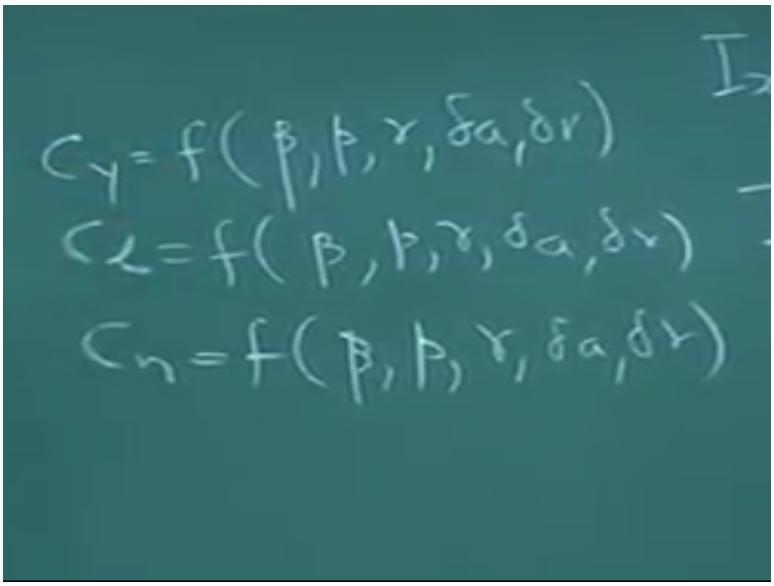

The CY is function of beta let be write this p and r delta e delta r minimum. The other function active is here complicated airplane and the normal airplane I you can write it like this and I explain you how this Armstrong is ok. Let us take as this as an airplane ok and you know this si the x transaction this is the y transaction ok. I am writing cy side force function of beta which is obvious the airplane is flying like this and it the wind is coming this direction is the positive beta vertical tail force in this direction.

So you have CY because of beta it is clear or not this is the beta vertical tail is there is force in generated. Will generate alone negative by direction is cy cans on of beta now the question is cy fraction of p let us see we are try to visualize with the cy the function of p or not . What is p? The p is rolled it the right wing going down like this please note down as the right wing is going down the vertical tail is also going down ok.

This is pushing the air in this direction there is also a push in this direction ok. The second thing is also understand that as it is rotating the rolling with p there is the relative air speed this vertical tail is that it will be see. That will be a p in to the distance. Whatever point to I saying that velocity and it is moving forward to actually give a beta so we could see for simple common if it is rolling like this the vertical tail will pushing the air this way how to experience force in the negative y direction it now we agree function of p also now for r what is r.

The r is a e rate, what is e rate? The e rate is it is like this the right wing is going back. Now you could to see as doing like this the vertical tail was a pushing the air in this direction so the express focus in side so again because a e r rate the force expresses direction the airplane is the y direction and the cy the y direction it function of all delta r it is obvious if I deflect the radar like this there will be a cy in the function of r.

As well as delta e concern you can neglect for as cy is concern this is very small and primary will be b delta b and delta r as you understand the cy there is rolling movement the primary contributed cy is the vertical tail for example whether there will be a rolling movement beta are not see beta is vertical tail so the force will action this direction and force in to distance of from center line will be rolling movement right there will be .

The rolling movement because of beta also you know that we there is high wing there will be cl beta and negative and the cl beta negative al thought thing will be there understood what about the p to see if I role the airplane like this wing it have additional air was here so there will be a force in the direction and give the rolling movement of these direction that is was damping nature so cl also function up p.

It also function up r could see that put airplane to that you right like this vertical tail is pushing the air in this direction experience a force in this direction and this force into this distance and give a rolling movement. So cl is also function of r version of r and you know c anywhere in delta e in the earlier on naturally the rolling movement is function in on. And the rolling movement is in the delta r that also you understand.

Similarly you can understand the for cn as a similar direction as long as you want understand that cy cl and cn function all the function motion variable and the control for life is simple as I can write I can the right hand side let take one example.

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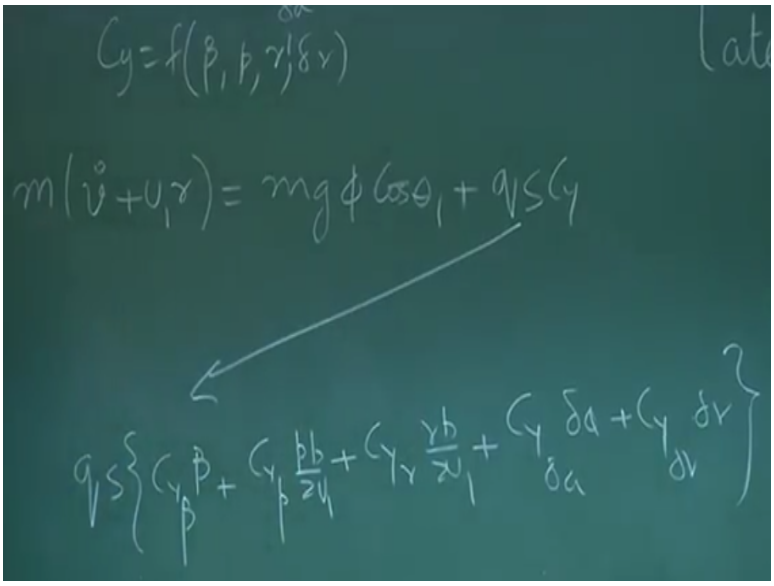

Let m v dot plus v 1 r the perturbed equations motion equal to mg phi cos theta one+q s cy the q s and cy and I write q s into cy beta into beta because I know. Cy is function of beta, p, r, delta r ok so I see to beta to beta cy1 p int p beta/z v 1 y p v 1 that we understand now why pv by 2v1 because this is the p dimensional q is the one dimensional and that same on the long dimensional for q one alpha dot.

So I write like this cyr int r b/z u 1+ cy delta a and delta a plus cy delta r into delta r although neglect for complete let me write this also and this is what basically here for this now. I divide every side in m and I can get the neat equation the from which will you like.

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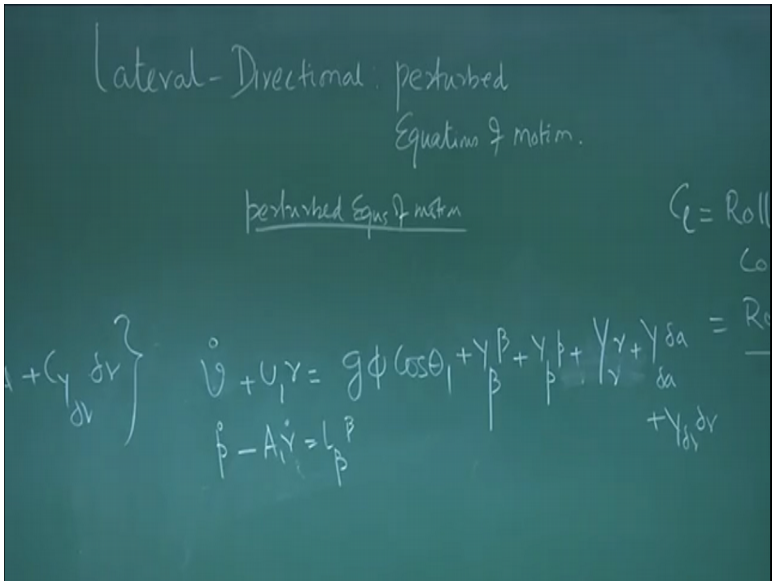

That all the dames will be nothing for lateral directional also P dot – let be right like me a1 r not = l beta int beta it from first equation is v not plus v 1 r is equal to g not cos theta one plus y beta and beta plus y p into p plus y r into r plus y delta a int delta a plus y delta r delta r and goes what we saying what is these y bête the y beta will be q s c y beta divided by l that is all with the dumas are nothing but the long divisional case also similar I will get.

The P not minus a one and let me write this r not is equal to l beta and beta let me write neater

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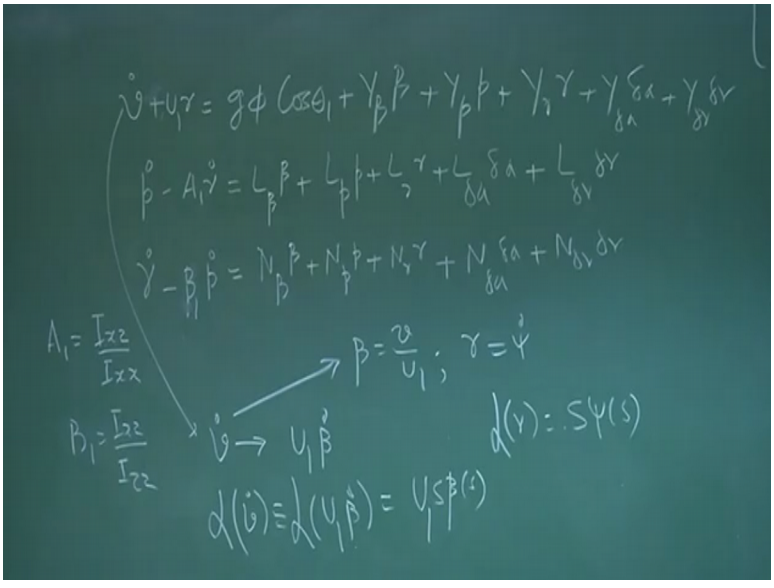

The first equation is The v dot plus u one r is equal to g five cos theta one plus y beta int beta plus y p int p plus y r int r plus y delta a int delta a plus y delta r int delta r similarly I can write p

dot minus a one r dot is equal to l beta int beta plus l p int p plus lr int r plus l delta a int delta plus l delta r int delta r similarly R dot minus p one into p dot is equal to n beta int beta plus n p int plus n r int r plus n delta a plus l delta a plus n delta r int delta r.

Were it is c from understanding and derive the expression find that just for a simplification, A1 is not but is for a simplification equal to ix square/ I xx, and B1 is equal to I x square / I x z/I z, z this is matter of I told of the you to that the last lecture just only the interactional apprise could you see that the similar thing for the approach will we see the similar thing we will see the longitudinal case what was the next the next will be Laplace transform everywhere all the three equation, also remember that beta is v / u1 that also is remember; r= slide dot.

if we take the last lecture transform here v dot that the Laplace transform v dot what I do the first lateral direction is equal to u one b dot by using this expression right now I am taking the Laplace transform tail in transform of tail , v not is equal to transform of u1, beta dot which is equal to u1 s beta of s. of as simple as that similarly if I take Laplace transform of r it will be nothing but s chi of s.

It will be nothing but and finally we will use take Laplace transform and write in the matrix form and try to see what is the characteristic and then we get the matrix as I will write it for you completion.

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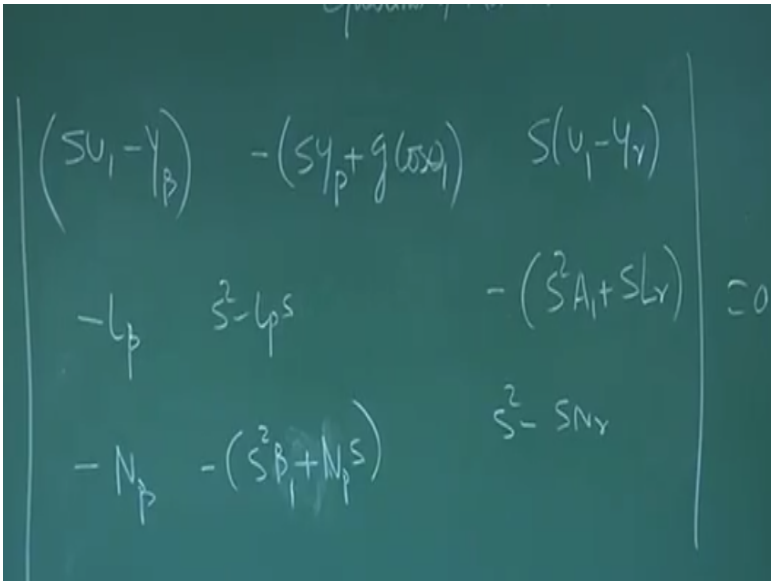

su 1 minus y beta one don't get upset that this see the so many expressions you need to derive it once but the purpose of this last lecture is given on a idea next should not difficult and follow the procedure and mechanically to happen - l p dot and s square – l p s and here it is –s square a1 plus s lr.

Here is have - n beta -s square beta 1+ n beta s here is the s square – snr is all determinant is equal to zero are the characteristic equation and here it should note that we have already use the fact Laplace form of s I of s and the find out take the determent is equal to zero and I will get equation of the from.

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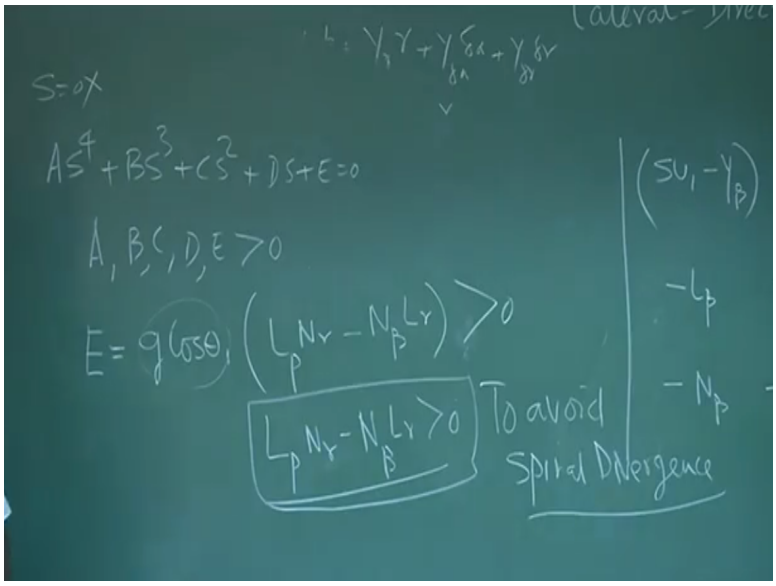

S= 0, as 4 + bs 3 +cs square + ds + e is equal to zero is get this a neutral stability type we can understand the airplane is going like this; is going like this; is going like this; size concern It is not sensitive right it is all same thing is going like this; is going like this is because all about the relative by speed ok.

It is taken out but this is the characteristic equation and again here as it did for a case we have a longitudinal case we have condition first condition is a, b , c , d , e > 0 in this second condition and this condition will be satisfied and we done for a longitudinal case to ensure that know notice in the right hand side the imaginable and the real plane because unstable here also so interesting the to see if a see the interestingly expression of the not derive here.

And do yourself and nothing to take to the determinate take . If e is something like a g cos theta 1 l p nr – n beta l r> zero right this is the expression of e got expensively terminate and write this from. This is extremely important so the theta 1 is a zero this is the positive number but the condition is l p n r – n beta l r > 0 as we see for the lecture case e greater than zero is the special actually meaning and if we see that case also e greater than zero is special meaning.

This is case also indicator zero zt the specially meaning you see that the e is not greater than zero this condition is not satisfy then the airplane is going to spiral diverse. What is spiral diverse see that the airplane like this disturbance of bank at it is bank the slide slipping as it is slide slips is turns like this s it turns like this velocity of the left will increase again in banks

so it is goes like this and a listen at tile this condition is satisfy ok. This is very important to avoid spiral divergence ok.

Now if we try to solve this equation long later motion role if find difficult most of the airplane we find there are three types of dusting roles.

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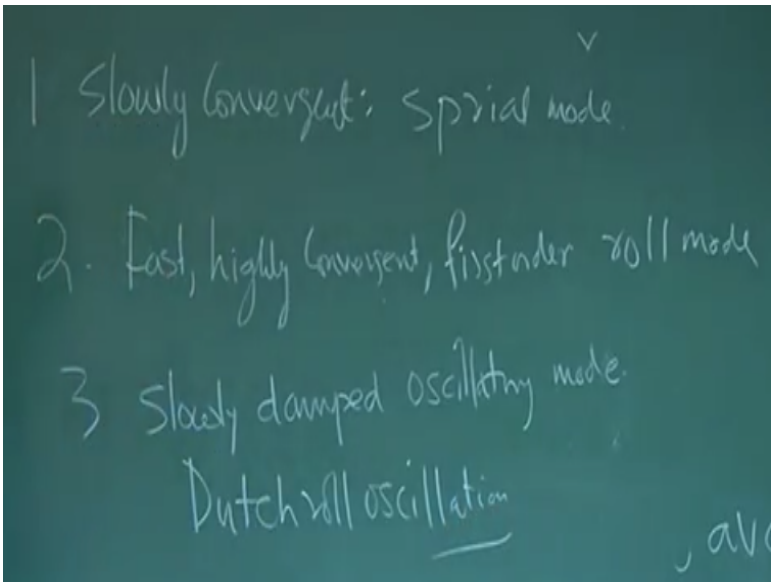

One is slowly convergent spiral mode could I explain more the number two is find very fast and highly convergent first order roll mode the third one slowly damped oscillatory mode I explain. I explain you in some manner the spiral mode will be understood the highly conversant you understand that because the huge wing that there is disturbance in bank this wing will give long of it try to comeback right ok has to damping wing and that is why it totally highly right the damping is very high.

In presents of the wing and that is role mode the third one is only damped actuality mode difficultly it is call Dutch roll oscillation is typically know it is just like this. I will be doing like this and moving like this as well as doing like this. And the one motion is like this the one combination of two motions is like this Dutch roll mode. The airplane direction is can get excited primarily I either of this mode of the combination of this mode right.

And there is why we distended the use to know what are the naturally convergent what are the time periods. The time period the similarly to the time to half two half how much time is take to one bank angle to another bank angle.

Then what is oscillation and natural frequency, natural mode to design the highly qualities when calculation ok I will told you that I will give you every short brief if an introduction for let a lateral case I knowing very well and it is ten lectures to make you understand and all of as a understand but we have decide it this dynamical separate ten hours course just to given an idea.

And without say please get a first field for this don't worry too much about examination from this part at least thank you very much I have hope you are enjoyed it a lot of expression in on but we cannot help we have to derived with this if you have any problem do write me a mail. Thank you very much.