Aircraft Stability and Control
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Lecture – 59
Tutorial – 4

Dear friends let me discuss few concepts from dynamic stability in tutorial 4. What was the dynamic stability?

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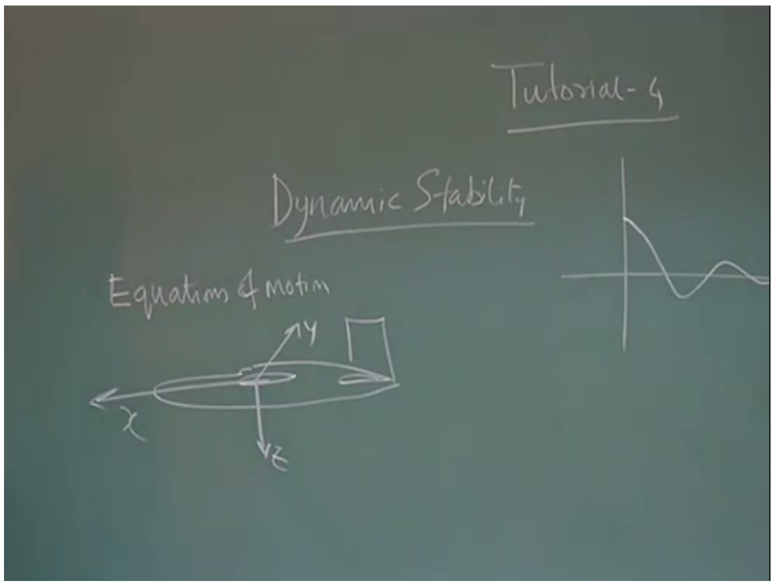

For static stability we realize that it is only the initial tendency that matters. If we try the initial tendency to come back to equilibrium we say statically stable but for dynamic stability not only initial condition but finite time it should come back to equilibrium right. So that is it if try to see like this suppose have disturbance system I tries to come back to the equilibrium and this. We are not talking about dynamic stability we have known reducing the concept of transient how it response.

It is coming back to the equilibrium or it is diverging it is possible. It has initial tendency to come back but every time it is further going up and up like this. So you could see always it has initial tendency as soon as it crosses the equilibrium so for sake equilibrium it goes again it stretch to come back to equilibrium again over source the amplitude goes on increasing. It is the typical case of dynamic in stability and typically you know these phenomena close to the resonance right okay.

Now for air craft how we postulated mathematical model to study dynamic stability. We said first we develop equation of motion so we started with equations of motions and the moment we talk about the equations of motion we realize that we fix axis system and we said okay. This is the X this is the Y and this is the Z which the body of axis system that is as the body rotates or moves axis also moves along with the body.

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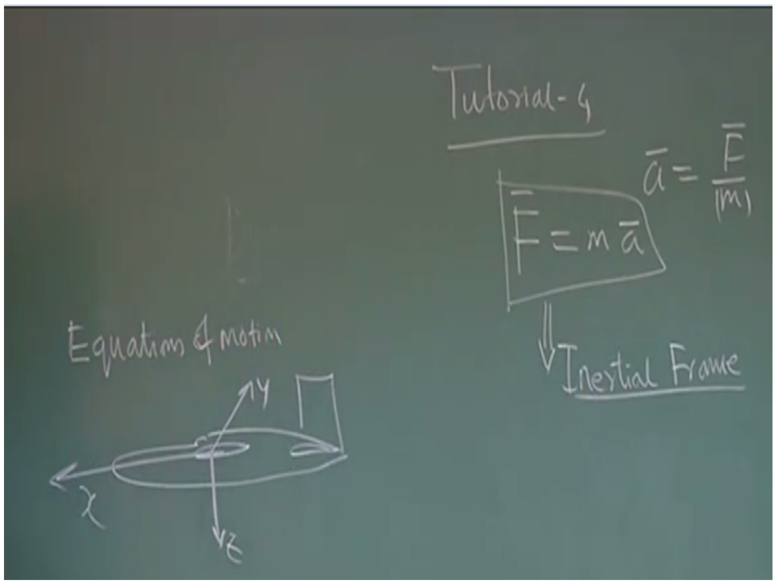

We wanted to develop equations of motion and you know that to develop equations of motion to newton's laws of motion. Where F is equal to m a. force is across the effect is acceleration and this acceleration is inversely proportional to mass property or inertia for the given force which you all know. But what is the condition, condition is this relationship is valid in inertial frame. But see we have chosen axis in the body fixed norms that is the axis is fixed the body rotates the axis rotates. So definitely this axis is not inertial frame.

So how to apply F is equal to m a. so it that okay. We will use Earth fixed axis system for our dynamic stability analysis as inertial frame what the logic behind it was. Although in axis rotating it is a non inertial frame how for dynamic stability analysis the duration is so short may be few seconds and we can assume during that time the acceleration or rotation of the axis negligible and all approximation we can assume to this to the inertial frame for our purpose of dynamic stability analysis.

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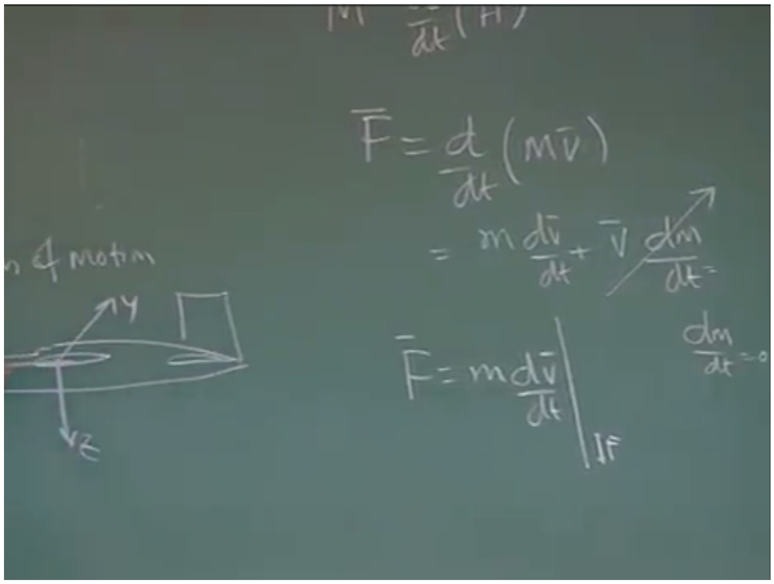

Lot of question was if I am trying to write the equations of motion in inertial frame that means what will happen. So we know that F is d by d t rate of change of momentum. And we know nothing but m d v by d t plus v d m by d t so another approximate value. As said we will neglect the second term and as if we assumed as d m by d t is zero but you know the air craft has fuel and there is huge fuel consumption.

And what is the logic again is given here it says since we are doing dynamic stability analysis it is for few seconds of response will be utilize for analyzing the dynamic stability of airplane for the small second will be assumed at the fuel has not changed or the mass of the whole airplane has not changed.

So we said F equal to m d v by d t but you know this has to be in inertial frame okay. But we are working in the fixed frame body fixed frame. Suppose we are walking in inertial frame one of the issues will be when I talk about the rotation if I use the moment equation the moment will be rate of change of angular momentum as the angular momentum vector. We have moment of inertia as inertial properties.

So what will happen if I work in inertial frame as the body rotates so the moment of inertia above the inertial frame will go on changing that will complicate the analysis right. Second thing you realize that the aero dynamic forces that depend upon the relative air speed. So from that angle body fixed frame is very very important so that speed of aircraft with respect the atmosphere.

Relative to wing relative to atmosphere will be very useful because after the aero dynamic forces depend upon relative air velocity.

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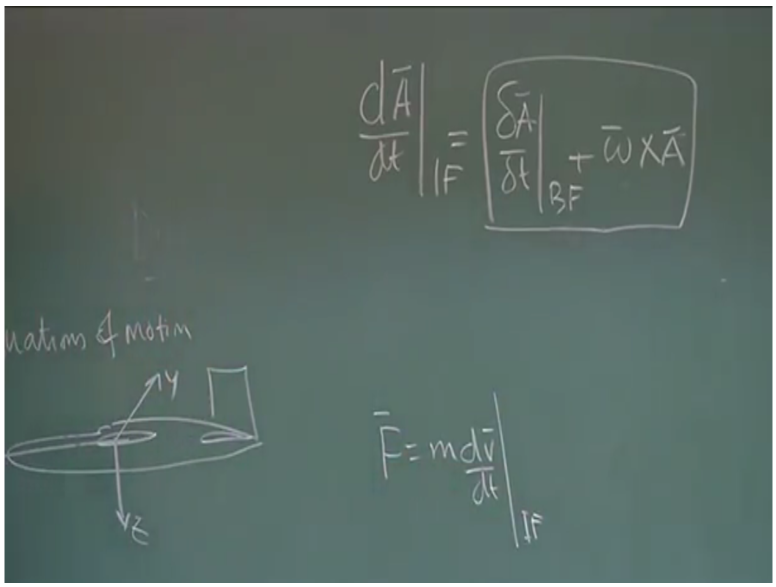

So you had a motivation for work in body frame this two reasons and how to do that need to work in inertial frame but we have motivation to work in body fixed frame so what is the trick you said after all d v by d t is the vector differentiation so we use this relationship d a by d t in inertial frame is equal to d a by d t in body frame plus omega cross A right.

This relationship helps us to work in body frame what is the message that either you work in inertial frame or if you want to work in body frame which is our motivation you can work wider you give this correction omega cross A right. And that is how the equation of motion are developed. This is the circle understanding you must have okay.

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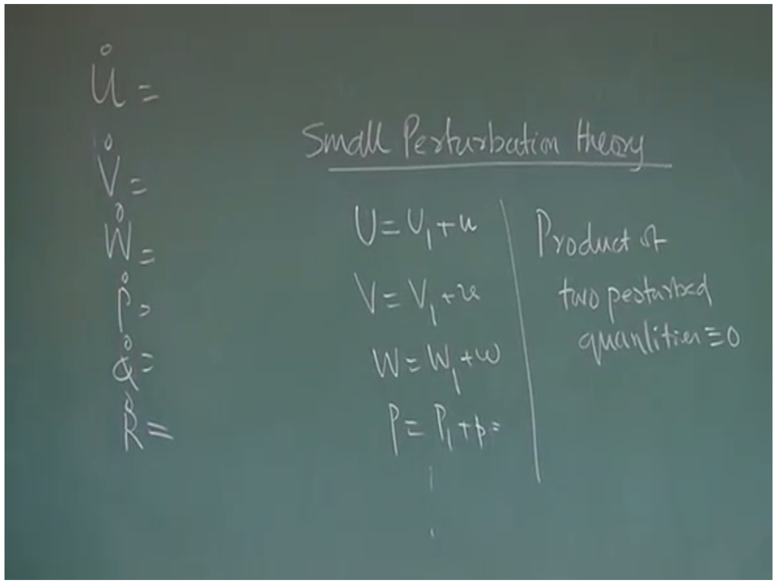

Now whenever we apply this equation of motion we will found v dot w dot p dot q dot r dot extra right. What we did we used small perturbation theory and what was the understanding of small perturbation theory. The perturbation is small so that we can write total U is u 1 steady state plus perturbed velocity U. You assuming that the changes assign that linearity is maintained similarly we write v equal to v 1 plus v and w equal to w1 plus w p equal top 1 plus small p like this.

When we use this equations and after modifying with the assumption that the product of two perturbed quantities are almost zero because neglected then we found out equation of motion which we refer as longitudinal perturbed equation of motion okay. So in that if we remembered when we develop longitudinal perturbed equation of motion. What was longitudinal, longitudinal means the motion is this U or along that Z, W and the pitching motion this three only y.

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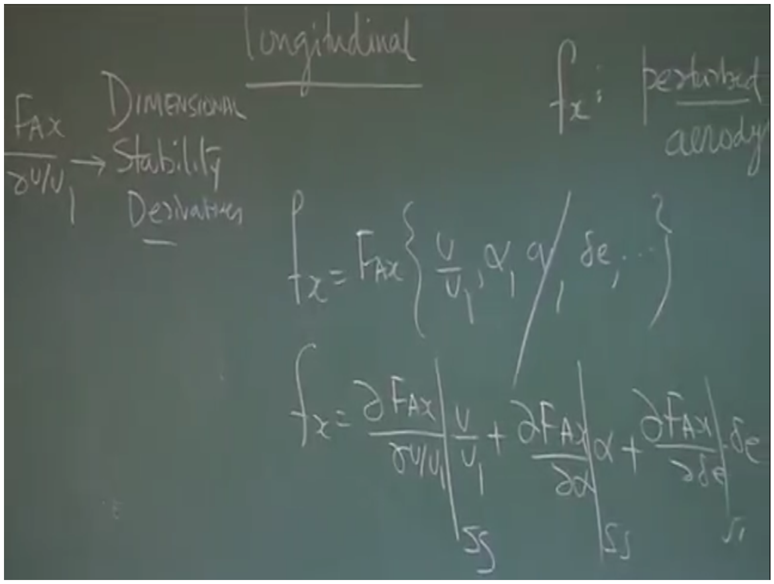

And we assume that this is the result of small perturbation so this is not going to influence lateral motion or directional motion in that if you check your notes. We have called f x and what was f x f x was termed as perturbed aerodynamic force right and when is your perturbed dynamic force we need to estimate this perturbed aero dynamic force and wrote f x is equal to F A axis the function of u by u 1 alpha q delta e extra for fs it takes q x is negligible.

And we wrote this f x as d f a x by d u by u 1 into u by u 1 plus d f a x by d alpha into alpha plus d f a x by d delta e into delta e like this. What is the understanding of d F A x by d v by v1 you want all this derivatives. This should be evaluated as steady state that is important okay and we derived some expression for those you could see that and this d F A x by d u by u1 these are termed as stability derivatives but these are dimensional there is the catch point, dimensional they have dimensions okay.

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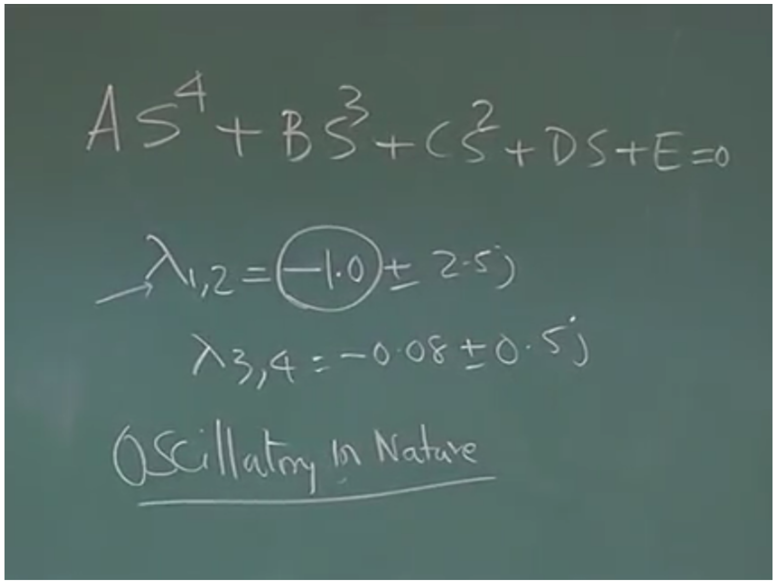

This one we did then what we realized once we develop the equations of motion which are called longitude final perturbed equation of motion which to Laplace transform right. And through Laplace transform we develop the equation of the form A s 4 plus b s cubic plus c s square plus d s plus e is equal to zero right. And numerically you can find the solution for this equation and you realize that most of the air plane.

When I solve this equation for longitudinal perturbed equation of motion we get roots this is lambda 1 lambda 2 and typically we can say I give number 1.0 plus 2.5 j is lambda 3,4 is minus 0.08 plus minus 5 j the important thing is to understand you get two pairs of complex conjugate that mean the roots tells you the response.

Which I will use to characterize its transient fine they are oscillatory in nature, one thing they are oscillatory in nature. You also realize one thing if I closely see this root the real part is one complex period is large negative as compared to other period. You could see from here so rigidly I know all the both are oscillatory this will decay faster than this okay.

Because the real part decides the decay it reverses theta omega and theta n. If that is okay you know true then we actually understand the physics to the airplane. We have seen that typical airplane when they are disturbed longitudinally then two distinct types of modes are generated one is called short period mode another is called phugoid mode okay.

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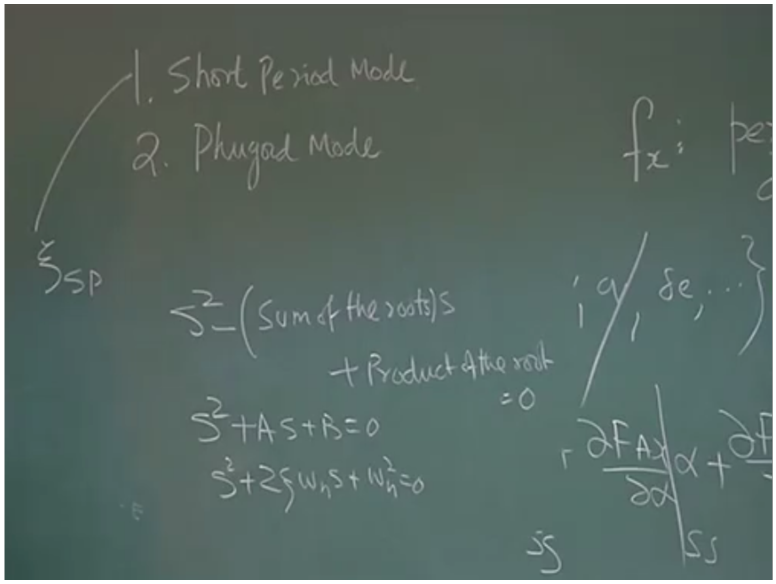

What is the short period mode, short period mode will correspond to this because this real term is large negative and typically if the airplane is moving like a cruse if I give a short elevator input one mode will be like this it will really come back to equilibrium it is the transient so that is the short period mode. Phugoid mode is if I exit phugoid mode is when I hold the elevator for long the airplane will come back to equilibrium.

Then it will go on doing like this this they need to come so it is long period mode that is the phugoid mode and we have asked to approximate to find out natural frequency and dumping ratios okay right and we have in deed seen the dumping ratio zeta for short period large compared to zeta for phugoid mode okay. Phugoid mode is weekly down to earth. And this should be very, very clear and you should also know if I have to find out.

The omega n for any complex period and the best way to do it is. You write the equation of motion is s square minus sum of the roots into s plus product of the roots equal to zero. You can use those roots and from there you get the equation in terms s square plus a s plus b is equal to zero and comparatively s square plus 2 zeta omega n s plus omega n square is zero is the typically secondary system then by its comparison you can find the value of omega n and value of zeta.

What is the understanding that over all longitudinal dynamics for most of the airplane can be dissolved into 2 second order systems one is the short period one is the phugod and once you know this I can easily find out what is the natural frequencies and what is the dumping ratio.

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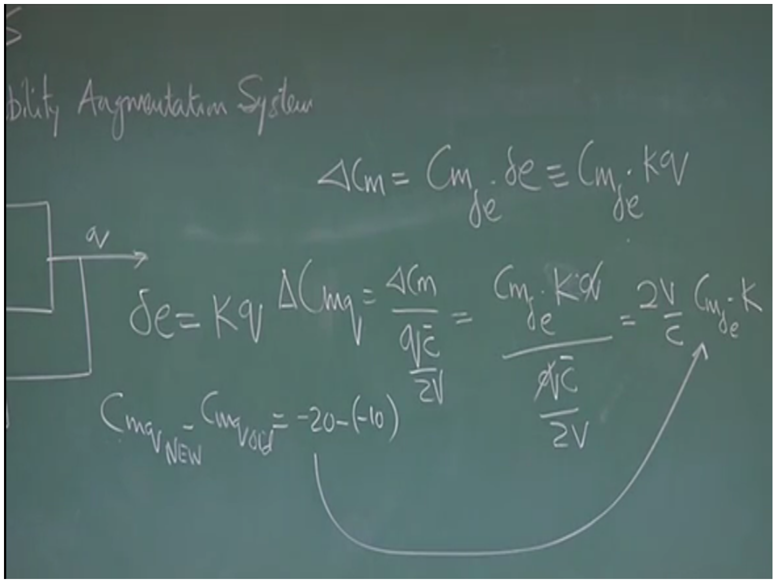

We also use this understanding and try to approximate the matrix and got expression for short period and phugoid mode okay. We also talked about SAS that is stability argumentation system what was that the understanding of this is? Suppose the airplane is having zeta equal 0.2 but we want zeta to be 0.6 one we change zeta change the volume ratio because this zeta largely depend upon c m q and you know c m q has v h tail volume ratio as one of the parameter variables.

If I have to increase get an I have to increase tail volume ratio but suppose you don't have the option to increase the tail volume ratio you want to change get an online. And what was the concept the concept was SAS and the understanding was like this this is the aircraft and list this is q I tab q this is k what I do I deflect elevator proportional to this q then what will happen as I deflect elevator what is delta c m?

I am getting because of elevator deflection that is c m delta e into delta e which will be equal to c m delta e into k q and c m q is defined as delta c m by q c bar by 2 v so I write this as cm delta e into delta e which is nothing but k q divide q c bar by 2 v so q, q get canceled so this become 2 v by c bar into c m delta e into k.

Now you could see that let take get a take c m q when calculate it say minus 10.0 let's say some number but to make get 0.6 will be required minus 20.0. so what is the procedure? The procedure is this could be written as delta c m q so sm q is minus 20 so c m q new minus c m q old this is nothing but minus 20 minus 10 this should be equal to 2v by c into c m delta into k c m delta is fixed velocity is fixed c is fixed.

So you have to alter the value of k we can go on changing the value of k and get that is the online value of c m q required to ensure zeta is point 6 and you could see how beautifully you can use SAS for dumping ratio.

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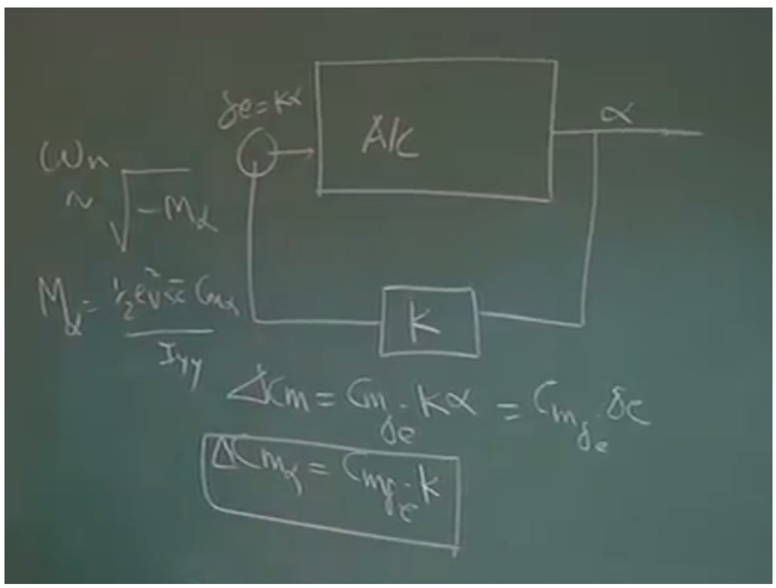

Similar thing you have to do for natural frequency you have to change natural frequency what we have to do? This is the aircraft this is alpha you tab alpha this is k and again delta is there for k alpha that goes to the plot so what happens I write delta c m equal to c m delta e into k alpha because it is nothing but c m delta e into delta e and delta e is k alpha.

So c m alpha which is nothing but delta c m alpha will be c m delta e into k and you know if, you want to change naturally frequency omega n, omega n goes with minus m alpha and m alpha is nothing but half rho v square s c bar c m alpha by I y, y so if you want to change omega n once want to change c m alpha change k so electronic will change the k value so get desired c m alpha to get required m alpha to have decide omega n.

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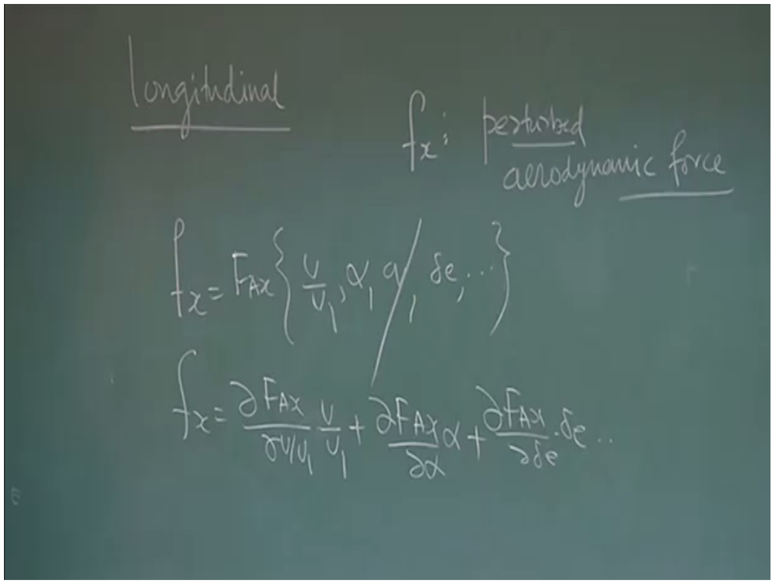

So that is also another way of utilizing SAS for the specific purpose but all these have find you do not forget that the moment I am using delta e is equal to k alpha see the c l equation that c l not plus c l alpha into alpha plus c l delta e into delta e so delta e equal to k alpha. So your c l also modified. Now c l becomes c l not plus c l alpha into alpha c l delta e into k alpha so c l also get modified because of this additional term. So you have to be careful you have to optimize total algorithm to see that the final equivalent are not disturbed much okay.

So I thought I will go for the revision on this I have requested one of the friend Dr. Deepu Philip to give one lecture in where he will speak about different types of aircraft and you should know that you should see, listen to the lecture and get an idea about why all this stories what we have achieved if you see the valuation of the aircraft please understand this evaluation rather possible unless our basic understanding was clear okay. Thank you very much.