

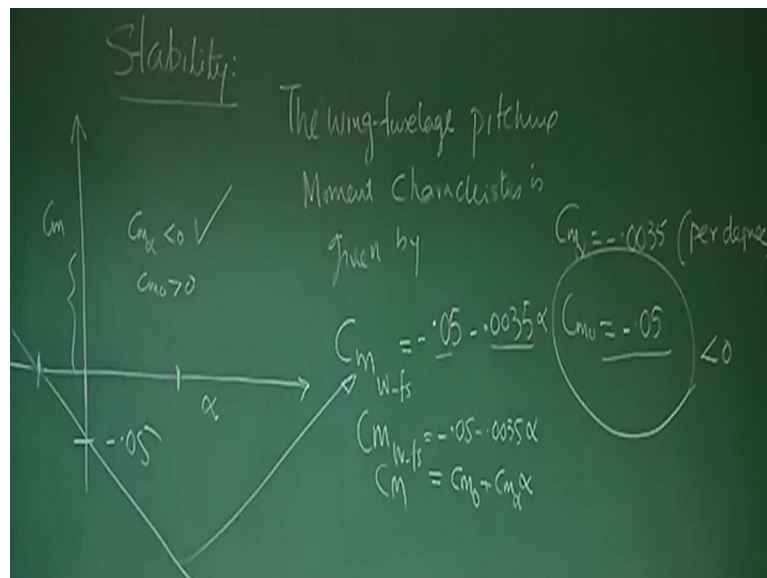
Aircraft Stability and Control
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Lecture No. 06

Problems: Stability and Wing Contribution

So dear friends, as I was promising you that we will soon try to solve a problem so that you get more clarity. But there is every possible chance that because of all those equations and terms you may lose insight of the airplane. So as I promised that we will be doing some problems so that you get physical feel of whatever equation, whatever expression we have developed. Today we will be solving a problem on stability.

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As this course title is stability and control, so today we will be solving a problem on stability so that you get a feel. Before I solve this problem, let me again reiterate the role of wing is not to provide stability. The role of wing is to give enough lift. That's very large aspect ratio or large wing area but who provides the stability? Whose responsibility is to give stability? It's the tail. For longitudinal stability it is horizontal tail, for directional stability it is vertical tail. So we are now talking about longitudinal stability.

We will try to solve the problem of longitudinal stability and try to see how whatever the expression we have developed how can we very conveniently use it for solving a problem okay? So let's start with a problem and this problem I've given from a book which is 'Flight Stability and Automatic Control' which is a second addition by Robert C. Nelson. It is a very

good book. I'm not campaigning for this book but it's my duty to tell you because I follow many books one of those is 'Flight Stability and Automatic Control' by Robert C Nelson and this problem is from that book only.

So we will be now using this opportunity and to use this beautiful example and try to communicate more on stability right? So what is the problem? Problem let me define the problem. The problem says, the wing fuselage, pitching moment characteristics is given by C_M by WFS means wing fuselage is $-0.05, -0.0035 \text{ Alpha}$. And please understand this part is what and this part is what? That is more important before you solve a problem.

What does it say if I try to understand by drawing a diagram? The fuselage has been designed and the wing has been designed okay and some CG we will soon know what is the location of CG of the aircraft is given. Now, the C_M characteristics is given by this for a wing fuselage combination. This is the fuselage and this is the wing. So do you think it is statistically stable? How do I check? I write this like this. C_M wing fuselage is $-0.05, -0.0035 \text{ Alpha}$ and I try to understand through our understanding of the expression we write C_M equal to C_{M0} plus $C_M \text{ Alpha}$ into Alpha .

So if I compare these two things, what is $C_M \text{ Alpha}$? $C_M \text{ Alpha}$ is coming out to be -0.0035 and for this we can assume that this is per degree okay. As per the problem states and what is C_{M0} ? C_{M0} is -0.05 . So this is the characteristics of pitching moment and we are seeing that this is only for fuselage and wing and the expression after whatever plane has been designed, for wing fuselage, it is something like this and when I try to interpret that we have understood we have seen that by comparing I find C_{M0} is -0.05 and $C_M \text{ Alpha}$ is -0.0035 per degree okay?

Now let us see what is the meaning of that? Are you happy with it or not? So if I try to draw a plot, C_M versus Alpha , right? What type of plot of variation you really expect for a stable airplane, statically stable airplane and which can be trimmed at positive angle of attack, we all know it should be something like this. That is slope should be negative. That is slope, $C_M \text{ Alpha}$ should be negative, and C_{M0} should be positive. Then only I can trim at positive alpha for most of the cases, right?

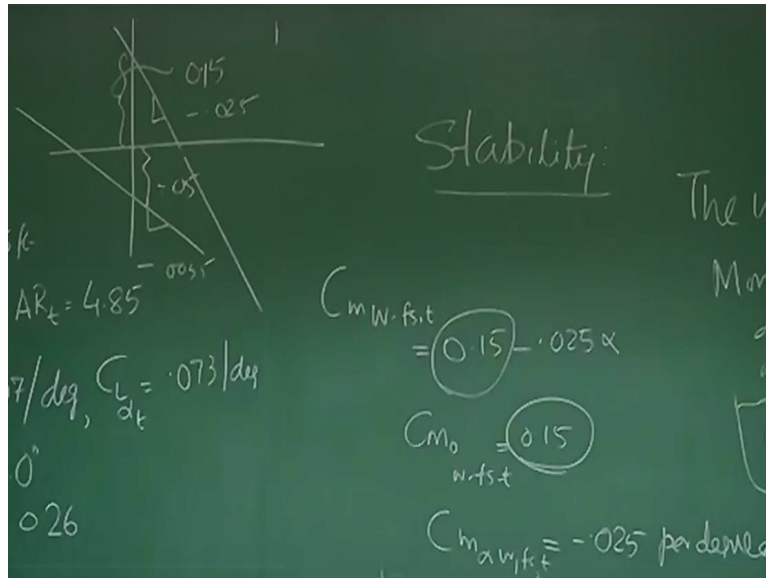
This part is clear? But if I try to see this, yes, CM_{α} is -0.0035 so $CM_{\alpha} < 0$ is satisfied. So this configuration whatever has been given for fuselage and wing, this is statically stable. To what degree stable, that is decided by the CM_{α} . So as far as static stability is concerned, this configuration is statically stable no doubt about it because CM_{α} is negative and that is the condition here.

But if you want to trim at positive angle, then what is the condition. That CM should be greater than 0. But what is happening here? Here CM_0 is less than 0. So, I wouldn't be able to trim this airplane at positive α . That is, if you see here, CM_0 some negative value here. Let's say this is -0.05 and statically stable so the line will be something like this for this variation. So what is the problem? Although this is statically stable, but I cannot trim it at positive angle of attack. It is having a $CM=0$ at negative angle of attack. So I don't want that right? So problem definition starts from here.

So what is obvious question for the problem. If I want to trim the airplane at a particular α , positive α and I may require CM_{α} little more, because I may require to design an airplane for a particular value of stability, static stability. So I may have decided number, CM_{α} should be -0.05 , -0.01 , -0.08 , depending on what sort of an airplane I am designing. So this problem has been designed to give that concept of CM_0 and CM_{α} .

Let us see what is the problem. Once you understood this, now we will read the problem and try to understand what is the problem. Problem is, although wing fuselage is giving a statically stable configuration.

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But we want to design a tail such that CM-wing fuselage and tail gives a combination, which should be something like this. This is a problem from Nelson. So please you can read it +0.15 -0.025 alpha. So what is the difference now? Try to appreciate the wing fuselage was like this where CM wing fuselage is -0.05 - 0.0035 alpha. That means, CM0 was -0.05. Now we want to design a tail for the same airplane so that overall CM0 becomes point, that is CM0 wing fuselage and tail becomes 0.15 and CM Alpha wing fuselage and tail, it becomes -0.025 per degree.

So what is the problem to a designer, I should do something so that CM earlier it was something like this. This value was -0.05 and now what I want? I want it should be like this where this becomes 0.15 and the slope also becomes -0.025 compared to the slope when it was here, -0.0035 okay? So not only we are changing the CM0 value to get a positive Alpha trim, but also we have to make it more statically stable. That is the problem.

Who will do that? We all know, this is the role of the primary role is of the tail, horizontal tail in this case. So we will be finding out what is the tail area required to change the CM-alpha from -0.0035 to CM-alpha equal to -0.025. Similarly, by giving some tail setting angle okay how can I change CM0 of the whole aircraft from -0.05 to +0.15? That is the question okay 0.15 this the value. This is we are designing we want to go for it. So let me repeat again and in a very small and crisp manner. What is the final problem to us?

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Handwritten equations on a chalkboard:

$$C_{m_{w-fs}} = -0.05 - 0.0035\alpha$$

↓
New

$$C_{m_{w-fs,t}} = 0.15 - 0.025\alpha$$

Through Design tail

$$\Delta C_{m_0} = 0.15 - (-0.05) = 0.20$$

Required from H-tail

$$\Delta C_{m_\alpha} = -0.025 - (-0.0035) = -0.0215 \text{ per degree}$$

It was given CM wing fuselage was -0.05, for CM wing fuselage was $-0.05 - 0.0035 \text{ Alpha}$. The new what I want CM fuselage and tail to be $0.15 + 0.05 \text{ alpha}$. So this is the problem. How this can be done? By through design of tail. Design of tail means what, what is the tail area? What is tail setting angle? Where should I locate the horizontal tail or what is the tail volume ratio I look for? So these are the question that will come into our mind okay.

So this is the basic simplified problem. Now how do I approach it? okay That is the question. From this you can understand how much delta CM_0 you require from tail. You required from horizontal tail. How much it will be? I want 0.15 CM_0 here 0.15, so it will be 0.15 minus already available -0.05. So this will be equal to 0.20 right? How much delta CM_α you required? See again if I see it from here $-0.025 - (-0.0035)$ and that is equal to -0.0215 per degree.

Again I repeat, this is the additional CM_0 and CM_α will be required to convert the pitching moment characteristics from this $-0.05 - 0.0035$ to $+0.15 - 0.025$. So how much we have to do? We have to generate CM_0 0.2 and CM_α additional should come 0.0215. And who will provide this? The horizontal tail. How do I get it? By appropriately selecting the horizontal tail area and the location of the horizontal tail from the series of airplane or together we call tail volume ratio and for CM_0 .

We also will try to use tail setting angle that is if this is the horizontal tail, instead of having it like this if I set it little at negative angle of attack, you know even at $\text{Alpha} = 0$, there will be force downward which will give a CM_0 positive. Which we are already

knowing. Now we will be using those formula to solve this problem okay. Let us go back to this formula, whatever we have realized. So let me erase this now.

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And we have the formula, CM Alpha tail which already we know Neeta VH CL Alpha tail into $1 - D \text{ Epsilon by } D \text{ Alpha}$ okay. Let me repeat again this is CM Alpha tail which is Neeta for this example we will take Neeta as 1 VH is tail volume ratio CL alpha tail is the lift cut slope of the tail based on the tail area and D Epsilon by D Alpha is the downwash gradient and you know already in first course you have seen D Epsilon by D Alpha I can compute as CL Alpha wing by π aspect ratio.

Approximately this is I am taking equal to 1 and if you see here in the initial slid I've given and now also you could see here CL Alpha is equal to 0.07 per degree and D Epsilon by D Alpha I can use it this also will be aspect of the wing. If I put those number I will get this is 2 into 0.07 into 57.3 because I have to convert into per radian. It was per degree the π aspect ratio is 7.3 and this we will get value as 0.35. So D Epsilon by D Alpha is purely mechanical, you put those numbers CL Alpha wing if you see here it is given CL Alpha wing as 0.7.

What I have is CL-alpha wing, please understand one thing. Here I have CL-alpha wing and fuselage. So there is an approximation we are neglecting the contribution of fuselage because this D Epsilon by D Alpha in the first approximation is primarily because of the wing okay no fuselage is there okay. But these assumptions here you cant take it that fuselage is negligible. So we are neglecting it okays as a first approximation. So I know all this term CL alpha tail is here . CL Alpha tail is given 0.73 per degree but note this is based on tail AD right.

That is why you see here when the formulation in the VH, it is ST_{LT} by S_{wing} C_{bar} wing this ST is here. So CL_{α} tail is 0.073 per degree and let us now see what will be the VH . So VH will be if I solve this equation from here, VH will be equal to CM_{α} tail required. How much CM_{α} tail will be required? We have seen already, this is how much? If we check, it will be around -0.0215. Let me write this there so that I can explain it divided by 1 then 0.073 into $1 - 0.03$, $0.1 - 0.35$.

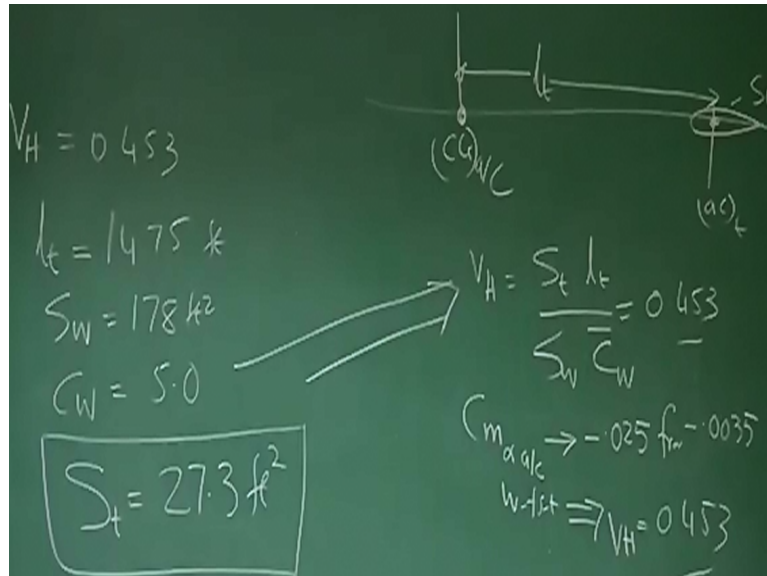
Wherefrom this has come? Let me again check, 0.0215 by 1, 0.073, $1 - 0.35$. You could easily see from here from this expression I could write VH equal to CM_{α} tail divided by this is minus sign divided by N_{eta} is $1 - CL_{\alpha}$ tail to $1 - D_{\epsilon}$ by D_{α} okay? Now you will ask me a question, where this minus sign has gone? So we have to be very, very clear that when we are computing this, there is a minus sign here so I put a minus sign here okay. Now it is clear? Again, let me repeat.

CM_{α} tail is N_{eta} VH CM_{α} tail $1 - D_{\epsilon}$ by D_{α} . You know how to calculate D_{ϵ} by D_{α} because this is contribution from wing, right? And this is $2CL_{\alpha}$ wing by π aspect ratio wing. And you know the value of 2 into 0.07 we are multiplying by 57.3 because 0.07 which is CL_{α} of the wing is per degree and so we have to convert to per radian. So we have multiplied by 57.3 and then divided by π aspect ratio wing and we have got D_{ϵ} by D_{α} as 0.35 which we have plugged here correct?

Now from this expression I can write VH equal to minus so let me make it neater, this is minus is here $-CM_{\alpha}$ tail by N_{eta} CL_{α} tail $1 - D_{\epsilon}$ by D_{α} . Here I have put those number and I have got the VH value as something like, how much 0.453. I have been telling in many lectures, when you are designing an airplane, VH should be around 0.5 to 0.8. That is a good initial guess number right? So this is a typical value of VH and what is VH ? Let us also understand before we use this number.

So let me erase this part. What was VH ? What is VH ? As a designer we would like to understand it.

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Remember if this is the central line of the airplane, and if this is the CG of the aircraft and if this is the tail, and this is the AC of the tail, right and this is the S tail area, then this distance is called tail moment on the horizontal tail LT and if you see LT here is 14.75 feet okay. And then what is VH? How VH is define? VH is defined as ST into LT by wing area into mean aerodynamic chord of the wing. So this ratio ST LT by ASW for this problem is 0.453.

So what is the result? Result is, if I want to change CM-alpha to 0.025 to -0.025, then I need to ensure that the tail volume ration is 0.453. Is it clear? I repeat again. What is the interpretation of this result? Is that if I want to change CM-alpha of the aircraft that means wing fuselage and tail to -0.025, right, from -0.0035 then I need to have VH equal to 0.453 correct okay?

What next for a designer? Let us try to understand little more as VH equal to -0.0453 and we have already LT equal to 14.75, also we have SW is equal to wing area is 178 ft square. Sorry for using feet unit. And CW is equal to 5. So what I can do? I can use this relationship VH equal to ST LT by SW CW and get the value of ST and if you do that you will get the value of ST as 27.3 ft sq clear?

Please you should do yourself this calculation if I commit some mistake or if there is some printing error, I will not be responsible. So it is very simple. We have to find ST VH you know 0.453 so ST will be 0.453 into SW into CW divided by LT so you will get S tail area this much. So what is the final conclusion? If I want to increase CM-alpha of this

configuration, from -0.0035 to -0.025, then I need to put a tail area of 27.3 such that the AC of the tail is 14.75 feet aft of CG of the airplane.

Is this clear? Or pictorially what I should say? I say this should be 14.75 feet and this is the AC of the wing and the S tail will be 27.36 square. If it is are done and the aircraft still remain at the same CG location then the CM-alpha of the aircraft will become -0.025 not -0.0035 right? And who has done this? It is the tail area has done this. For a designer what he will tell? It is the appropriate tail volume ration of 0.453 has done this change okay?

Now this is as far as slope is concerned it is static stability is concerned. Now I'll be talking about the trim. The CM0. Remember the problem was wing fuselage, CM0 was negative, so we want it has to be positive, and we realized that Delta CM0 which is required is around 0.15. So how do I get this Delta CM0 0.15 using the tail? Primarily through tail setting angle, and that is the problem we will be doing now.

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Let us again use those formula whatever we have developed. CM0 tail equal to VH Neeta IW plus epsilon0 minus IT right? So IT equal to -CM0 tail by VH Neeta CL Alpha tail. Here also CL Alpha tail will be there no. That is why, CL Alpha tail, yes. This is this minus IW plus IT. IT we are finding out so this is a single minus Epsilon 0 okay? Please be careful here. Initially I missed this CL Alpha term here. CM0 tail as we have derived is VH Nita into this term into CL-alpha tail.

So if I try to find out what is the tail setting angle IT required, I can ultimately show that IT will be equal to minus CM_0 tail by VH Neeta CL -alpha tail minus IW wing setting angle minus ϵ_0 . I hope you remember what is wing setting angle? What is the value of CM_0 tail? How much CM_0 tail required is $0.15 VH$ we know, Nita we are taking 1 . CL alpha tail is also given here right? IW wing setting angle $IW 2$ degree is given here.

Epsilon-not right? How do I find epsilon 0? You know for finding epsilon 0 you know epsilon 0 is $2CL$ not by PI aspect ratio and CL not is given how much? Let us see CL at alpha equal to 0.26 . So I put this value here. So epsilon not is 2 into 0.26 by PI and what is aspect ratio is around 7.3 . So this gives you a value equal to 1.3 degree correct. This is the why this epsilon not is present? Because the wing is cambered. So given at alpha equal to 0 , there will be a lift and lift mean there is a pressure difference between lower and the top surface.

Lift means there is difference between pressure between bottom and top surface. So there will be what we say and that will induce downwash. That is why epsilon at alpha equal to 0 . So this is definitely cambered aerofoil. We should appreciate that. So now if I put every value here I can easily see IT as minus 2.7 degree. It is straight forward you can plug in all those values here so you are getting IT equal 2.7 degree and VH required is 0.453 . So this is the solution.

So now to conclude through this problem what we learnt, we learnt there is a wing fuselage combination which had CM alpha negative. That means It is very statically stable and most probably AC of the wing is why most probably, if CM alpha is negative for a wing fuselage combination, it is sure wing AC is behind CG of the airplane right? However we wanted to increase static stability as well as to ensure that it can be trimmed at positive angle of attack.

So we found out what is that tail size required, what is the tail setting angle required to ensure that now the CM versus alpha for a wing fuselage and tail is what exactly we are looking for. Is it clear? This problem is only to give you a better feel for all those expressions we have developed.