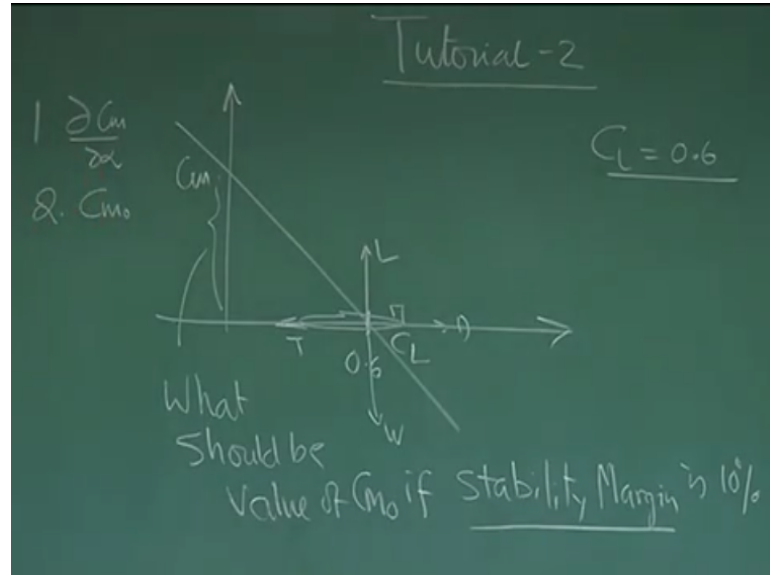


**Aircraft Stability and Control**  
**Prof. A.K. Ghosh**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology-Kanpur**

**Lecture-57**  
**Tutorial-2**

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Good morning let us do one problem let us solve one problem I say tutorial -2 and we like to see whatever we understood can you apply this by solving simple, simple problems are not ok. What you understood one thing  $d C_m / d \alpha$  with static stability parameter and that is  $C_m$  not. So let me create a problem. This is  $C_m$  let see this is  $C_L$  and it is let us frame the problem that we want to fly an airplane such that.

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$$C_L = 0.6$$

$$L = W$$

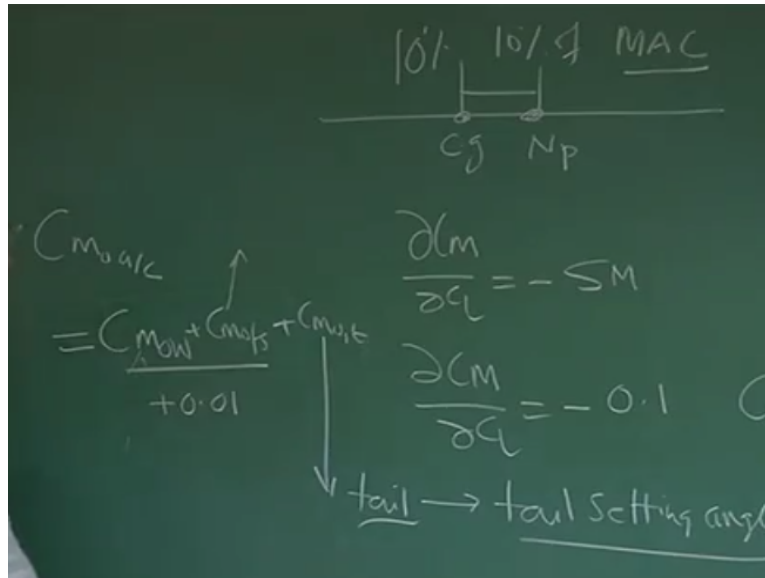
$$\frac{1}{2} \rho v^2 S C_L = W$$

$$C_L = \frac{2W/S}{\rho v^2}$$

The lift coefficient or  $C_L = 0.6$  where from you get  $C_L = 0.6$ . After all what is point is cruise that is your equilibrium. So lift=weight I know  $\frac{1}{2} \rho v^2 S C_L = W$ ,  $C_L = 2W/S / \rho v^2$ . Suppose I am designing an air plane whose wing loading we have decided, we have decided what is the cruise speed and we have decided what is the altitude at which you are cruising. So if I put those numbers here I will get a value  $C_L$  and that value  $C_L$  let's say all those geometrical parameters values 0.6 got it.

So if I try to visualize that this at this point. Lift and weight are equal and of course it is the cruise, thrust and dragger also equal ok. And it goes with the saying  $a_m$  is also equal to zero. Now the question is what should be the value of  $C_m$  not if stability margin or static stability margin is 10% this is the question what is the question what should be the value of  $C_m$  not if stability margin is 10%. And I am going fly at  $C_L = 0.6$ . The movement i mention stability margin it should come to your mind that.

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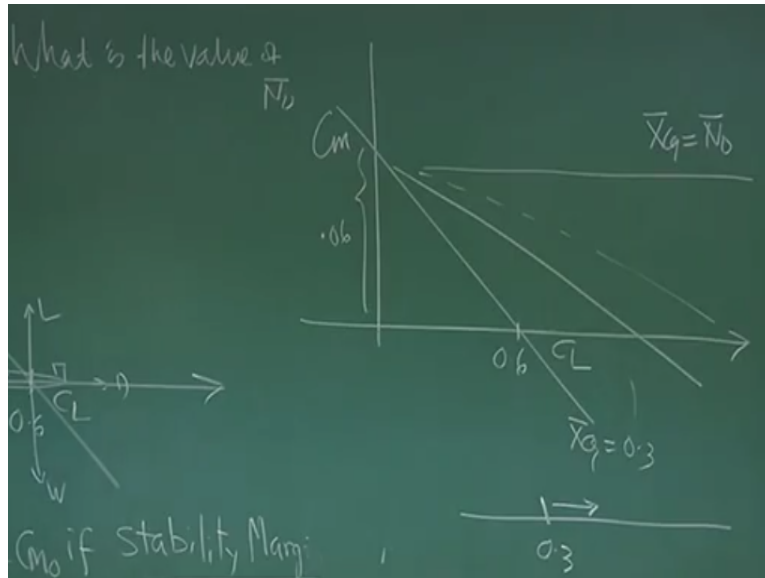
dc  $m/dcl$  is nothing but = - static margin we are developed this relationship with some approximately that we will carry on with. So what is the  $dcm/dcl = -0.1$  is 10%. The 10% means what? it actually tells you what is the meaning of the 10% if this is the neutral point and this is the cg location and this distance is 10% of main aero dynamic chord ok. I know  $dcm/dcl$  is -0.1

So I know the slope which is here which is linear and same distance -0.1. So what is the  $c_m$  not then?  $C_m \text{ not} = 0.1 * 0.6$  that is 0.06. So I need  $c_m \text{ not}$  0.06. So if I have an air plane I should ensure that  $c_m \text{ not of aircraft} = c_m \text{ not wing} + c_m \text{ not fuse} + c_m \text{ not tail}$ . What you understand?  $c_m \text{ not} = 0.6$  means because contribution from wing fuse large contribution the tail target up to 0.6.

Suppose this two contribution is not giving the 0.6 if it gives 0.6 fine plus .06 which is very difficult you understand mostly the limbered airfoil and that  $c_m \text{ fuse}$  large is negative all those small. What is the option? Suppose we have said this is 0.01 let us say this. Then remaining point .05 must come from tail and how to get the  $c_m \text{ not}$  from tail. You know for tail setting angle.

You can use those formula and find out what is the tails setting angle required ? for making this whole  $c_m \text{ not}$  to 0.06 and catch point is here the right now let us further look in this problem.

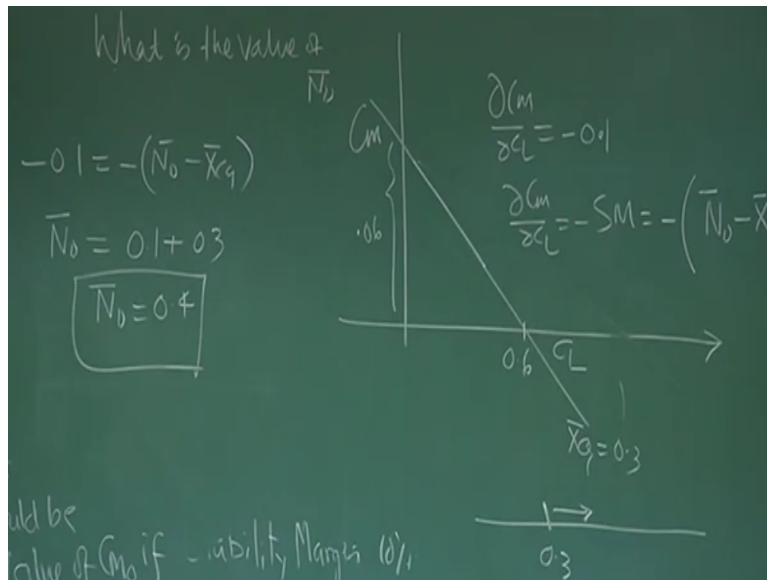
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This is point 0.6 this is point.06 and let the  $\bar{x}_{cg} = 0.3$   $\bar{x}_{cg}$  means  $\bar{x}_{cg}$  location non-dimensional at which a mini aero dynamic chord. Now if I move the  $\bar{x}_{cg}$  backwards if the  $\bar{x}_{cg}$  is here 0.3. Now I am moving the  $\bar{x}_{cg}$  backwards what will happen as I move the  $\bar{x}_{cg}$  backwards the slope will go on the reducing the time will come when the slope will parallel to the x axis and that  $\bar{x}_{cg}$  location you all know that neutral point.

So what is the definition of neutral point is that  $\bar{x}_{cg}$  location at which is  $dc_m/d\alpha$  is zero right of the aircraft is neutrally stable in static sense. Now if I am given this plot  $\bar{x}_{cg}$  and  $C_L$  and  $C_m$  not. If I try to find out what is the value of  $\bar{n}$  not I can easily find out that.

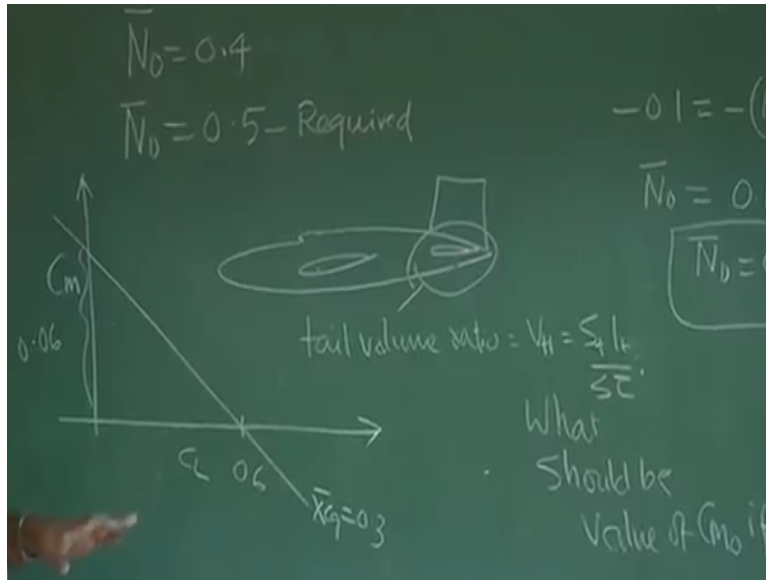
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If this is the combination this is given what you know you know  $dC_m/dC_L = -0.1$  and you know  $dC_m/dC_L$  minus static margin =  $-(\bar{N}_0 - \bar{X}_{cg})$  and you know  $dC_m/dC_L$  how much,  $dC_m/dC_L$  is static margin 10% we have mentioned here  $dC_m/dC_L$  is  $-0.1$ . So what I can write we can simply write  $-0.1 = -(\bar{N}_0 - \bar{X}_{cg})$ ;  $\bar{N}_0$  bar. And so  $\bar{N}_0$  will be,  $\bar{N}_0$  bar will be  $0.1 + 0.3$  plus  $X_{cg}$  is given  $.3$  so your  $\bar{N}_0$  bar neutral point is  $0.4$  that is simple the reason is given like this  $C_m$  versus  $C_L$   $X_{cg}$  is given.

Here this values was given here usually find out what is the neutral point is in the airfoil ok. This is one of the simplest this which you are suppose to do in the sense that automatically come to your mind that this is such reasons. Now let us try to we little more creative.

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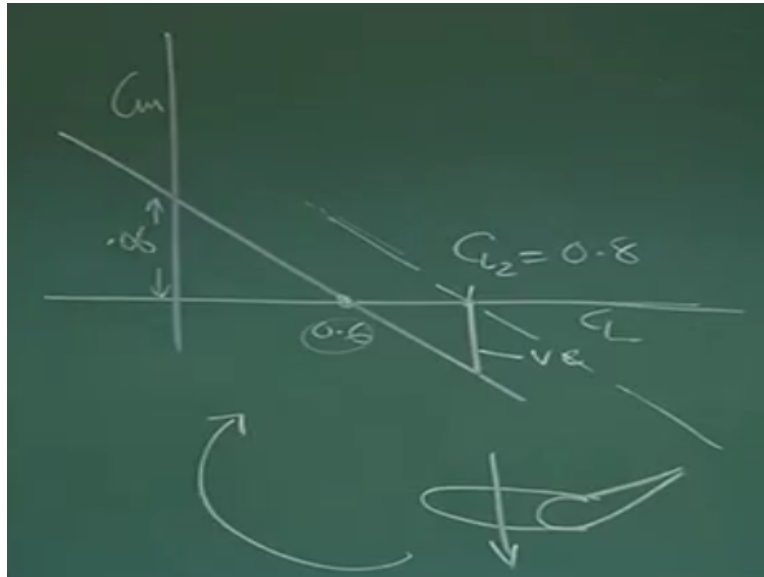
Let say we have got  $n_{not} = 0.4$  as given the example but you want the airplane  $n_{not}$  is equal to 0.5 that is what is required I am talking about the right what is to be done again visit this diagram. Graph this is  $C_m$  this is  $\alpha$  this is  $\alpha_{cg} = 0.3$  and this is 0.6 and this is .06 most of the student start thinking change the  $C_g$  and change the  $n_{not}$  do is correct. Answer is no because  $C_g$  nothing to do with  $n_{not}$ .  $N_{not}$  is decided by whom.

By the tail, tail sides aspect ratio with by wing size aero dynamic properties. Once the configuration is fixed  $n_{not}$  is fixe. So by changing  $C_g$  you cannot change  $n_{not}$ . You can change the stability margin that is different issue. But our problem is we want to change  $n_{not}$ . So what is to be done. So the best way to do is. This is the airplane this is tail this is the wing I am not changing the  $n_{not}$  and the  $n_{not}$  whatever number it is.

And the best way to deal this is change tail volume ratio that you remember that is  $V_H = S_T l_t / S_C$  bar. What is happening geographically? Please see this I want to change the 0.4 to 0.5. So, what I will do I increase the tail size. The increases the tail size drag also increase right. Better you take the tail little back. If it take further back then the condition of the whole airplane length will be increased even at the right click. The little bit of area little bit of length in effect re effect the volume ratio.

This will immediately change the  $n$  not value. So we can find out corresponding change in the  $v_h$  required to make it from 0.4 and .5 best way to handle is, change the tail volume ratio ok. That you should not forget ok. Once this is done. Let's also see another problem so that we can understand all this construction very clearly. Ok.

**(Refer Slide Time: 12:02)**



Let us see let this point is .6 let this point is 0.06. We are flying this  $C_l$  now there could be recall that a now fly in the same altitude at high  $C_l$ , this  $C_l$  2. Let's say the equation as .8. So what will happen the moment you try to fly out the air plane is .8 the air craft will generate the negative pitching mode right? But you want to trim this air plane at this  $C_l$  without changing the stability all  $C_g$  remains same. If you want to that mean's this negative pitch movement should be nullified. So who will nullify this?

We will nullify putting the elevator up. Elevator up means it gives force like this moment about  $C_g$  nose up able to give elevator which will neutralize this negative pitching movement at  $C_l = 0.8$  and how do I am do that we allow use this relationship  $\Delta e = \Delta e_{not} + d \Delta e / d C_l$  trim into  $C_l$  trim.

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$$\delta e = \delta e_0 + \frac{d\delta e}{dC_{L_{trim}}} \cdot C_{L_{trim}} = -(\bar{N}_0 - \bar{X}_{CG})$$

$$= \frac{C_{m_0}}{C_{m_{\delta e}}} + \frac{(-\frac{\partial C_m}{\partial \alpha})}{C_{m_{\delta e}}} \cdot C_{L_{trim}} \rightarrow$$

Elevator Control Power

10%

3

0.8

Where  $C_{L_{trim}} = -\frac{C_{m_0}}{C_{m_{\delta e}}} + \frac{(-\frac{\partial C_m}{\partial \alpha})}{C_{m_{\delta e}}} \cdot C_{L_{trim}}$ . And you could see easily for the aircraft the  $C_m$  not will be available  $C_m \delta e$  elevator control power. Control power otherwise you find out the formula  $C_m \delta e = -C_{L_{trim}} \frac{C_{m_0}}{C_{m_{\delta e}}} + C_{L_{trim}} \frac{(-\frac{\partial C_m}{\partial \alpha})}{C_{m_{\delta e}}}$ . Which you can check your notes  $d C_m / d \alpha$  as 10% given here and  $C_{L_{trim}}$  is now it becomes point 8.

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Suppose aircraft  
cruising

$V_c = 100 \text{ m/s}$   
 $\rho = 1 \text{ kg/m}^3$   
 $S = 20 \text{ m}^2$   
 $W = 2000 \text{ kg}$   
 $C_{m_0} = 0.06$   
 $C_{m_{\delta e}} = -1.0$   
 $SM = 10\%$

So simple define from the other problem also as discusses suppose aircraft cruising.  $V_c$  cruise is 100 meter per second  $\rho$  is =1 kg/m cube that is the altitude where the density of mode 1 kg per meter cube.  $S = 20 \text{ m}^2$ ,  $W = 2000 \text{ kg}$ ,  $C_{m_0} = 0.06$ ,  $C_{m_{\delta e}} = -1.0$ , static margin = 10%. What is the



elevator required if this question is asked us what is the elevator required. How can I do that in I come back to that.

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$$\delta e = \delta e_0 + \frac{d \delta e}{d c_{l \text{ trim}}} \cdot c_{l \text{ trim}}$$

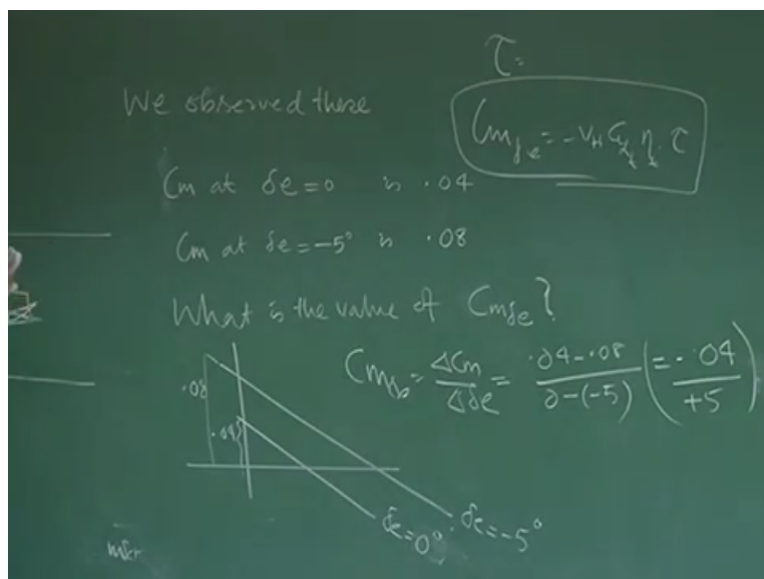
$$c_{l \text{ trim}} = \frac{2 w/s}{\rho V^2} = \frac{2 \times 2000 \times 9.8}{1 \times 100^2 \times 20}$$

$$\delta e_0 = -\frac{C_{m0}}{C_{m \delta e}} = \frac{-0.06}{-1.0}$$

$$\frac{d \delta e}{d c_{l \text{ trim}}} = \frac{-\frac{\partial C_m}{\partial c_l}}{C_{m \delta e}} = \frac{-(-0.1)}{-1.0}$$

Relationship  $\delta e = \delta e_0 + \frac{d \delta e}{d c_{l \text{ trim}}} \cdot c_{l \text{ trim}}$ ,  $c_{l \text{ trim}} = 2 w/s / \rho v^2 = 2 * 2000 * 9.8 / (1 * 100^2 * 20)$ . So that will give the  $c_{l \text{ trim}}$ .  $\delta e_0 = -C_{m0} / C_{m \delta e}$  the values are given  $= -0.06 / -1.0$  then the  $d \delta e / d c_{l \text{ trim}} = -\delta C_m / \delta c_l / C_{m \delta e}$  again you know that  $-(0.1) / -1.0$ . So we get  $d \delta e$  plus  $d c_{l \text{ trim}}$ . And  $c_{l \text{ trim}}$  we found from here plugging here and get the answer. That is as simple as that ok.

**(Refer Slide Time: 17:22)**



Let us take another problem. Suppose we conduct the wind tunnel test and observe this. In wind tunnel test we found that  $c_m$  at  $\delta e = 0$  is 0.04 cm at  $\delta e = -5$  degree is 0.08. So we are contacting a wind tunnel test here put modal this is the elevator here. One test will content the  $\delta e$  is 0 measure the  $c_m$  string x balance and it is found .04 and second case we will put the elevator let this up – 5 degree and then measure that came out to be .08.

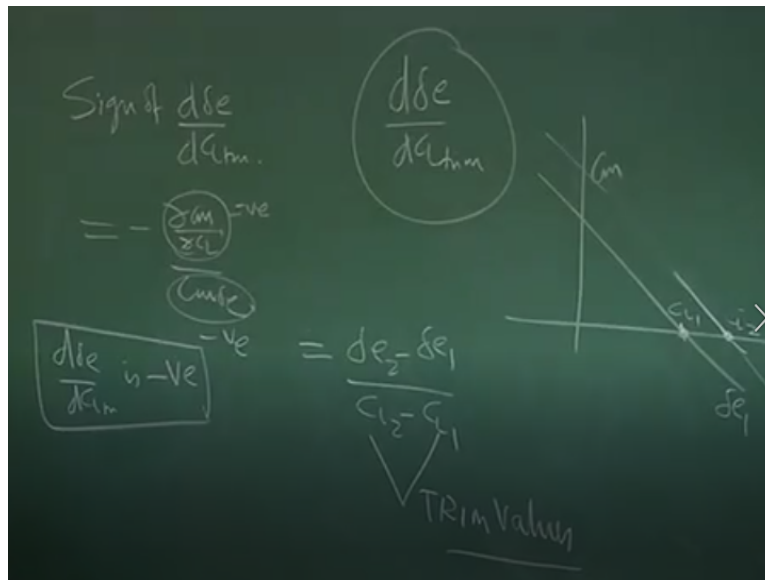
What is the value of  $c_m \delta e$  1 so the  $d \delta e / d \delta e$  1 trim the  $d c_l$  trim is nothing but  $d^2 \delta e / d \delta e^2 / c_l^2 - c_l^2$  these are the trim value. This is important ok and what is sign of  $d \delta e / d c_l$  trim you should know sign of  $d \delta e / d c_l$  trim = -  $d c_m / d c_l / c_l \delta e$  and statically stabile at the this sign is negative. This sign is negative sign of  $d \delta e / d c_m$  trim is also negative ok and unit wise we see that this is demarcate is in there per degree then the  $d \delta e / d c_l$  trim degree per radiant  $d \delta e / d c_l$  trim is a radiant generate will given when degrees ok.

How will find out but the potion what will be happen here this is one  $\delta e$  one to  $\delta e = 0$  degree this value 0.04 is  $\delta e$  is -5 degree from the value is 0.08 ok. What is volume of  $\delta e$ . the  $c_m \delta e = \delta c_m / \delta \delta e$  if we take this  $0.04 - 0.08 / 0 - (-5)$  be careful at the sign and that will be give the value  $-.04/+5$  is per radian. To give this much of value per degree not per radian because the angle is in degrees. There is should be very clear right.

So from tunnel test seeing this result immediately find out what is the  $c_m \delta e$ . Once I know is somebody asking  $C_m \delta e$  what is the corresponding angle of tau? Remember tau is  $d \alpha / d \delta e$ . rho c m  $\delta e$  is expressed c l alpha t Neeta t into tau right. Please check the formula so you have to only use this formula here.

If you know the tail volume ratio if you the alpha tail you can manipulate the relationship to find out the value of tau given other change this is way you can go on calculating information based on the other information. We do a on the relationship on the ok right.

**(Refer Slide Time: 21:30)**



Another important thing we should be known typically  $d\alpha/d\alpha_{trim}$  what is the meaning of the  $C_m$  vs.  $C_L$  right from one  $C_L$  under played at  $C_L$ . This is  $C_{L2}$  this is  $C_{L1}$  and the  $\alpha_1$  and the  $\alpha_2$  you want to fly  $C_{L2}$  and I need  $\alpha_2$   $C_L$  is trim for flying at the  $C_L$  trim 1 it was  $\alpha_1$ . So  $d\alpha/d\alpha_{trim}$  is nothing but  $\alpha_2 - \alpha_1$  by  $C_{L2} - C_{L1}$ . Where these are trim values this is important ok. And what is the sign of the  $d\alpha/d\alpha_{trim}$  also you should know the sign of  $d\alpha/d\alpha_{trim}$ .

You know this is nothing but  $-dC_m/d\alpha$  by  $C_m - C_{L,trim}$  and for the statically stable airplane this sign is negative, this sign is negative. So total sign of  $d\alpha/d\alpha_{trim}$  is also negative. And unit wise you could see that it will be this is dimensional s, this is either per degree. If it is per degree then  $d\alpha/d\alpha_{trim}$  is per degree.

With this understand I am sure to able to under the life problem including an example at us I should all the best will we also try to see that will given lectures of the evolution the shapes. How the shapes all is change at the white class as change and could understand what every you studied here how they translated into real product and today you will fly so many high air carft and the understanding this so simple ok. Thank you very much.