Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 1
Lecture No 01
Introduction to Dynamic Stability

Good morning friends. As you all know that we are going to venture into the concepts and understanding of aircraft dynamic stability and we need to know the purpose of this course. In the last lecture, I had given you a brief indication but let me a little bit more focused and when I write the purpose for this course title, primary title, Aircraft dynamic stability.

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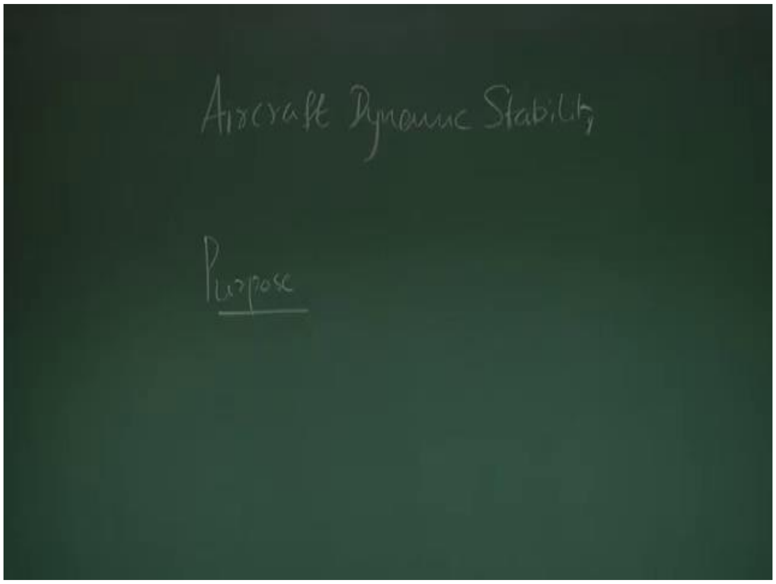

The purpose if I write, the purpose is to understand. The dynamic stability is of course there and what goes along with dynamic stability, the response of that. This response and dynamic stability, they are somehow linked. Okay? And if we talk about dynamic stability immediately, something will come to your mind, how the airplane is going to respond or in more precise term, how will be its transit. That is what we look in aircraft dynamic stability.

However, we would like to see dynamic stability behaviour or a response when the aircraft is disturbed from its equilibrium which goes along with the definition of dynamic stability. It is response, when it is disturbed from the equilibrium. But what type of a disturbance? It should be a small disturbance. So that should be kept in mind. That is, I would like to study dynamic stability of the airplane in terms of small perturbations about steady-state, about the equilibrium state. And you all know that Cruise could be one of the study states, steady climb could also be a steady-state. But mostly we would be focusing around Cruise.

So what is the understanding? We want to see the response of the airplane when the airplane is given small perturbations about its equilibrium state. Okay? Now what is dynamic stability? If I try to now again revisit, dynamic stability, the tendency of the airplane to ensure that the disturbed amplitude, that is when the aircraft is supposed moving, cruising at a steady-state and if I give a small disturbance say in terms of angle of attack and if this disturbance is withdrawn, I would like to see how the airplane is coming back to the equilibrium state. Right? And this part will be addressed all throughout in our dynamic stability analysis. Okay?

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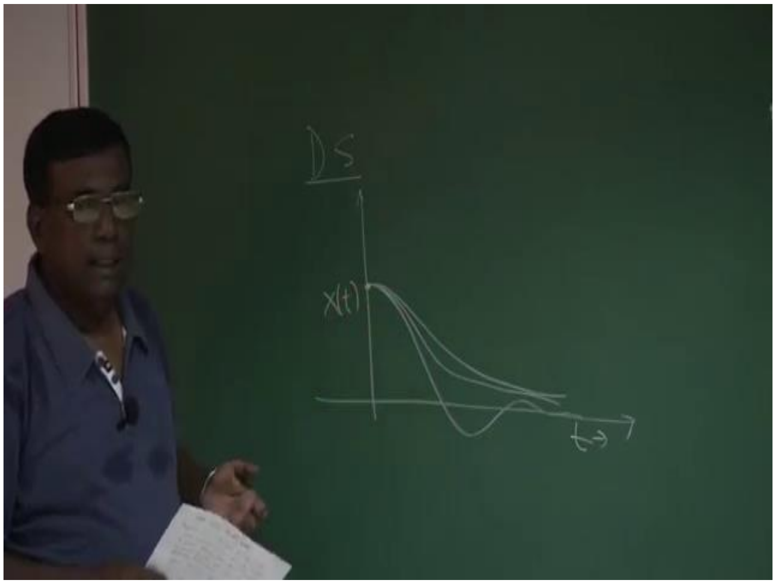

For example, if I say something is dynamically stable or the system is dynamically stable, graphically I show it like this. Let us say this is the disturbance. So I have given it a disturbance and if I withdraw the disturbance, whether it comes like this, whether it comes like this, the question is, how it is transient is behaving with respect to time and whether it is coming back to its equilibrium or not? Right? That is the area where will be addressing under the area of dynamic stability.

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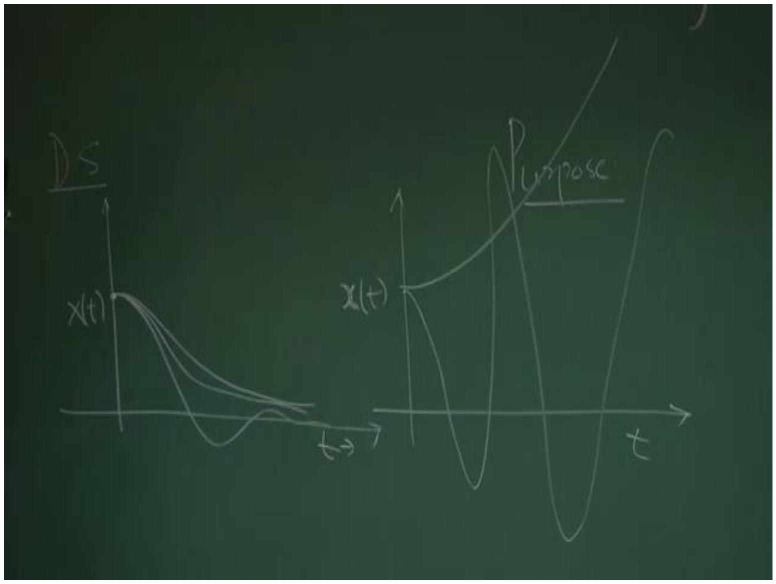

Even system is dynamically unstable. Then if I have given a disturbance, it may go like this, it may go like this, it may go like this, it may go like this. All are simply representing that system is dynamically unstable because in finite time, the disturbed element or disturbed motion variable is not coming back to the equilibrium.

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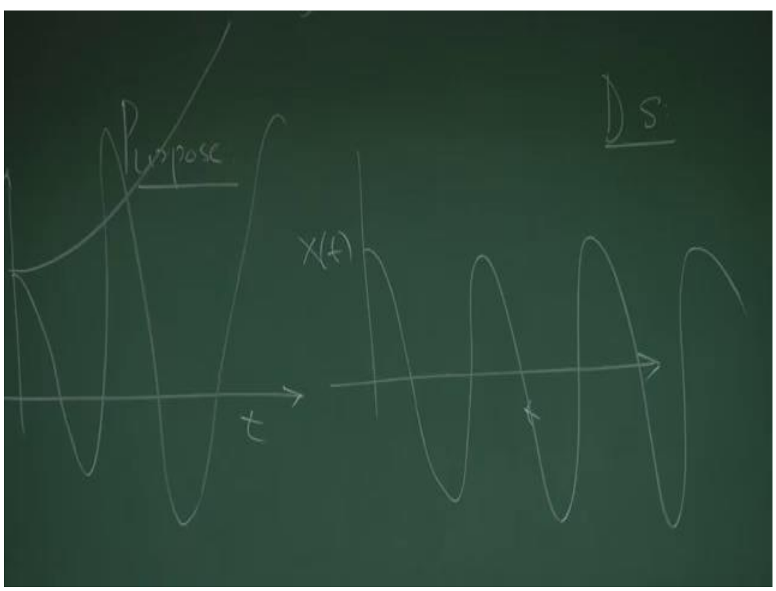

There could be a case where see, this is the disturbed quantity. This is time and this man goes on oscillating like this. You see that amplitude is not increasing but it is going on holiday. So it is actually speaking, dynamically neutrally stable. So this sort of cases, we will be discussing and this sort of understanding of the definition is required and what is the response here?

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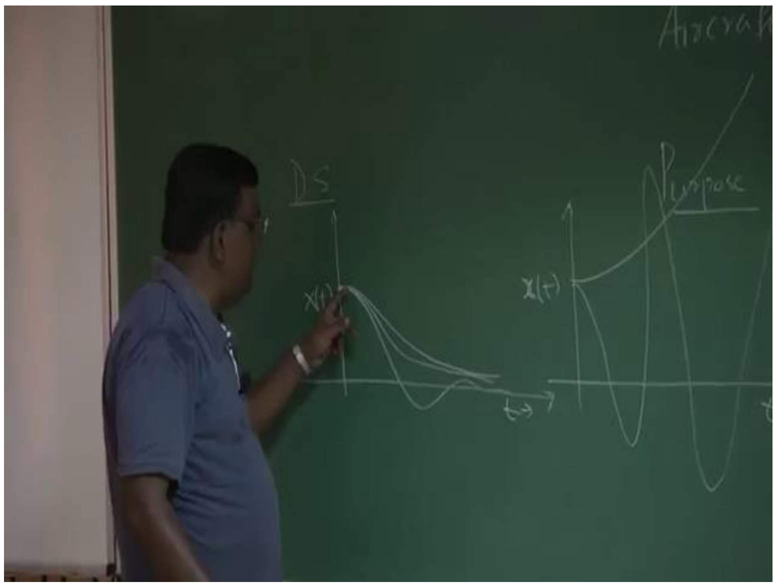

How this x is changing with time, that was the response. That is the transient for our case of analysis. Right? Please understand, when I write XT, this is the disturbed variable or perturbed variable.

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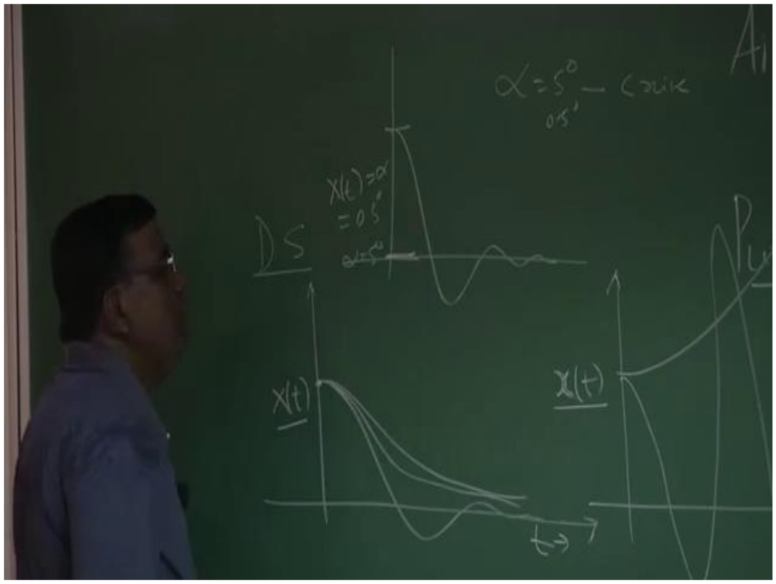

In the sense, suppose the plane was flying at alpha equal to 5 degree and it was maintaining a cruise. I give a small disturbance in alpha. Say, maybe I give 0.5 degree of disturbance. So this XT is 0.5 degree. Now if it comes like this, I say, the disturbed quantity is vanishing all the disturbance which is causing the disturbance to be withdrawn and it is coming back to the same. This was alpha equal to 5 degree and this XT is alpha and this line, equilibrium was 5 degree. It is again coming back to 5 degree. So that shows, the aircraft is dynamically stable.

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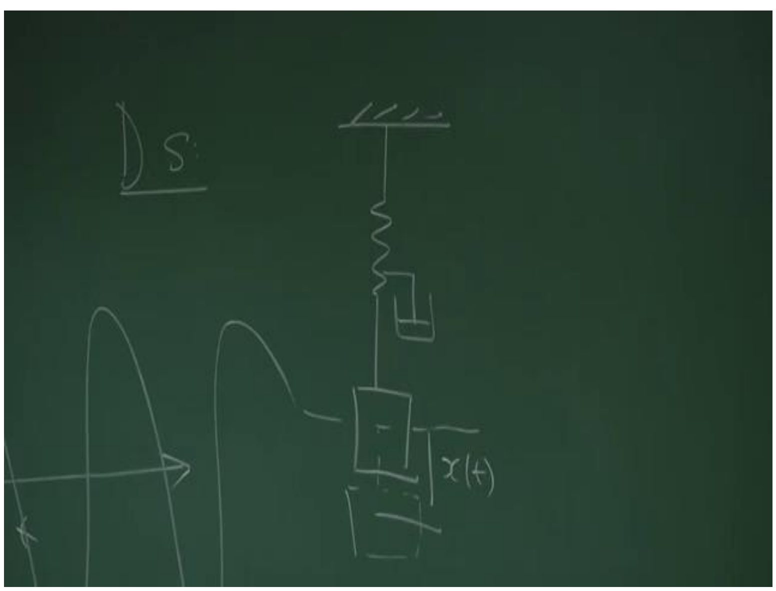

If we see a similar example for a mass spring damper system, once I disturb it, this distance is XT. This is a perturbed point, small perturbed. Right? The perturbation should be such that three should not be any gross stage in the whole system behaviour. In the sense, I should not stretch it so much that the spring constant will change, the length of the spring will change, the material cross-section will change. All these things are not allowed. Okay? That is a small disturbance.

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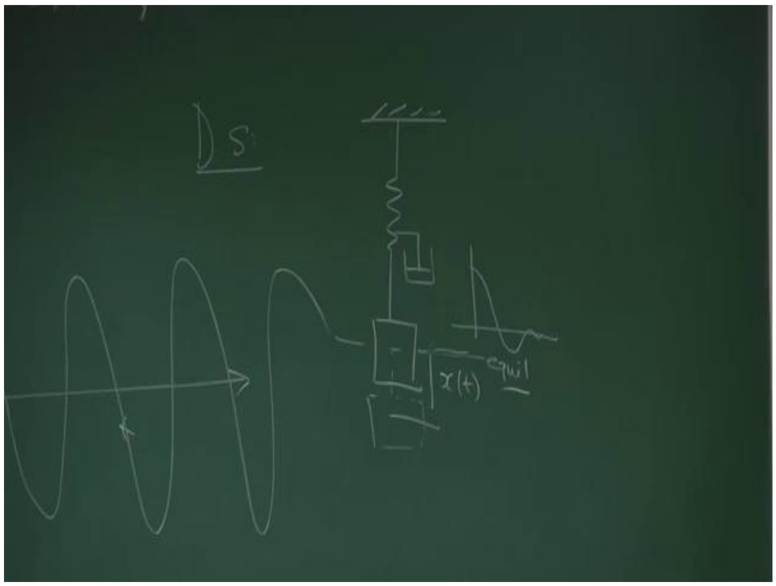

And as I release this, it comes back. Again if you see, it comes back to equilibrium which is here. Then I say, the perturbed quantity or XT has vanished, it has become 0. It has come back to the original equilibrium condition. Then you say that it is dynamically stable. So you could see that when I am talking about dynamic stability for aircraft, suddenly I am forced to bring this mass spring damper system for discussion. Why this is so? Let us also try to understand.

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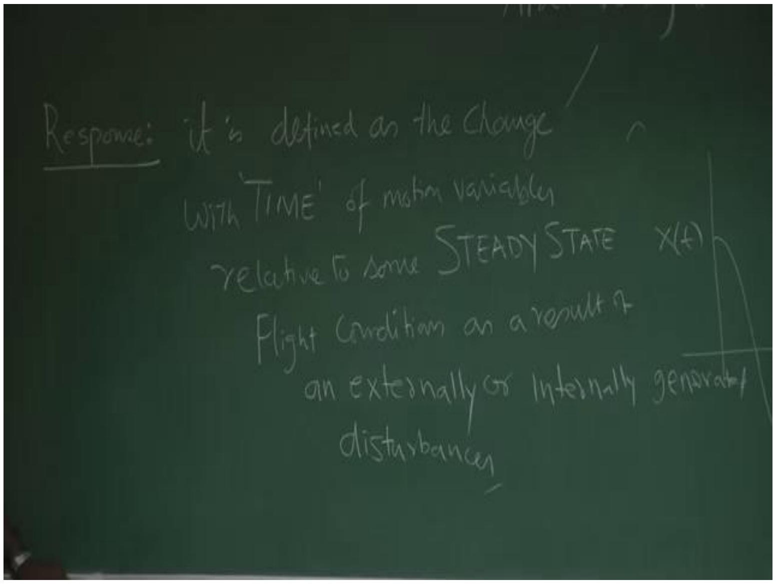

Before I go to that, let me write a classical definition for response. And you should know this. Although we have discussed as a matter of understanding, let me write the classical definition. It is defined as the change with 'TIME' of motion variable relative to some steady state flight condition as a result of an externally or internally generated disturbance.

Very important. Your aircraft, we will try to understand this. This is clear? It is defined as the change with time. That is we are talking about time history of motion variable. What are the motion variables? When the aircraft is moving like this, one of the motion variable is angle of attack, one of the motion variable is Q, picture rate, speed, so many things, right? Relative to what? Relative to some steady-state flight condition.

Let us say we have taken a cruise. At cruise, it is flying at alpha or angle of attack equal to 2 degree and the disturbance across it, additional 0.1 degree. We are seeing, how this 0.1 degree is changing with respect to the steady-state which is 2 degree. So relative to steady-state flight condition, why this has come? As a result of externally or internally generated disturbances.

What is is an externally generated disturbance for aircraft? I am going, moving like this and suddenly a huge crowd is coming or a huge density variation is there, right. There may be an upward gust, turbulence. So they become external disturbances. Now what about internal disturbances? I could always try to change from one equilibrium to another equilibrium by giving an elevator input, additional elevator input. Suppose I am moving at alpha equal to 3 degree and I am given Delta is 2 degree.

Now I have to go to alpha equal to 5 degree. So I have to increase the elevator angle. So now, from this steady-state, and given either 2 degree or 3 degree. So that is the externally generated disturbance.

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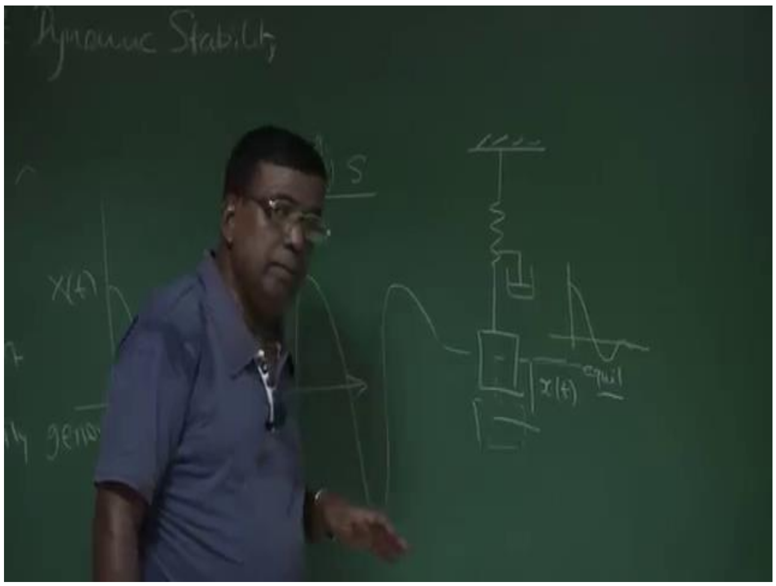

In all the cases, it should have the tendency to come back to the equilibrium once this is withdrawn. That is important. That is why I thought, the response part I should write carefully so that you know what is happening. Now I come back to the statement whenever I am talking about dynamic stability of airplane, why am I bringing mass spring damper system? Okay? So let us first understand that.

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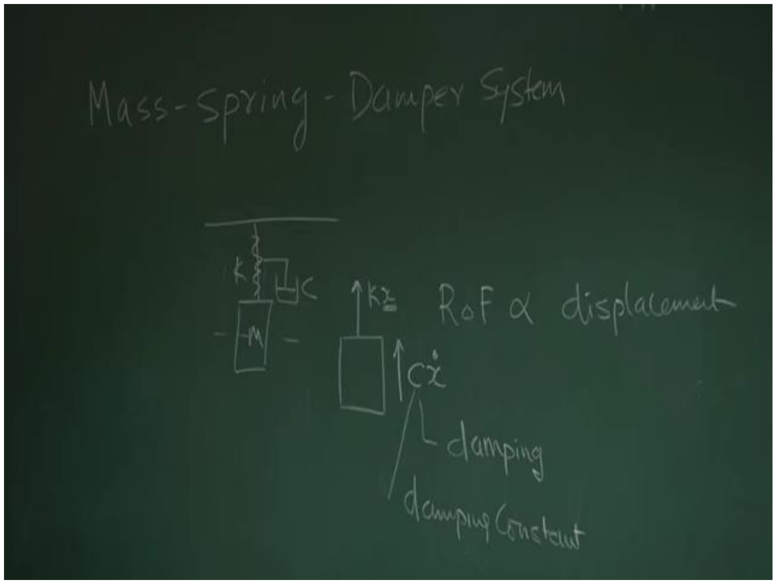

If I come to a mass spring damper system, you know typically the diagram but this is mass M and there is K and there is damper. This is K. You know, as I stretch it, the mass, there will be a restoring force, KX. X is the disturbance, a small perturbation from equilibrium. So this will try to take it back to equilibrium. So there is a restoring force proportional to displacement. Also you know that there will be a restoring force proportional to the rate of change of this small disturbance. And this we call as damping and C is called the damping constant.

Please understand here, damping is basically part of friction damping we are talking about. That means, energy is lost because of friction in terms of heat, primarily in terms of heat. And there is another assumption that we are talking about linear damping, that is this is CX dot. This is linear. There could be non-linear damping also. But we are talking about linear damping and we are having the liberty to assume it linear damping because we are talking about small perturbations. So what is understood here?

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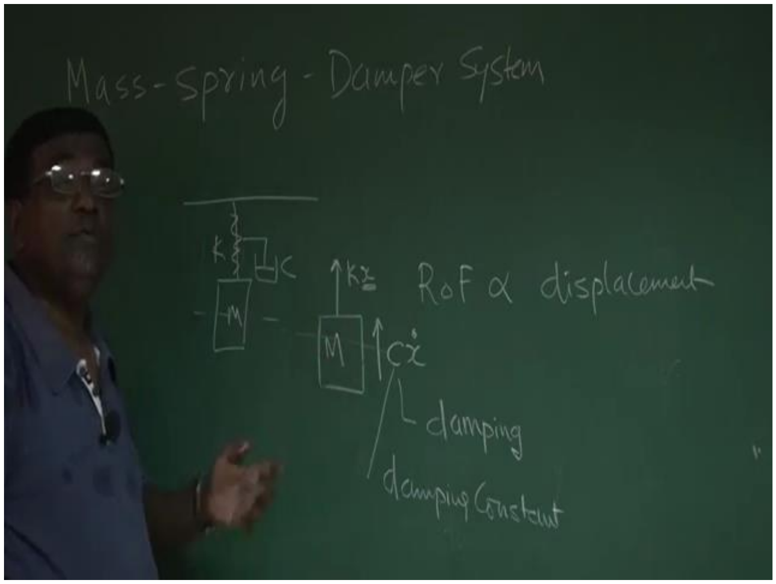

If this is the mass, there is one restoring force as soon as you try to take it out from the equilibrium condition and also depending upon what rate you are taking it out, there will be again a restoring force which will be damping in nature. These 2 forces combined, will try to oppose this motion or try to bring it back to equilibrium, tendency to bring it back to equilibrium. Now think of an airplane.

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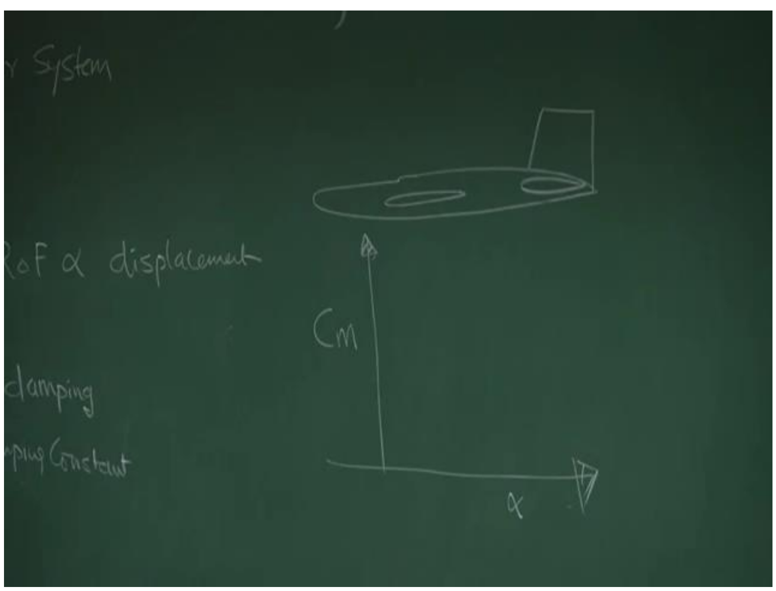

If I see an airplane, I am sure you have done your second course on static stability. Remember CM vs alpha. What was CM? CM is the pitching moment coefficient.

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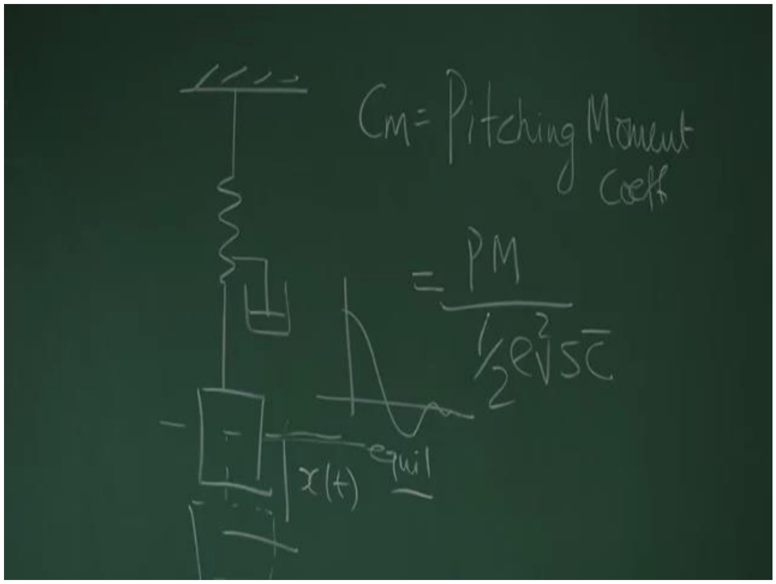

So let me write it here, CM as pitching moment coefficient and which is defined as, pitching moment (inaudible 14:20) by half lo V square SC bar. All these things you know. And what is the convention of pitching moment? Its nose down is negative and nose up is positive. Clear?

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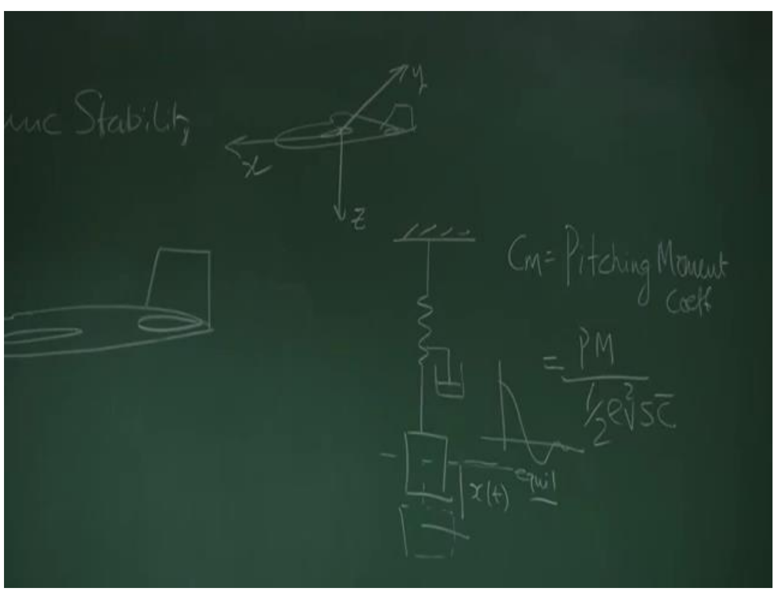

And to be more explicit, if I draw the airplane like this, if this is the x-axis and if this is the y axis, and this is the z-axis which is the body fixed axis? Pitching moment is motion about Y axis. Nose up is positive, nose down is negative.

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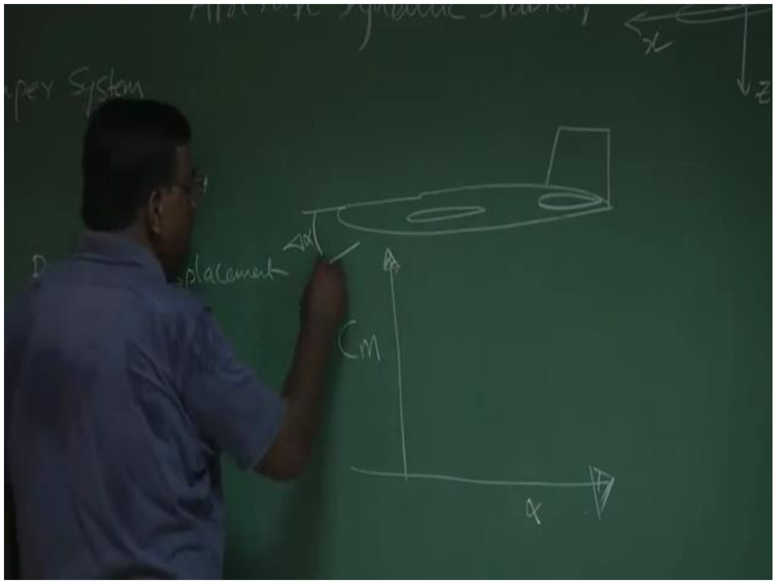

Now coming back to the static stability for airplane, we realise that if there is a disturbance Delta Alpha that is, the airplane was moving at this equilibrium and suddenly there is a Delta Alpha. If it was flying at 5 degree, then there is a 0.1 degree of disturbance, then dynamic stability, we tell that in finite time, it should come back to 5 degree. Okay, one of the disturbances is withdrawn.

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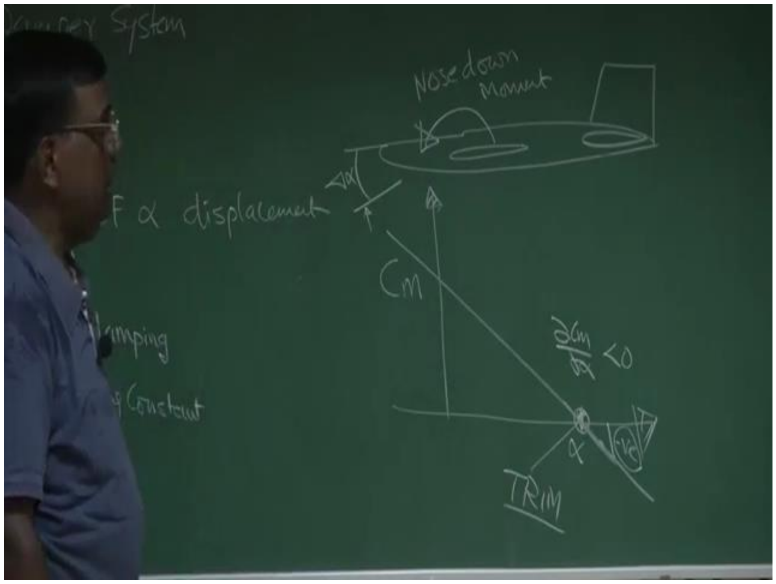

So if this is is more than 5 degree, it should generate a nose down moment. We all know that and that is how we draw CM vs alpha and the slope, DCM by D alpha negative. If at equilibrium, DCM by D alpha is negative, we say the body is statistically stable. No doubt about it. We can again crosscheck here. If the airplane is flying at this trim, why this is trim? Why do you define this as trim? Trim means, the CM is 0.

The airplane is going like this. No such motion. It is going, cruising like this. Pitching moment is 0. And of course for the cruise, thrust is equal to drive, lift equal to weight. Net force is also equal to 0. So now suppose, the airplane is flying like this, because of some disturbance, angle of attack has increased. Now what this slope will tell you? It will immediately generate this negative pitching moment.

So the airplane, if alpha has increased because of some reason, airplane will give a nose down moment. That is the significance of DCM by D alpha less than 0. But what is the meaning?

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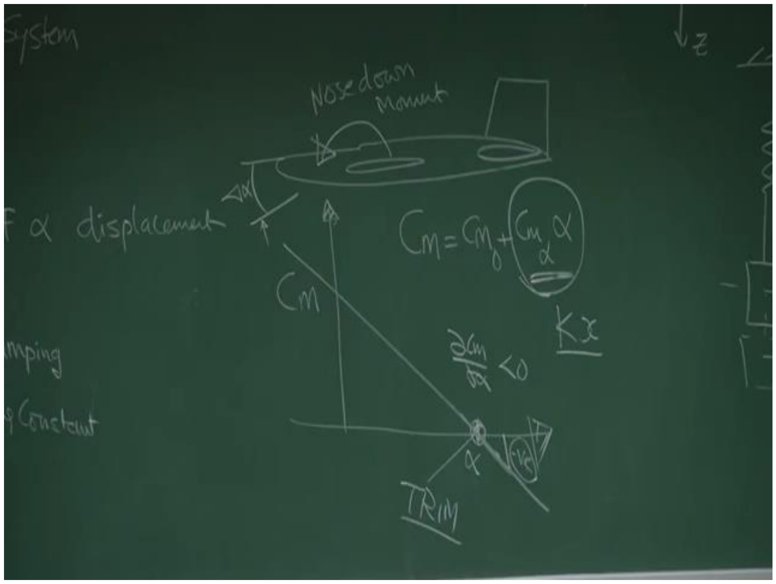

The meaning here is we understand, when I write CM as CM not + CM alpha into alpha. Remember? That is DCM by D alpha into alpha. So if this is negative, this contribution is a restoring moment. Like you have for the spring case, K into X, the restoring. You see, alpha is the displacement, x was displacement. So this is restoring force for spring and this is restoring moment for the airplane. So this CM alpha into alpha, KX into KX, they are analogous. They have a spring action. That is why, you will see that with CM alpha, we try to relate the stiffness of the air plane along which (())(17:28) stiffness of the airplane.

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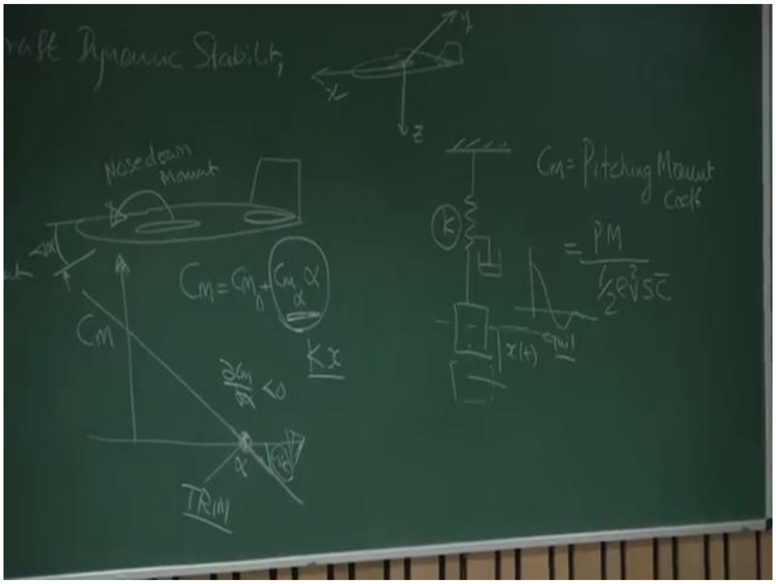

Because in a spring K, we always talk in terms of spring stiffness. Right? So there is similarity. For a statistically stable airplane, because of CM alpha into alpha, now what about damping? Let us see about damping now. These things, we have discussed in my second course. You might have seen that. But I am just revising that so that you feel comfortable.

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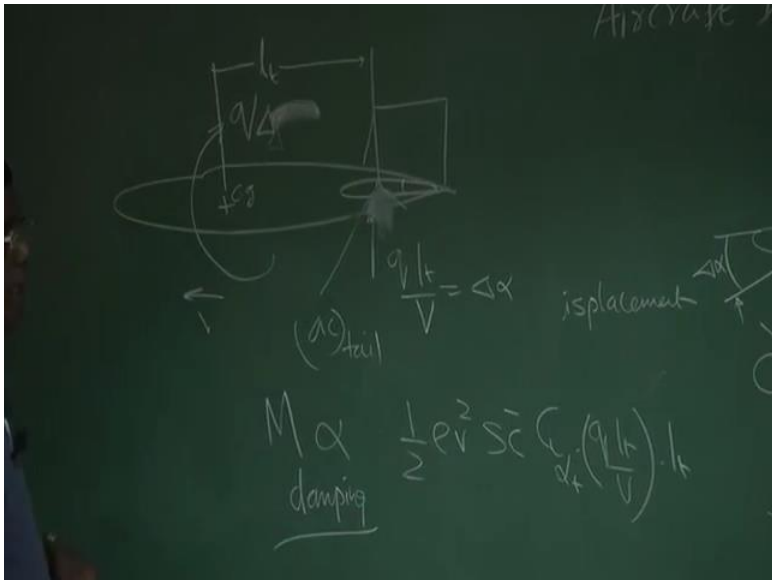

This is a tail. Right? And this is at CG. Let us say the airplane is going for a pitch up q.

As it goes for a pitch up like this, this tail goes down. So there is a relative air speed which I can write, QLT by V and LT is the distance between CG and the AC of the tail. Right? QLT, omega R and it is moving forward. V, this is the angle of attack introduced at the tail because of pitch rate. Now what this angle of attack will do, this will give a force in this direction which will give a moment is this direction. The moment here will be proportional to half ro V square SC bar CL alpha tail into Q LT by V that is alpha into LT.

That is the moment will come, will be generated by the airplane because of Q. Now you could see, this is also restoring moment because the airplane was disturbed for a Q like this but its tail will try to put it down. It will try to restore it and the restoring moment is proportional to the rate, not to the alpha. But to the rate, pitch rate. So this is also in damping. Earlier was M, CM alpha into alpha was stiffness. So this is damping.

This is analogous to CX dot. X dot is nothing but it is like Q. So I see that the dynamics of an airplane can easily be modeled in a restrictive sense if I understand a mass spring damper system. So that is why, we always give example of mass spring damper system before become to air aircraft dynamic stability. As far as these derivatives are concerned, you already know and I am sure when I talk about damping, you should recall there is some derivative called CMQ.

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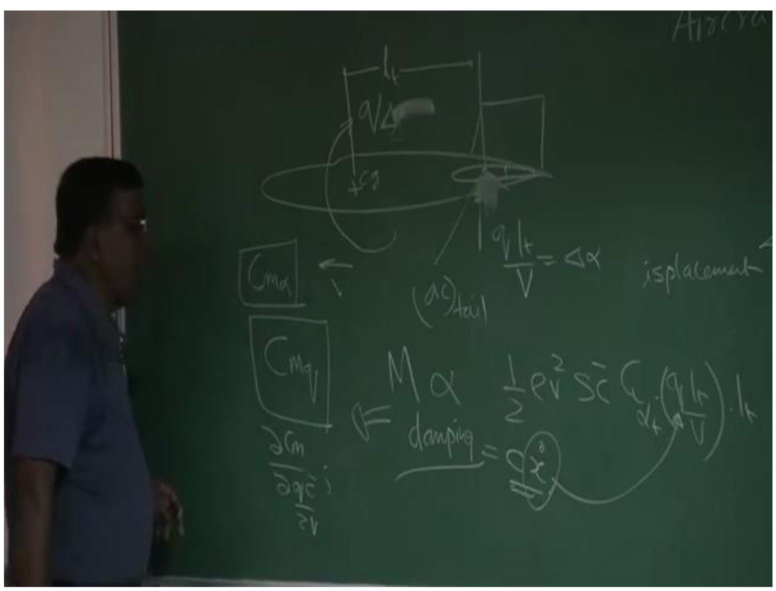

Like, for stiffness, we have CM alpha. CM alpha means DCM by D alpha and CM Q means DCM by DQ C bar by 2V. Do not worry. We will be again deriving these derivations so that you understand in complete. But to build this course, I would like to orient your mind and see what is our roadmap? Where we are going?

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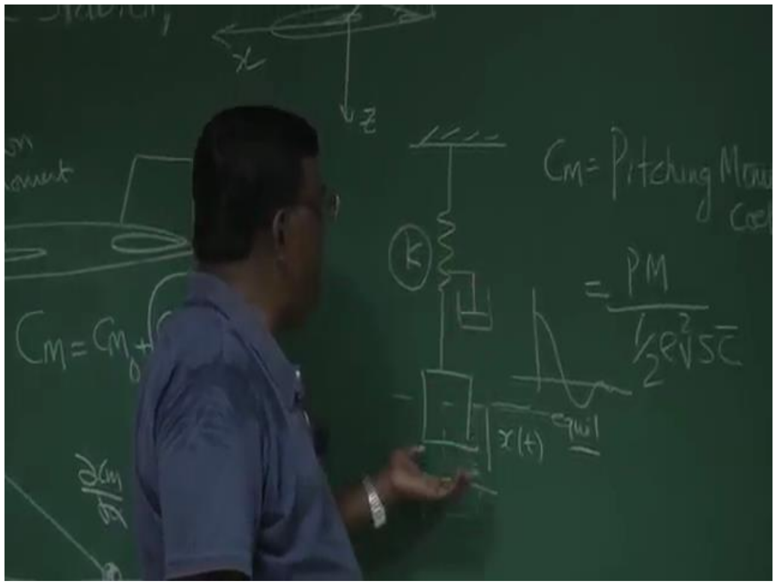

The basic question is, why for an aircraft, I am giving this example of mass spring damper system. Remember one thing, very important, in a mass spring damper system, there is a stiffness. That means, the restoring force is proportional to the displacement.

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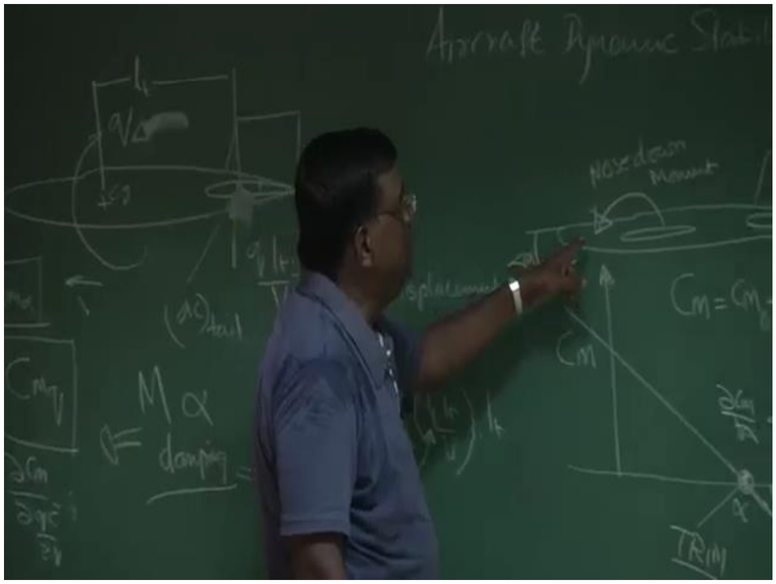

Similarly for an aircraft, restoring moment is proportional to the angle of attack which is linear displacement.

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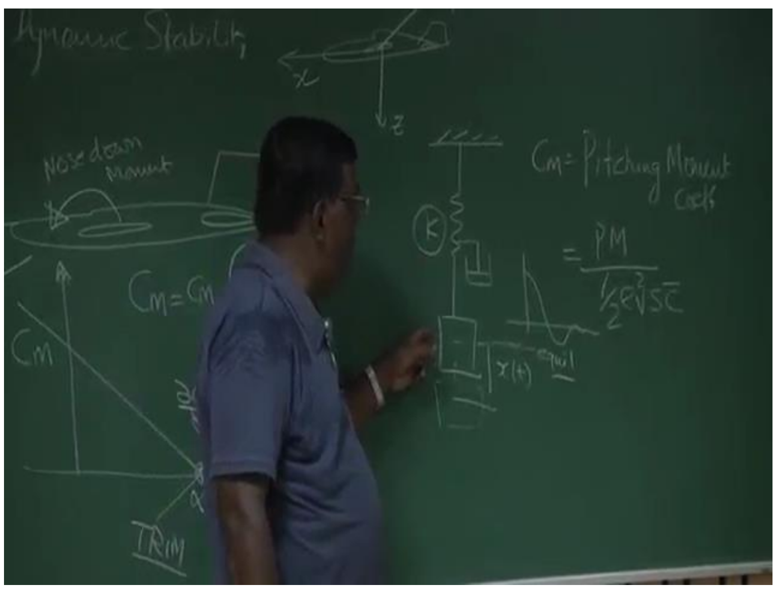

Also, in a spring mass damper system, restoring force is also proportional to the rate, CX dot.

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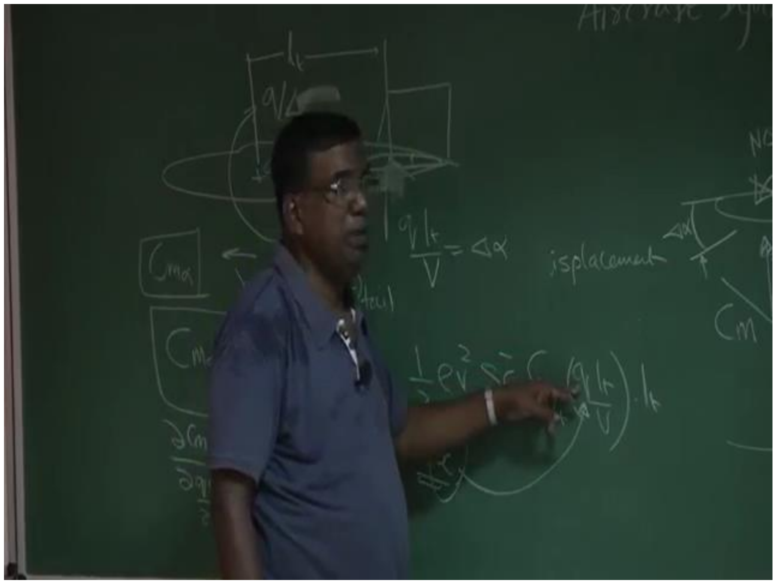

Similarly for airplane also, restoring moment, damping is also proportional to the rate which is Q pitch rate.

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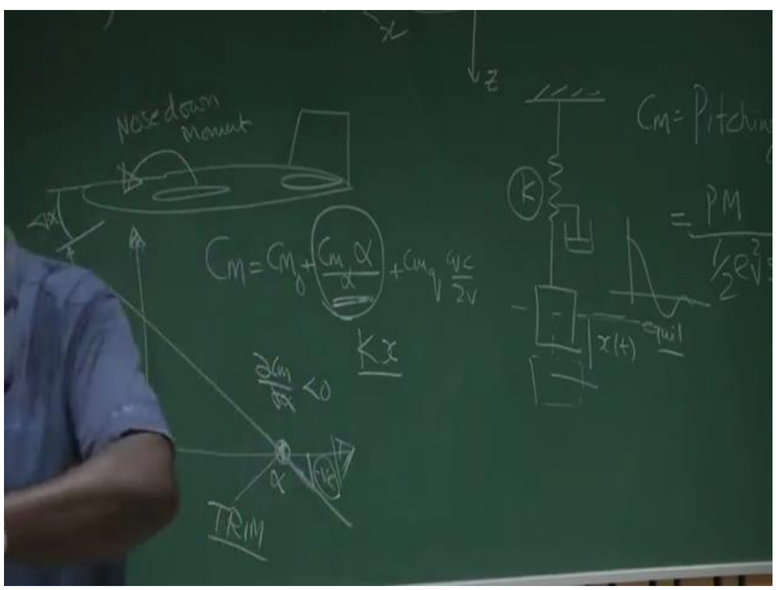

And here we write it as CM Q into QC by 2 V. So these are the restoring tendencies and because it has both, restoring moment has both, functions of displacement as well as rate. So this is attributed towards stiffness and this is towards damping. And there lies the similarity, there lies the necessity to understand the mass spring damper system dynamics. Once you understand that thoroughly, we will come back to aircraft again. Is it clear?