Aircraft stability and Control
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Lecture- 09
Longitudinal Control

Today in this session, we will be discussing about longitudinal control. What is the meaning of longitudinal control?

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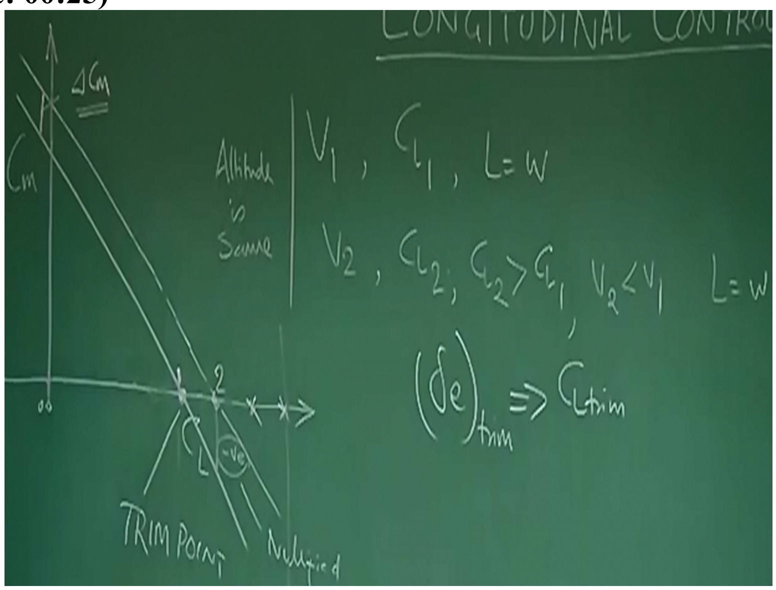

Let us again try to understand, from our very popular diagram, this is CM versus CL and you know at this is the equilibrium point or trim point okay. This is the trim point, this is the equilibrium point that is net force and movement are zero, and you could see from here CM is 0, this is zero, here as far as.

CM is concerned and what is typically is that one of the examples will be let’s say the aeroplain is cruising okay. So what is happening lift = weight, thrust = drag, and net moment is 0, so cruise is also a trim point. And we see that at trim, at equilibrium, the slope DCM by DCL is negative, so it is statically stable. This much you understand right. Also If I little bit of stretch further, I see that it by some disturbance.

If CL from here it gets increased, to another point let’s say CL at 2, the CL has increased, then the airplane being statically stable will automatically generate a negative pitching moment, right.

And it will try to reduce the CL so it will have initial tendency to come back to the CL trim, so this is another way of explanation that the airplane is having static stability, okay.

Now our problem of control is how to actually fly at a different CL, we want to fly at different CL the physical situation is suppose, I fly that is speed V1 at a CL CL1, let say V1 at a CL CL1 and I am maintaining the lift = weight correct. Let say that is here CL1 has flying at V1 and lift = weight. Now I want to fly it at another trim in another cruise condition where V1 is V2 and CL is CL2 and CL2 is greater than CL1.

So that naturally V2 will be less than V1 because weight is same, because lift = weight I am maintaining at the same altitude, the point is all these things I am doing where altitude is same right. This you could easily understand this aircraft is flying at altitude set density low the V1 and CL is CL1. Now, at the same altitude the airplane wants to fly at a lower speed V2, so he has to fly with a different CL I call it CL2, since lift as to same because weight is same.

So CL 2 has to be more than CL1 So, pictorially I want to fly at a trim at two this point, but we know by now the moment I try to take it here immediately some negative pitching moment will come, because it is statically stable so, if I want to really fly at this point two. So I have to ensure that this negative moment is nullified by a positive moment, and how I can get that positive moment, positive moment will come, when I put we call this, this is the tail, this is the elevator and CG is somewhere here.

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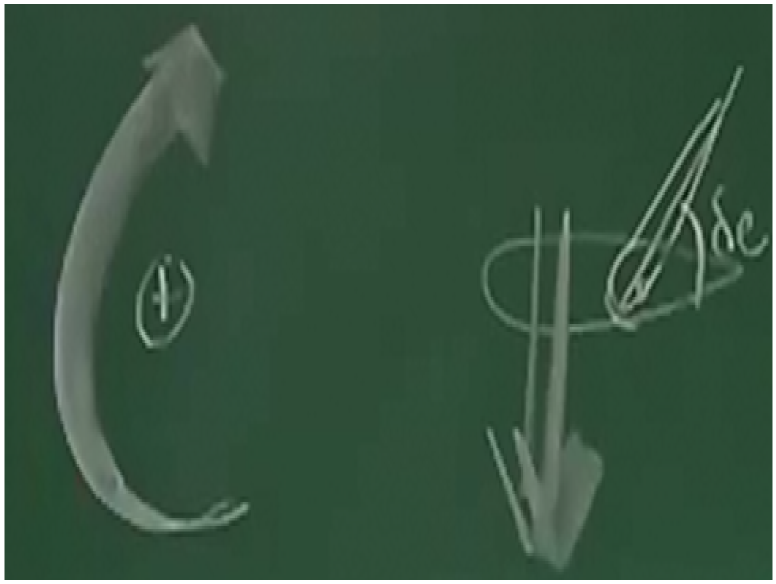

So if I want to generate the positive moment I have to put the elevator up, if elevator is up force will come downward which will give me a positive moment. That means I have to deflect the elevator such a manner at CL 2, such that whatever negative moment is generated here that gets nullified by this positive moment, correct. So what we have seen? By doing that if I draw this straight line as I have just extended I see there is an increase in Delta CM because, by giving

The elevator deflection that static stability is not going to change right okay. Elevator will not change anything on stability because its a control thing okay, Elevator doesn't decide the stability it is the total tail which includes the elevator area which decide the static stability, since that is same so this line will be parallel the slope will be same.

Only thing if I want to fly from one I want to step up to two I've to generate this much of CM Delta CM, and who generates this who helps us to generate this elevator deflection. So one question will be how much elevator I have to deflect to get equilibrium at CL2, that is one of the questions we will have okay. So that is the basically what you are looking for this longitudinal control what is the Delta E Trim required for different CL trim.

That is if I now want to fly here then how much Delta is required. If you want to fly here how much Delta is required and final questions comes can I really fly here. Because suppose if I am flying here and If I draw a line like this much of negative moment is to be nullified, by a positive moment generated by elevated deflection, it may happen we will see that you do not have that much of elevator deflection, because elevator you can deflect it to certain extend.

Beyond that say 15, 20 degrees normal case the elevator may stall. So we have to be also careful that I need to have an appropriate elevator deflection to trim, whatever CL I want to fly at. There is a limit of CL because, If I go on increasing the angle of a attack it may stall, also even if we generate that much of CL by some method you need to check whether you have got realistic elevator deflection possible or not.

Because elevator also beyond 15, 20 degrees will may stall in normal cases, right. So these are the issue we will be talking about, and we will try to develop a mathematical formulation, before we exclusively answer these questions, and we will be solving a problem to get better inside into what is happening here okay.

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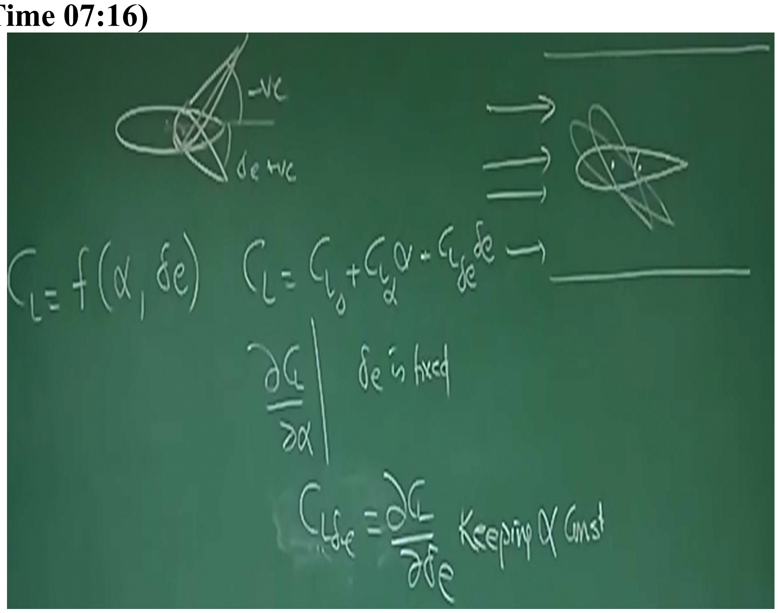

Let us start doing the mathematical formulation, for this is the elevator as per the convention. This is Delta E positive Delta E or elevator down is positive, and elevator up is negative this is a matter of convention okay. Now, if see CL is not only function of angle of a attack, it also function of Delta E, because as I am changing the elevator deflection the CL on the tail is changing, so overall CL of the airplane also will change right.

So far we were talking about CL the function of Alpha now see elevator is being deflected it also become function of Delta E again we have assume linearity and we expand CL as CL = CL0 + CL Alpha into Alpha + CL Delta E into Delta E, we know what is CL0 or what is CL Alpha so we know it is a partial derivative, that is DCL by D Alpha okay, where Delta E is fixed that is a partial derivative and changing one variable at a time so that is CL Alpha.

Similarly CL Delta E is DCL by D Alpha, I am sorry similarly CL Delta E CL Delta E is DCL by D Delta E, keeping Alpha COS Delta. Because these are partial derivative okay. If I want to generate CL Alpha in a tunnel, how do I do? I go inside a tunnel, I put mount the model inside the tunnel, and this is the velocity vector, so I can always put this at different orientation, and find out what is the lift and I know what is the angle of attack.

So I can find out CL Alpha similarly if I want to find CL Delta E then I will not change Alpha at all. What I will do? I will take this model I will keep this axis, and then I will give deflection some Delta E different deflection, and I will measure what is the lift force is coming and that additional lift.

Force at Alpha = 0 is attributed to Delta E, so from there I find out what is CL Delta E right. So also note one thing here when I am writing like this, we are assuming that the airplane is at there no pitching motion the airplane is at equilibrium. All are called all these modeling is done, where no such pitching motion or auxiliary motions are there, It is at an Alpha particular Delta E eliminated deflection. What is the CL it will see okay. For example when I actually deflect an elevator.

The airplane will take some time some oscillation and then go to this steady state condition type, fixed Alpha and fixed Delta E. So we are only analyzing that final point we are not talking about the transient, A transient we talk in terms of dynamic stability okay, So these are typically like static performance type okay. So now let us have a closer look what is this CL Delta E, we are very smart now we know what is CL Alpha.

We know what is CL0 so let us try to understand what is CL Delta E, How can I calculate CL Delta E? I repeat here that all these formulation what we are doing, you try to understand it but however the clarity you will become master, when you solve some numerical, you will find oh my god you are so straight forward thing right. So have patience and try to follow whatever I am telling.

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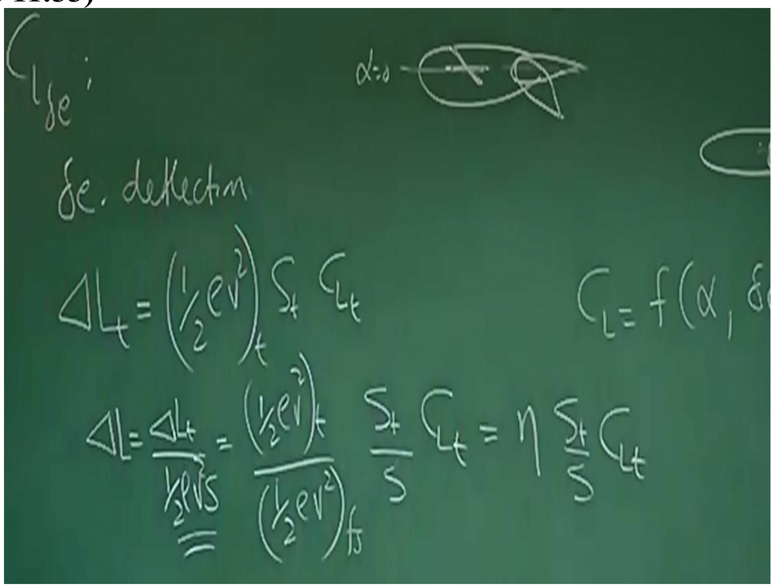

We are trying to see what is CL Delta E, see because of Delta E deflection, what is the Delta lift on tail that is = half row V square at tail S tail into CL tail because of Delta E. We are talking about Delta E deflection right. So, what is Delta L? That will be Delta L lift by half row V square S free stream right because this lift is proportional to the ST, horizontal tail area. But when I non-dimensionalize, I do it not on limit free stream but also referrals area, which is general wing area correct. So if I do that again I get half row V square at tail, that is dynamic pressure at tail half row V square free stream into ST by S, S is from airplane.

It is the wing area so this I can write as Neeta ST by S, CL tail, What is this CL tail? CL tail is change in the lift CL, lift coefficient CL tail because of elevator deflection, How can CL change? CL change if by giving elevator deflection, you are changing the angle of attack at the tail. See one way to visualize this if it is symmetric like this the 0 lift line is here.

That is at Alpha = 0 lift is 0 the moment, I deflect it by Delta E I am actually introducing a cambered, so I can say my 0 lift line has some angle right. So, actually I can say the Alpha at the tail is somehow changed because of elevator deflection right or aerodynamics will say the circulation around that oriented tail has changed so; there will be change in the lift okay. So this I can again write Delta L = Neeta ST by S and DCL T by D Delta E into Delta E.

What is the meaning of this? As I was explaining this CLT is because of change in Delta E. So I want to know how much CL tail changes with Delta E, if I know this gradient I multiply with D Delta E to get CLT, so I have written Delta L like that.

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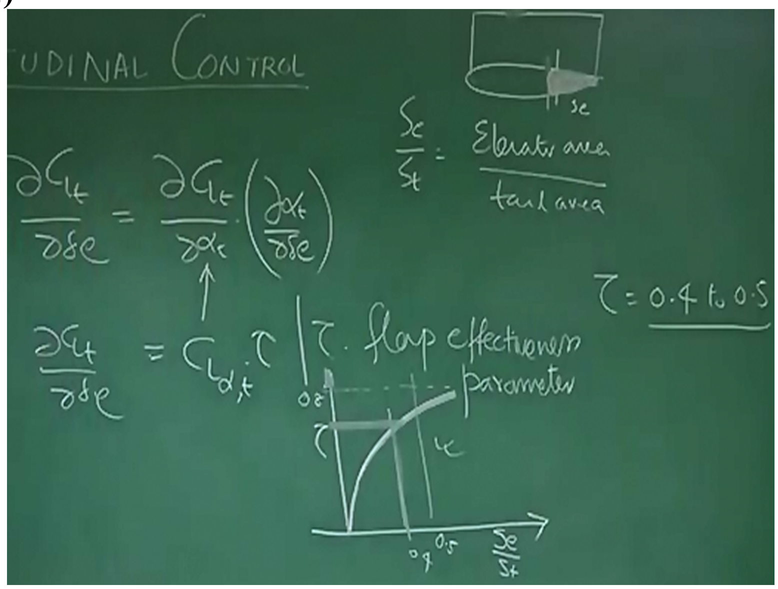

Now, let us carefully see D CLT by D Delta E could be written as DCLT by D Alpha T into D Alpha T by D Delta E. Do you see now what is happening, what is DCLT by D Alpha T what is this? DCL by D Alpha of tail this is CL Alpha tail and this D Alpha T by D Delta E, physically what does it imply it implies power unit change in elevator deflection, how much Alpha tail effectively is being changed, because that causes the additional lift.

And this is a parameter tow so, I can write this as DCLT by D Delta E, and CL Alpha T into tow. And this tow I can in text book you see this tow is called flap, effectiveness, parameter, and typically this you can get through some empirical graph, if you see most of text book will give you this variation is something goes like this.

And this is typically point eight, and somewhere around here it is point five and what is this? SE by ST. I will explain you what is SE by ST. And this is tow. Is this diagram clear to you? Typical variation so you can see Nelson book you will find out this graph this could be point four, so typically you are finding that tow value also will be for this sort of point four of SE by ST the value might be around point five point six at typical numbers right.

And these are easily you can find it out by using this graph, we want to know about SE by ST, where SE will be the elevated area, typically it is from the hinge line or even for all approximation is the elevator area you take okay. This is an horizontal tail area ST the complete of this. Typically SE is taken from the hinge line but for all these studies we take SE is the elevated area only, let's forget about hinge line I don't to confuse you because there are no floating tendencies.

We have not discussed any floating tendencies. So let's convert SE is the elevated area and ST is the tail area correct, if we know this ratio by using this graph we can find tow typical value of tow will be around point 4 to point 5, so tow will be around point 4 to point 5 okay. So what is interestingly we have understood that DCLT by D Delta E can be computed through the geometry of the tail.

Geometry will tell you what is CL Alpha tail depending upon the air foil, tow will tell you what is the ratio of SE by ST that is what you should appreciate okay.
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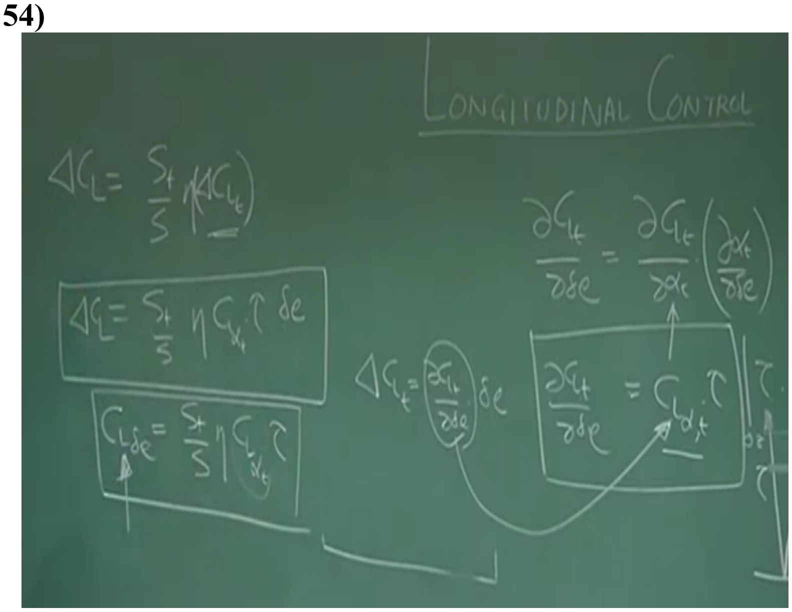

So again going back to Delta CL Delta CL was given as ST by S Neeta into CLT, which is typically is Delta CLT, If I am more appropriate and more correct in the last equation. I have mentioned CLT but you understand that, this is a change in additional CL you are getting because of Delta E okay. So, this I can write now as ST by S Neeta for this which is here, it will come as CL Alpha T into tow into Delta E. Let me repeat this, this Delta CLT is what? This Delta CLT which is incremental lift because of Delta E will be DCLT by D Delta E into Delta E.

What is DCLT by D Delta E? It is nothing but CL Alpha tail into tow, so I have put them here to get Delta CL equals to this clear okay. So now from this I can write CL Delta E = ST by S Neeta CL Alpha tail into tow. Right this divided by Delta E, so you will get this expression, So now you know how to calculate CL Delta E. Once the geometry of the tail is given okay the tail area tail wing reference area you know Neeta you know CL Alpha tail you know and tow also you know.

Neeta will be typically one around one for this case, we can take it as one also. Neeta you know as usual, it is more than one if some jet is exhausted blown into the tail or sometime it will be less than one but typically it is values around 1.9 to 1.05 something like that okay. This is clear you know what is CL Delta E okay. So what is the meaning of CL Delta E? What is the change in the CL because of elevator deflection Delta E.

What is the sign of this? you see CL Delta E, if I want to know the sign, I know if I I'm putting the Delta if positive is like this, Delta is positive, so what type of lift it is giving, it is giving lift upwards CL positive, so CL Delta is actually greater than zero sign of CL Delta is positive okay clear.

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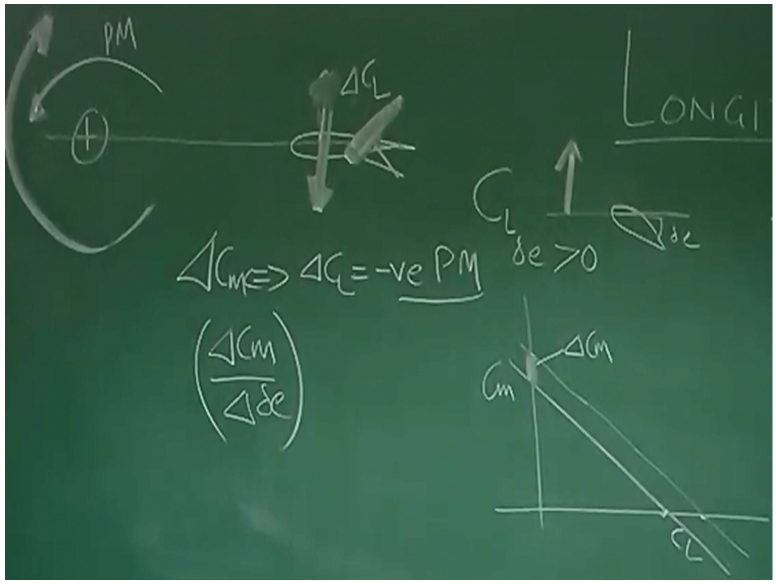

Let's do now what is CM Delta E, Again you see If this the CG of the airplane and here this is the tail and elevator is deflected like this, this is giving CL additional CL which will give a pitching moment nose down so this positive Delta CL because of positive Delta E will give what type of moment positive or negative. How do I check it, what is the sign of the pitching moment? Nose down is negative nose up is positive.

So this is giving what type of moment about CG nose down so whatever moment is coming Delta CM because of Delta CL right, it is giving negative pitching moment okay. This clear as far as sign conversion is concerned, so let us find out what is Delta CM by Delta Delta E that is per unit changing elevator deflection, how much pitching moment is being generated. Remember why that is important?

Because we have seen, if I go back to CM versus. CL, if I has I was flying here if you want to fly here, then it equivalent saying that I need to generate this much of Delta CM. Who generates this Delta CM? It is to the elevator, what is the mechanisms? Elevator if you deflected down it give Delta CL, then it gives pitching moment, In this case its negative if you want to increase positive, then you have to do other way like here.

I want to increasing CM right, so I have to ensure that elevator is up so that this force is down and that gives you nose up pitching moment okay. But in the nutshell this Delta CM is caused because of Delta E which gives Delta CL from CL I get Delta CM, So let us see the expression for a CM Delta E Delta CM by Delta E, So we are looking at for an expression for CM Delta E, CM Delta E we have already finished the expression for CM Delta E is done.

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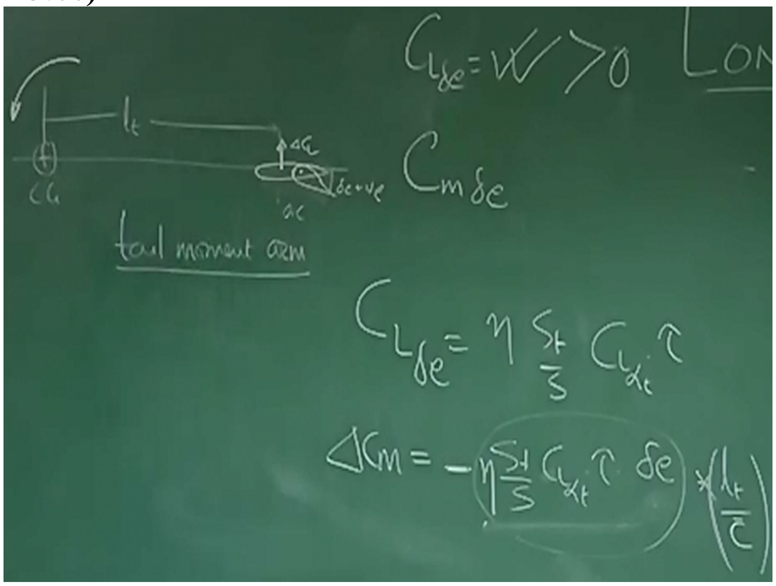

We now understand you also understand SIN is positive. We are now looking for CM Delta E. Who is caused CM? Is comes through elevated deflection, elevated deflection give CL and CL is the force and moment about CG will give CM that will be cool in saying CM per unit Delta E. So I want to now formulate for Delta CM. I know CM Delta is Neeta ST by S CL Alpha tail into tow.

So Delta CM because of this CL Delta E which is coming because elevator deflection that will be ST by S Neeta will be here, so CL Alpha T with tow into Delta E because this CL Delta E So what is the CL because of Delta E, CL Delta E into Delta E, so this portion is CL Delta E into Delta E, so this is the CL okay. Now it will give a moment, so let me write this Delta CL because of elevator deflection.

So moment means it has needs to multiplied with momentum LT, What is the LT? We know if this if this the CG and if this is a tail, horizontal tail. If this the AC so this distance is LT it is called tail moment arm okay for horizontal tail. So if I write like this do you think I am correct or something I am missing? When I am writing Delta CM please understand I need to write a non-dimensional, so this is non-dimensional null with C bar okay.

This should be very very careful about it, because I am talking about non-dimensional Delta CM is LT by C bar, but still this is not correct there is something wrong in this. What is wrong in this could you see remember, what I am in the moment and all, we are also bothered about the sign.

Please see here if Delta E is positive is like this, what sort of force it will generate, It is elevator Delta is positive by convention, so that will give force in this direction Delta CL at the non-dimensional quantity that will give what time of moment it will give nose down pitching moment, So I have to put minus sign here clear, just to take care of the sign convention, So now if you see Delta CM = - Neeta into ST by S into LT by C into CL Alpha T into Tow into Delta E.

So CM Delta E which is Delta CM per unit Delta E, it things are linear so I can easily use this is = - Neeta ST by S LT by C CL Alpha T into TOW, which I can address - Neeta VH into CL Alpha tail into tow. This is CL Delta E. What is VH? By we know VH is tail volume ratio right, VH is tail volume ratio STLT by SC bar. Do you see one interesting here? The interesting here is if I want to increase the controlled power this is also called for information elevator control power okay.

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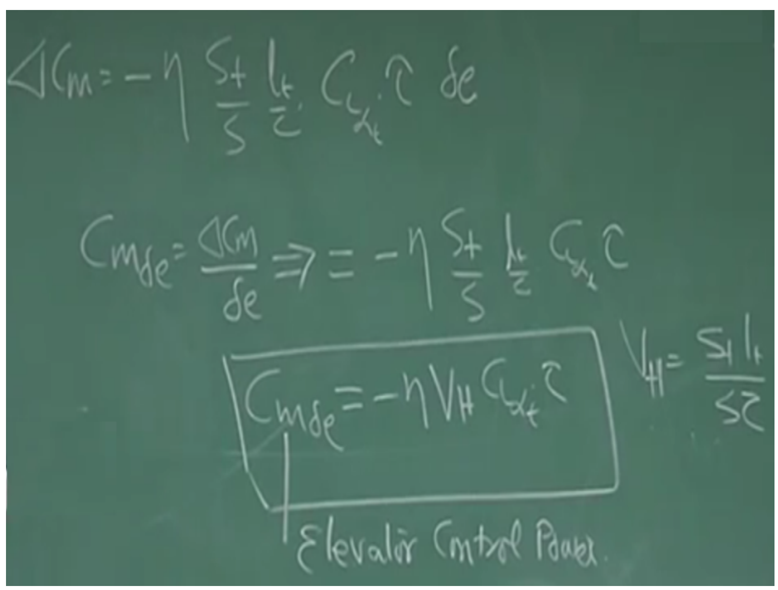

You can very obvious because if CM s Delta e is strong it will give a large moment for a smaller elevator deflection and you can easily pull pitch up or pitch down the aero plane. But if you see here I can increase CM Delta E by increasing the VH value that is tail volume ratio meaning there by if I want to increase CM Delta E then I have option like I increase tail area I increase tail momentum on I increase both of them or in a combination.

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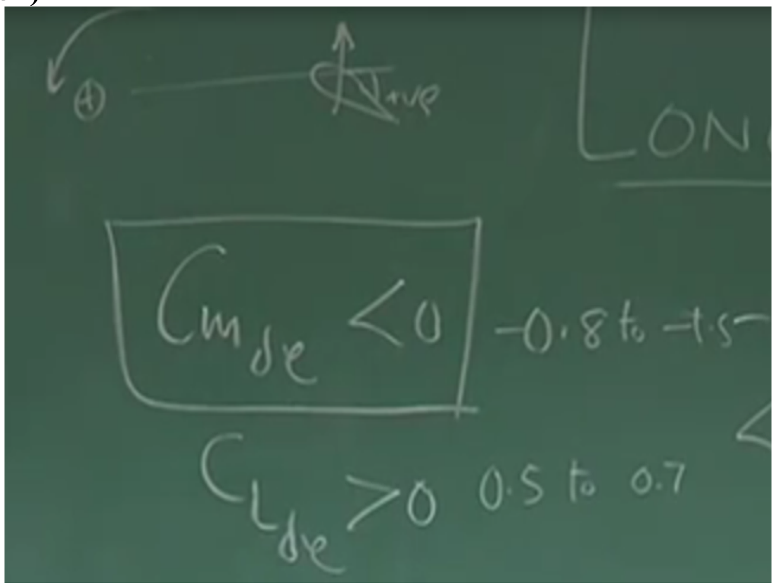

So this tail volume ratio is extremely important parameter which is evolving to our discussion and that's why I told you If VH, I can take initial value to decide from point five to one point zero given but you are clear if you have taken one point zero the CM can take value to very high. So we have now understood what is CM Delta E what is CM Delta e and it goes without saying from the expression and from the understanding you are very well cleared the sign of CM Delta E is negative right.

Physically also you know, If I deflect the elevator down which is positive and CG is here it produce a lift like this which give you the a nose down nose down moment which is negative so sign of CM Delta E is less than 0 and sign of CM Delta we have already check it is greater than 0 okay. Typical value of CM Delta E for light weight aircraft limit from - point eight to minus one point five CM Delta E it could be point five to point seven around all this type of range will get it right.