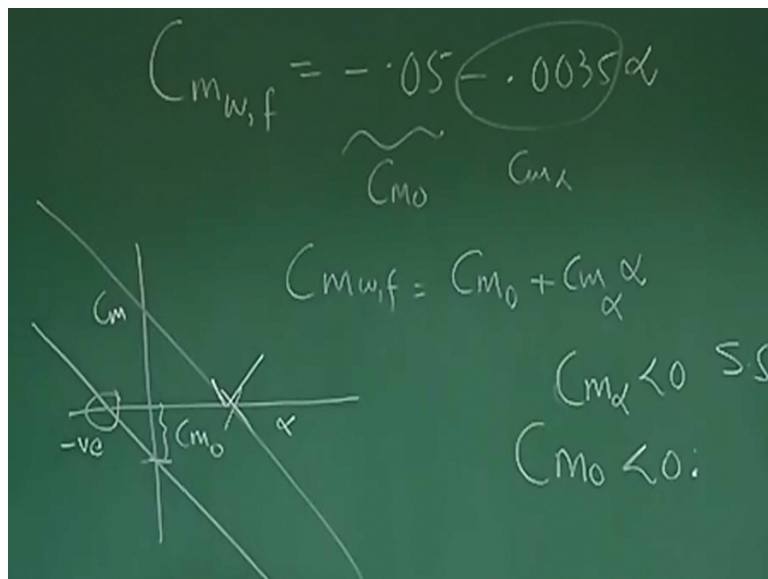


Aircraft Stability and Control
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Problems – Stability Tail Contribution Completed

We were trying to understand whatever in our theory class. We have explained regarding stability of airplane, that is static stability in particular and more specifically stick fixed static stability. And what was the problem? We solve a problem.

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Where it said there is a configuration, wing and fuselage with CM versus alpha graph, alpha plot was based on this linear equation and we know that this is nothing but C_{m_0} , we realize this value is C_{m_α} . So it is basically $C_{m_{w,f}}$ it is represented in this fashion. And when you check this number we found that C_{m_α} , C_{m_α} is negative, so it is statically stable configuration. However, we found that C_{m_0} is also negative. So it is not possible to trim it at a positive angle of attack right.

Barring that simple understanding we always remember that for a cambered aerofoil there could be a negative angle at which we can trim. However, in general we try to trim an airplane at positive angle of attack. And if C_{m_0} is negative, then we know that it is not possible to trim. This is $C_{m_{w,f}}$ and this is C_{m_α} , so if C_{m_0} is negative, this is C_{m_0} , then the trim is at a negative angle of attack, which we do not want. We want something like this.

So we trim at positive angle of attack. This is what we are looking for, so this we understood that this cannot be trimmed at positive angle of attack with this sort of a number.

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$$C_{m_{w,f,tail}} = 0.15 - 0.025\alpha$$

$V_H = \checkmark$
 $S_t = \checkmark$
 $l_t = \checkmark$

$C_{m_0} > 0$

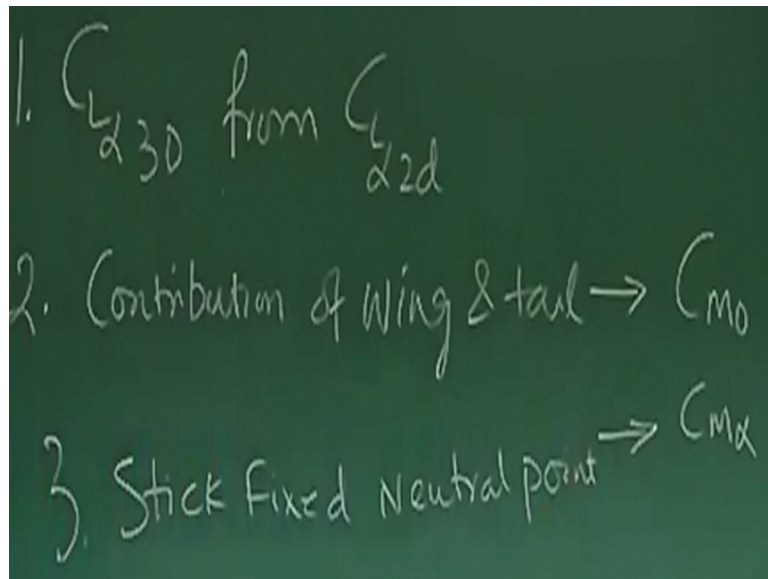
W. Lift
T: Stability + Trim

So the problem was given, why don't you change this whole CM wing fuselage and now we have to put a tail and so that it is represented by this expression 0.25α where you could see that first we are not talking about tail that means we have to add a tail. The tail will do two things. One is, it is not making CM Alpha more negative from -0.0035 per degree to -0.25 per degree. So it is becoming more statically stable and also now the tail should be such that C_{m_0} also becomes greater than 0, so that I can trim it at positive angle of attack, right.

And we solved this problem and we found that what is the tail volume ratio required or tail area required we found out and also we found out what is the tail setting angle. So this things were done. This is one of typical real life design problem. Generally, you design a wing thinking in terms of how much lift it should be able to generate because the area of the wing is very important, as the lift is the primary component, which produces lift to balance weight, or to produce enough lift more than weight whenever manoeuvre is required.

So, we are now very clear that wing is primarily to generate lift and tail for stability, stability plus trim. This is the primary role of a tail okay. It does not mean in longitudinal case you will see that wing also contributes to trim, provided you very smartly you locate aerodynamic centre and the CG of the whole airplane in a particular fashion. However, that is not the primary aim of the wing okay. So that part of the problem was more focused towards stability and trim. When I say stability, means I say static stability okay.

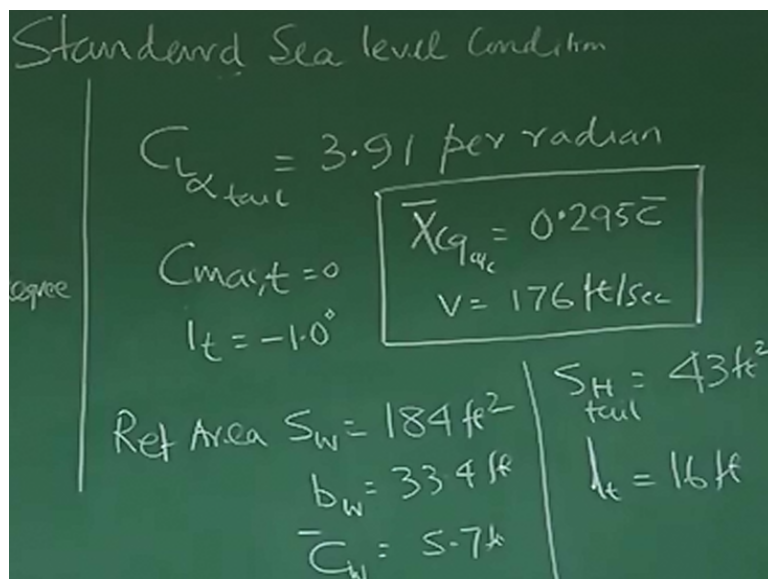
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Now the second problem we will be doing which is more a general case. This problem will help you to understand, to refine your understanding in terms number one CL Alpha 3D from CL Alpha 2D of an aerofoil number two it will also contribution of wing and tail towards CM0 towards CM Alpha and third stick fixed neutral point. Remember, I have constantly telling you I will be solving some practical problems so that you don't get lost into all those expression.

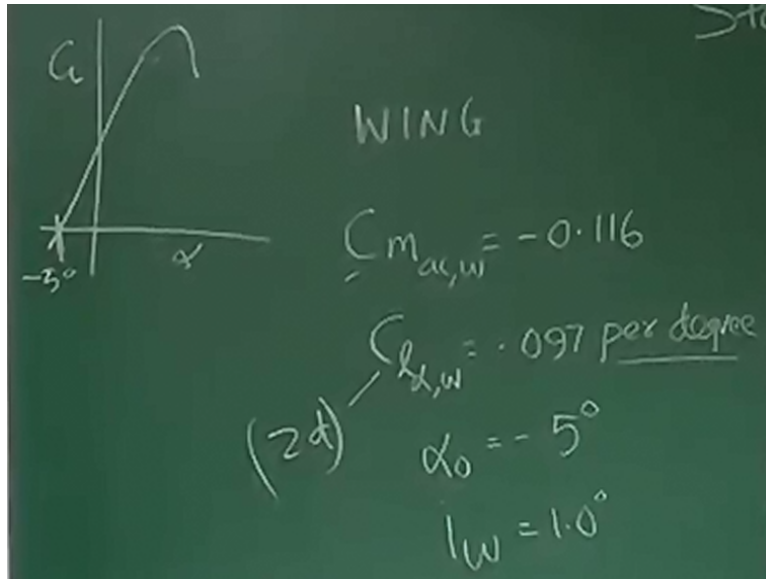
At once expressions you know for the first time you know how it is to be derived. Then more important it is how do I use it where finally I'm going to design an aircraft okay. So with this in mind now I will be solving a problem and let me define the problem.

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Let us concentrate on the problem. So I have given some number for a general aviation aircraft. Typically, this aircraft are below 1500 Kg class. It could be Cessna 182, it could be Piper Saratoga, around that family of aircraft. What is the information we have, let us first understand this information about wing? What we said, CMAC wing is equal to -0.116. What is the meaning of that? What inference we get about from this information is that it is a cambered aerofoil correct.

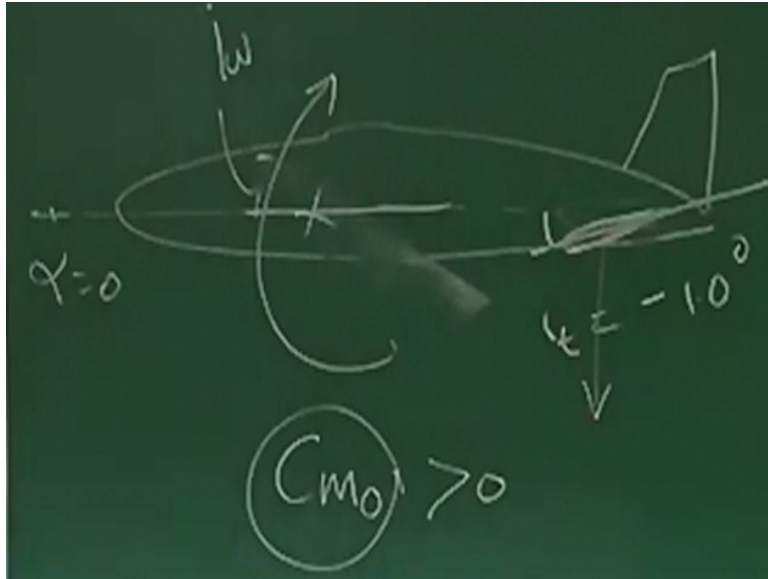
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Because there is a CMAC wing present had it been a symmetric aerofoil CMAC wing would have been 0. So one thing we understood, it is a cambered aerofoil. For a lift co-efficient this is 2D, 2D value which is 0.097 per degree. Be very careful generally, we operate in terms of per radian. right. So many book also will be using both per degree and per radiant at their convenience. But we should be very clear that this is per degree so if I want to convert it into per radian so I have to just multiply it by 57.3 okay? That is 180 by π .

Then again alpha 0 means alpha at lift is equal to 0 or $CL = 0$. Since it is a cambered aerofoil from where we have inferred, this is -5° . That is if I try to draw it, CL versus alpha, this is CL, this is alpha, this is that -5° okay. This is typically of cambered aerofoil. IW is the wing setting angle. What is this wing setting angle?

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If you again recall one of my lecture I have explained this. If this is the fuselage reference line, the wing is set at some angle with fuselage reference line. This is called IW or wing setting angle okay and that wing setting angle is 1° .

Now come back to the tail. This is the tail portion. Let me write tail here. CL Alpha, now this is it is a capital L Alpha of tail. That means, it is 3D value. Already 2D effect has been 2D aerofoil value has been converted into 3D value. So this is directly given 3.91 now it is given in per radian okay. SO CMAC tail is given as 0. That means the tail aerofoil is symmetric aerofoil. Then IT is -1° and what is IT? Then you can check my lecture.

If this is the tail, I'll be putting the tail like this and this is IT equal to -1.0° . So the tail is not like this, tail is set at the minus setting. Why this tail is put as minus, you know by now that at $\alpha = 0$, suppose this is not here. Suppose this is wing is like this and at $\alpha = 0$, tail will experience force in this direction and about CG, it will give a value of positive CM so we say, we'll get some amount of CM0 positive by giving a tail setting angle, negative correct.

Why this CM0 positive I'm talking about now? Because we know that if I want to trim an aircraft, at a positive angle of attack, I have to ensure that CM0 is greater than 0 okay? So this is the description of the airplane. What is the task?

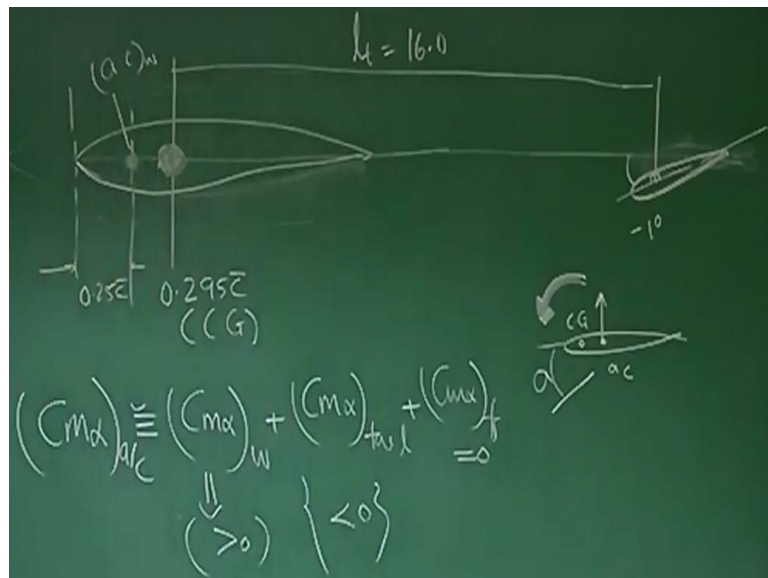
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Calculate Contribution of Wing & tail

- (I) $C_{m\alpha}, \frac{\partial C_m}{\partial \alpha}$
- (II) C_{m0}
- (III) Neutral point stick fixed

Task is to calculate contribution of wing and tail for CM Alpha or DCM by DCL, whatever you want to compute and second for CM0 and third, just need to find neutral point, neutral point, stick fixed. We are talking about stick fixed question is clear. With this configuration of an airplane, we want to calculate, the contribution of wing and tail. We are not including fuselage. We'll have one session on fuselage. So we are now focusing on contribution of wing and tail to our CM Alpha or to our static stability, CM0 was stream and find any neutral point for stick fixed case okay. So let us solve this problem.

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Let us again come back to this configuration. One of my advice to all of you would be just don't start solving a problem seeing some numbers, using some equation. Try to look through those numbers and visualize an aircraft okay. That will tell you many more things than what I have been telling you or what the numbers will tell. If this is the wing, tail is somewhere here.

Tail is symmetric, but tail is not only symmetric, tail is set at -1° . We should correctly draw it like this. This is -1° and this wing is cambered so I should at least pictorially draw as a cambered aerofoil okay.

We are neglecting fuselage. What is more information we have? We have let's say it is the fuselage, I'm just drawing dotted line. We have location of CG somewhere here which is 0.295 Civa and AC at 0.25 Civa okay this is clear. And further you could see LT is given here. LT is what? LT is the distance between CG of the airplane, this is CG of the airplane and the AC of the horizontal tail.

So this is LT and this is given as 16.0 clear? So you are going to picture what information you get from here. From this drawing we understand that wing is a cambered wing tail has a setting angle. What more? Very important thing is here since we want to calculate CM Alpha for the whole aircraft as equivalent to CM Alpha because of wing plus CM Alpha because of tail of CM Alpha because of fuselage we are putting it to 0 and neglecting it.

Transfer CM Alpha of the wing, one thing you should immediately get and you should be very clear and that will show how much you are understanding it. If this is the CG and this is the AC of the wing, one thing I know that the AC of the wing is ahead of CG of the wing.

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$$1. C_L \rightarrow C_{L_\alpha}$$

$$(2-d) \quad (3-d)$$

$$C_{L_\alpha} = \frac{C_{L_\alpha_w}}{1 + \frac{C_{L_\alpha_w}}{\pi A R_w}}$$

$$C_{L_\alpha_w} = (0.097) 573$$

$$1 + \frac{(0.097) 573}{\pi \times 606} = 4.3 \text{ per radian}$$

$$R_w = \frac{b^2}{s} = \frac{(334)^2}{184} = 6.06$$

So the contribution of wing towards stability or to CM Alpha will be stabilizing or destabilizing we know that for it to give stabilizing effect, the AC should be behind CG right.

If I again try to help you think, remember if AC is behind CG, if there is a disturbance, the force will come here which will give a nose down moment. So for a positive alpha there will be a negative pitching moment. So $C_{m\alpha}$ will be negative so that leads to static stability. Or in other terms I say because AC is behind CG, so the moment there is a disturbance, it has it will generate this force C_L which will give you a pitching.

Pitch down moment or it has a initial tendency to come back to equilibrium again. That is alpha equal to 0 or alpha equal to alpha desired or alpha star. $C_{m\alpha}$ sign for wing we will find it should come positive. Because this is destabilizing. Second thing what you understand, for tail, for tail you could see AC of the wing is behind CG. So it will have stabilizing components, so $C_{m\alpha}$ will be negative.

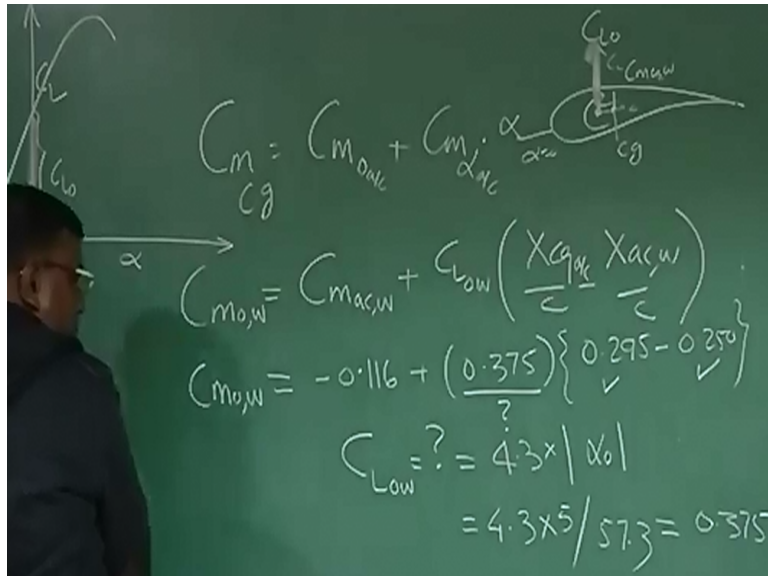
You could see from here also, if there is an alpha, then the force upward which will give nose down moment above CG, so this aft tail which we are talking about now and I'll be addressing it as tail, is always stabilizing. The moment you put a canard or something here, which is called forward tail, then it becomes destabilizing because the AC is now ahead of CG. okay. We are now do a calculation how $C_L \alpha_{2D}$ can be converted to 3D right.

The formula which you already know $C_L \alpha_{3D}$ is equal to $C_L \alpha_{2D}$ by $1 + C_L \alpha_{2D} / \pi$ aspect ratio. I'm taking E as 1. right. Now let us do it.

What is the aspect ratio for the wing? We have to calculate the aspect ratio for the wing and we know aspect ratio is B^2 / S . What is B for the wing? Somewhere it is 33.4 feet. 33.4^2 by S is also given 184. So you will get the value which is equal to roughly 6.06 right. So this is aspect ratio of the wing.

Now it is forward. $C_L \alpha_{wing 3D}$ will be what about $C_L \alpha_{2D}$ value. Let me put that 0.97 this is per per degree, so I'll multiply it by 57.3 to make it per radiant $1 + 0.97 / \pi$ into 57.3 divided by π into aspect ratio which is 6.06 and if we do the calculation, if I am not wrong, this is 4.3 per radiant. This problem is also from Nelson which I showed you yesterday. So those were interested they can see this problem there. But let us understand, this problem I'm using to synergize whatever I have talked about stability. Similarly C for control as well. okay this is fine. Now, what is our aim?

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We know C_m equal to C_{m0} , C_{mCG} of the aircraft, C_{m0} of the aircraft, plus C_m Alpha of the aircraft into alpha right? We are not talking about C_{m0} wing because C_{m0} aircraft has C_{m0} wing plus C_{m0} tail plus C_{m0} fuselage. So we are try to calculate C_{m0} wing and this we know if we check is $C_{mac,w}$ wing plus C_{L0} wing into X_{CG} by C minus $X_{ac,w}$ wing by C . I hope you understand.

This expression, remember just ot make things clear to you if you have a cambered aerofoil at AC, the C_L , C_D and there will be a $C_{mac,w}$ wing which has come because of transferring the forces and typically for cambered aerofoil this value is negative. And now suppose you are CG is somewhere here, so at alpha equal to 0, there is a C_L . So what will happen. This is C_{L0} , C_{L0} into this distance will give you pitch of moment. So here is typical case. So what is $C_{mac,w}$ wing?

I put the number which we already know. -0.116 plus 0.375 into 0.295 minus 0.250 which is nothing but this is X_{CG} and this is $X_{ac,w}$ wing. If I do this I can get C_{m0} wing. What question is coming to your mind? If you say this is given 0.116 minus $C_{mac,w}$ wing, this is X_{CG} of the airplane. It is given. This is F aircraft right. It is given. $X_{ac,w}$ wing is 25.25% is also given. But wherefrom I got this number? Suddenly I have written it. wherefrom I got this number? So let us see what is this number.

This number is nothing but C_{L0} wing. How do I find out C_{L0} wing. Let us see what is C_{L0} . We draw here. This is cambered aerofoil. This is C_L , this is alpha. So at alpha equal to 0, this value is C_{L0} . So how do I find C_{L0} . Simply this will be since linear, this will be C_{L0} into

absolute value of this alpha right? So CL0 will be CL Alpha which is 4.3 into absolute alpha 0. That is equal to 4.3 into 5 degree divided by 57.3. Why I'm dividing by 57.3? Because this 4.3 CL Alpha is per radian. And 5 is in degree, alpha-0. So I have to convert degree into radian so divided by this and thus we get 0.375 correct? This part is clear.

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$$C_{M_{cg}} = C_{M_{DAL}} + C_{M_{\alpha}} \cdot \alpha$$

$$C_{M_{\alpha,W}} = C_{M_{\alpha,W}} + C_{L_{ow}} \left(\frac{X_{cg,W}}{c} - \frac{X_{ac,W}}{c} \right)$$

$$C_{M_{\alpha,W}} = -0.116 + (0.375) \{ 0.295 - 0.250 \}$$

$$C_{M_{0,W}} = -0.099, \quad C_{M_{0,t}} = 0.199$$

$$C_{M_{0,W}} + C_{M_{0,t}} = -0.099 + 0.199 > 0$$

So let me erase this. I now erase this, I erase this also because it is clear what from it has come. So this value will come or to be I advice you to do yourself this so that even if some mistakes are committed, you should not get a wrong number. okay. So this gets to -0.99 CM0 wing right. So are you happy if CM0 wing is -0.099, then you know one thing for sure that finally I want CM0 to be positive. And with major contribution towards CM0 should come from where horizontal tail by giving a tail setting angle and that is perhaps 1° tail setting angle was given. Let us check that.

We have seen we have got CM0 contribution from wing as -0.099. Now we want to find out what is CM0 tail for this configuration. But we know one thing that, if you want to trim the airplane at positive angle of attack, total CM0 should be positive. That means that whatever CM0 I get from tail, that should cancel this, nullify this plus add some positive value okay? So let us see how much CM0 tail we get from this configuration.

Going back to our expression, CM0 tail was given Neeta, VH CL-alpha tail Epsilon 0 IW minus IT. What was IW? Wing setting angle. IT, tail setting angle, now what was Epsilon 0. Epsilon 0 was downwash at alpha equal to 0. And when Epsilon0 will be there when there is a cambered aerofoil. Because at alpha equal to 0 there will be still lift for a cambered aerofoil

wing. Lift means there is a pressure difference between bottom and the top surface and there will be vortices which will give downwash value.

And the expression to calculate ϵ_0 is $2CL$ by π aspect ratio and for α it is CL_0 . So I'm putting 2 into 0.375 divided by π into aspect ratio of the wing. This is very important. We need to calculate ϵ_0 using this. The values are known. CL_0 is 0.375. Already you have computed that, aspect ratio you have computed to be 6.06. So if I use this expression, I get ϵ_0 as 2.3° okay? So if I put all those things here, I could check Neeta VH 0.66, CL_{α} 10 3.9, ϵ_0 2.3, calculated from here.

1° is the wing setting angle given IT is -1° , so $-IT$ means minus -1° . But remember, this whatever degree is there, that has to be converted into radian by divided by 57.3 because this whole term is getting multiplied by 23.9 which is per radian. right? So if I do that I get CM_0 tail as 0.194. This is a wonderful number. You could see, if CM_0 is 0.194, so CM_0 wing plus CM_0 tail becomes $-0.99+0.194$. so you could see this is greater than 0 right. So how by giving a tail setting angle of 1° .

We have not only neutralized this CM_0 contribution because of wing which is negative, we have also added some CM_0 value so that I can trim the airplane at positive angle of attack. That is pictorially if I see, if this was the CM_0 wing once I put the tail, what we have done, the tail setting angle, this CM_0 has now become positive okay. And something, α can be trimmed at positive angle of attack okay. This is clear.

So this is because of wing and then somewhere because of tail is here, when I add this one and two, somewhere I get here, which is this value. So indeed it is a design where appropriate tail setting has been given which is 1° and also wing setting angle given so that you will find finally here CM_0 is positive okay. This is very important. So we have seen the contribution of wing, contribution of tail on CM_0 . We now know how to use these expressions. Right.

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Contribution of wing
to: C_{m_α}

$$C_{m_{\alpha,w}} = C_{L_{\alpha,w}} \left\{ \frac{X_{CG}}{c} - \frac{X_{AC,w}}{c} \right\}$$

$$= 4.3 (0.295 - 0.25)$$

$$= +0.1935 \text{ per rad}$$

+ve

We now see how to compute the contribution towards stability. Now we are talking about contribution of wing towards C_{m_α} or static stability. One thing prior you know that AC of the wing for this configuration is ahead of the CG of the airplane. So C_{m_α} sign will be positive. Let us see what happens. If I write C_{m_α} wing which I have derived is equal to C_{L_α} wing into X_{CG} by c minus X_{AC} wing by c .

If I now put the numbers then it comes to be 4.3 into 0.295 - 0.25 and this is 0.1935 per radian. So indeed this is positive. It should be positive because AC of the wing is ahead of CG okay.

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Tail

$$C_{m_{\alpha,t}} = -\eta V_H C_{L_{\alpha,t}} \left(1 - \frac{\partial \epsilon}{\partial \alpha} \right)$$

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{2 C_{L_{\alpha,w}}}{\pi ARW} = \frac{2 \times 4.3}{\pi \times 6.06} = 0.45$$

$$C_{m_{\alpha,t}} = -1.42 \text{ per rad}$$

(-ve)

Now see for tail let's see what is the effect of tail so we write again expression for C_{m_α} tail and that is given by minus $\eta V_H C_{L_\alpha}$ tail into 1 minus D_ϵ by D_α .

Neeta I take 1, VH I know CL Alpha tail I know. I don't know what is D epsilon by D alpha. So I calculate D epsilon by D alpha as 2. If I calculate D epsilon by D alpha by using this expression, π aspect ratio wing.

If I put this number I will get this as 2 into 4.3 by π 6.06. This is 0.45. Let me tell you 0.45 D epsilon by D alpha is pretty high. You could see that what is this expression. This CM-alpha tail? What is its role? It is a minus here. So this is basically stabilizing effect. If this number is more. If this number becomes equal to 1 then this will become 0.

So we will try to design an airplane such that this value is as low as possible for stability point of view right. So when I once I put that I get CM Alpha tail as -1.42 per radian. Now I could see CM Alpha wing was +0.1935 per radian, CM Alpha tail is 1.42 minus 1.42 per radian. So, this positive which was destabilizing, this is negative stabilizing.

So when you they have a combined effect the aircraft because of wing and tail has CM Alpha negative. That means it is statically stable right. That is in a way we have now seen what is the role of horizontal tail. We could see in destabilizing configuration has been converted into stabilizing airplane by using appropriate tail. And also tail has a contribution towards trim by giving setting angle in the tail, the CM0 has been made suitably positive okay. This is the beauty of this term, designer. You need to use it more frequently okay.