Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 2
Lecture No 12
Trajectory of the Aircraft.

Welcome friends. In previous classes, you have studied the 6 degree of freedoms equation of motion. Today we will try to find the trajectory of the aircraft with the help of these equations. Before we proceed to find the trajectory, we must have to be familiar with all the forces and moments acting on the aircraft. However, you already know but before proceeding, let us take a quick review of these forces and moments.

You know all the dimensions and all the inertial criteria of all the inertial properties of the aircraft and you know all the aerodynamic coefficients. Using these coefficients and the inertial property, you can find all the forces and moments. You know all the dimensions of the aircraft.

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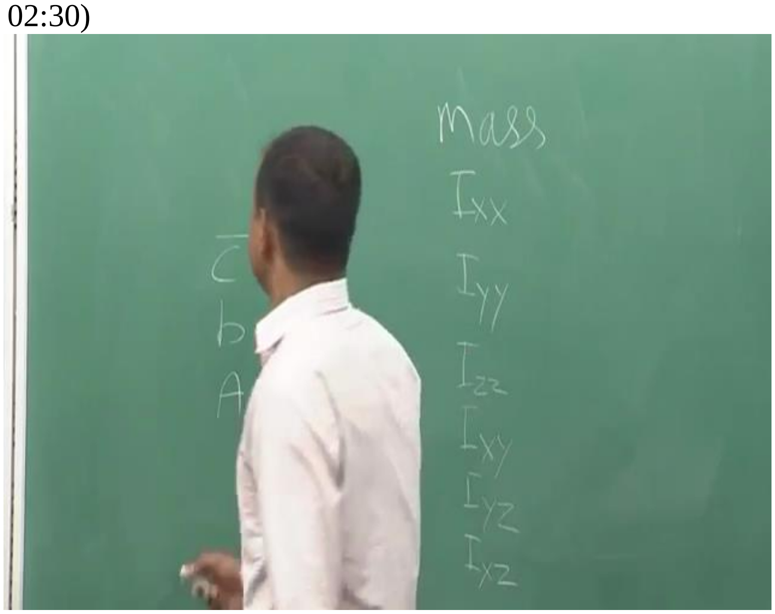

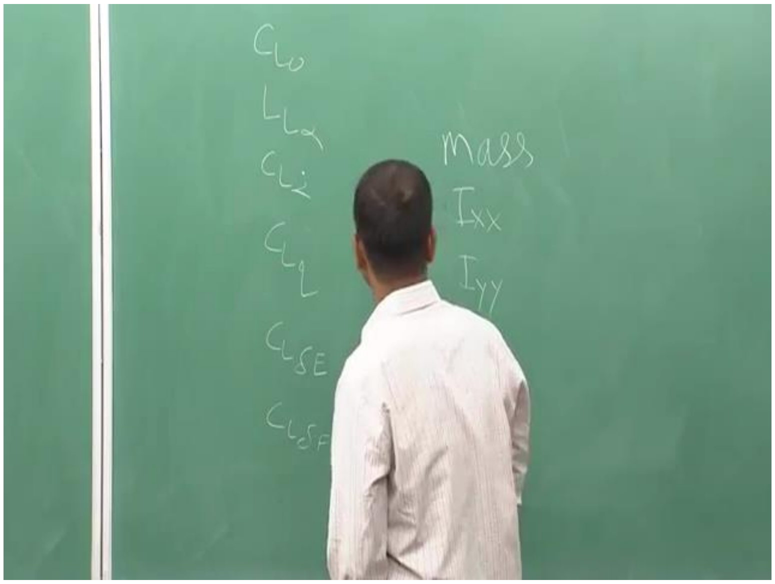

You know dimensions means, you now C bar, you know the wingspan, you know the area of the aircraft and you are familiar with the inertial properties means you know the mass, inertial components that is IXX, IYY, IZZ and the cross products, IYZ and IXZ. And you already know all the aerodynamic characteristics of the aircraft. Means you know all the aerodynamic coefficients. Aerodynamic coefficients, CL not, CL alpha, CL alpha not, CLQ, CL Delta E, CL Delta F.

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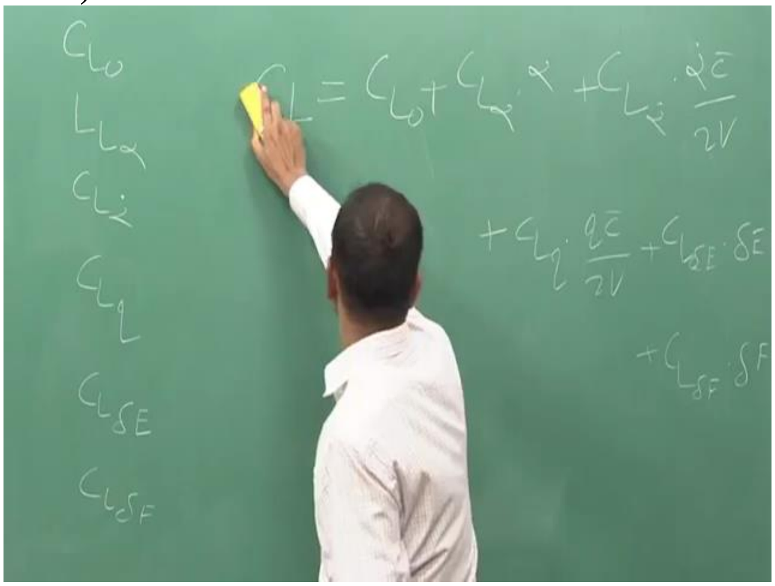

If you have all these aerodynamic coefficients you can find the lift acting upon the aircraft because within these coefficients, you can find the CL and CL is given by CL not + CL alpha into alpha + CL alpha dot into alpha dot C bar by 2V + CLQ into QC by 2V + CL Delta E into Delta E CL Delta F into Delta F.

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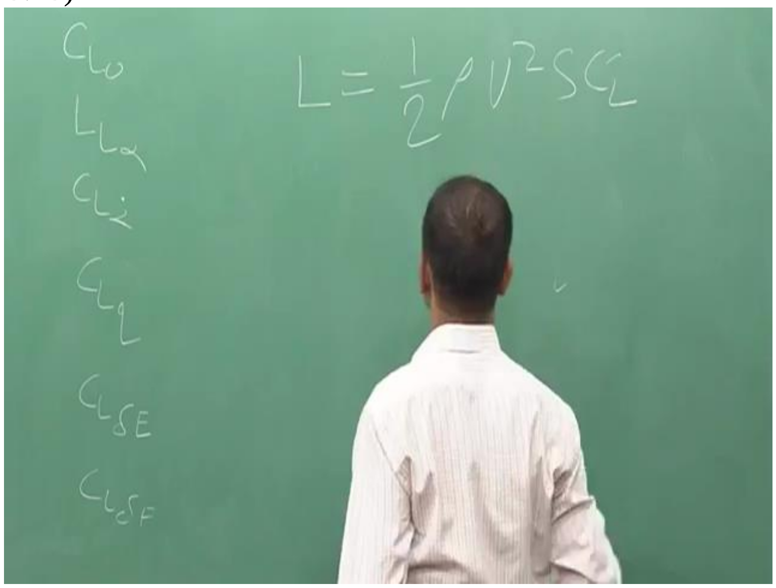

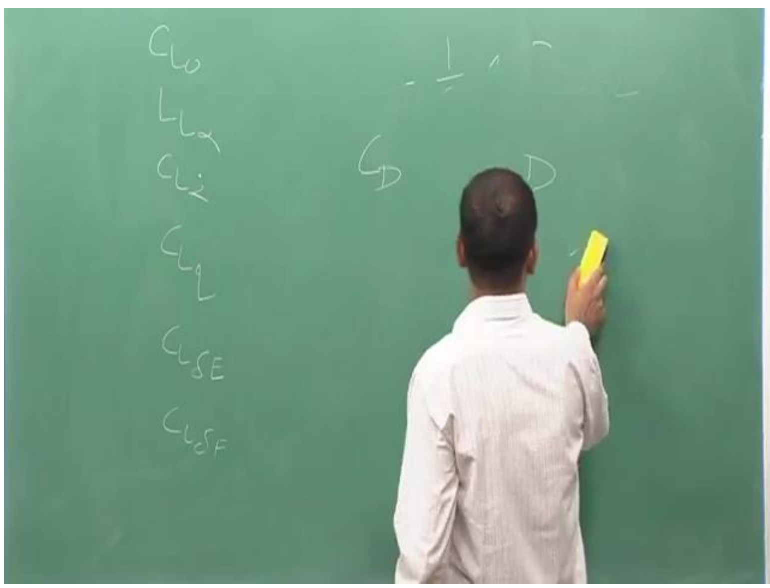

Using these coefficients, you can find CL and once you have the CL, you can find the lift acting on the aircraft by half UV Square S CL. Similarly you have all the drag coefficients. You know CD not, CD alpha, CD alpha dot, CDQ, CD Delta F, CD Delta A, CD Delta R and all other remaining coefficient and from there you can find the CD and once you know the CD, you can find the drag.

And you can find the Y forces also acting on the aircraft. Once you have all the forces, you can decompose them into body frame, X, Y and Z axises. How you do this? You have the angle of attack and side slip angle.

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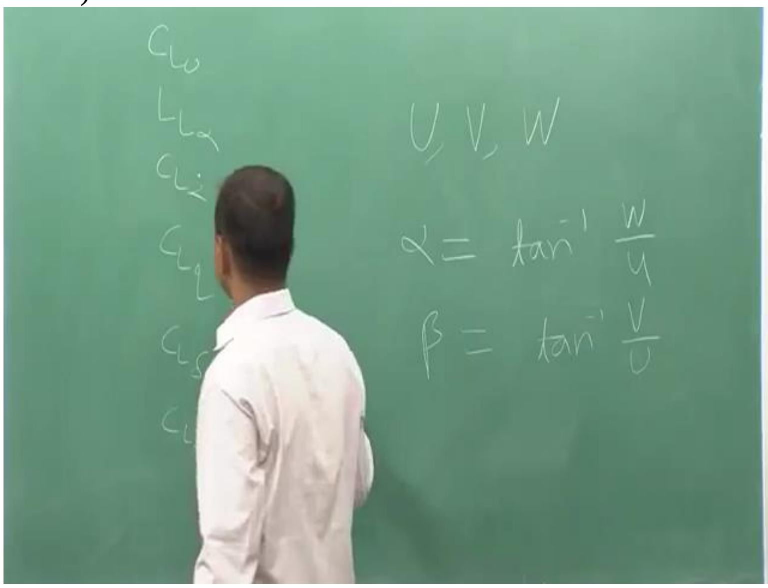

If the velocity component of the aircraft in the body frame U, V and W, the angle of attack, alpha will be Tan inverse W by U and beta will be Tan inverse V by U. So now if you have the alpha and beta and all the forces, lift force, drag force and Y force, then using these 2 angles, you can find F aerodynamic X and F aerodynamic Y and F aerodynamic Z acting in the body axises in the aircraft.

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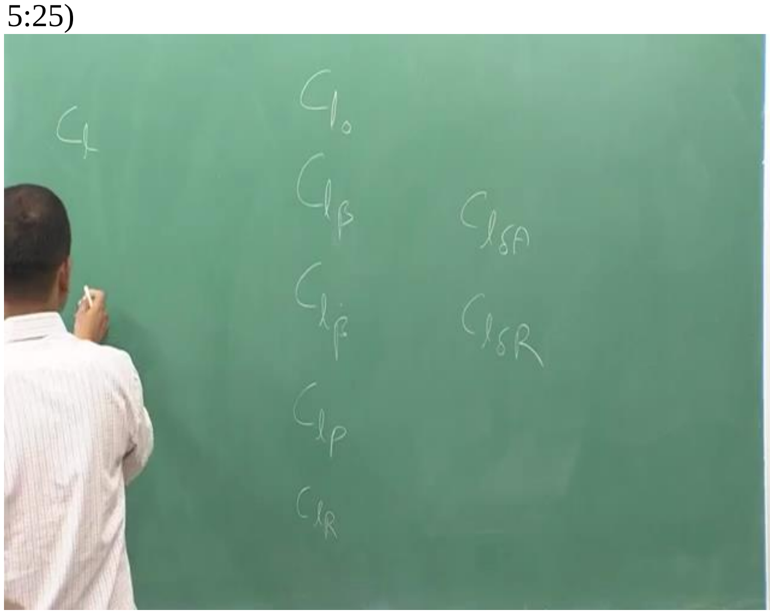

Similarly, if you have all the moment coefficients, CL not, CL beta, CL beta dot, CL P, CL R, CL Delta A, CL Delta R, you can find Cal. And once you have Cal, you can find the rolling moment, L which is given by half V Square S CL into C bar.

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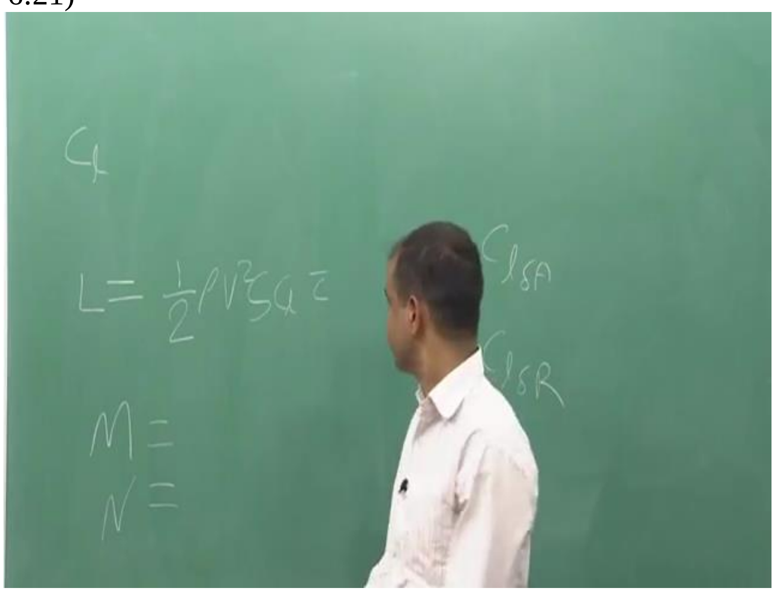

Similarly, you can find the pitching moment and yawing moment also. Now you have all the aerodynamic forces and moments acting on the aircraft. Now the task is to find all other forces and moments acting on the aircraft. What are the other forces? The thrust force and the force due to gravity. In case of propeller aircraft, the force basically depends upon the throttle and the velocity of the aircraft and if you can model it, you can find it, there are a lot of methods given in the book or you can prepare a lookup table for the different velocities and the different throttle levels.

And then you can pick a value of thrust for your particular aircraft, for your particular throttle level and velocity. And the third force is the gravity force.

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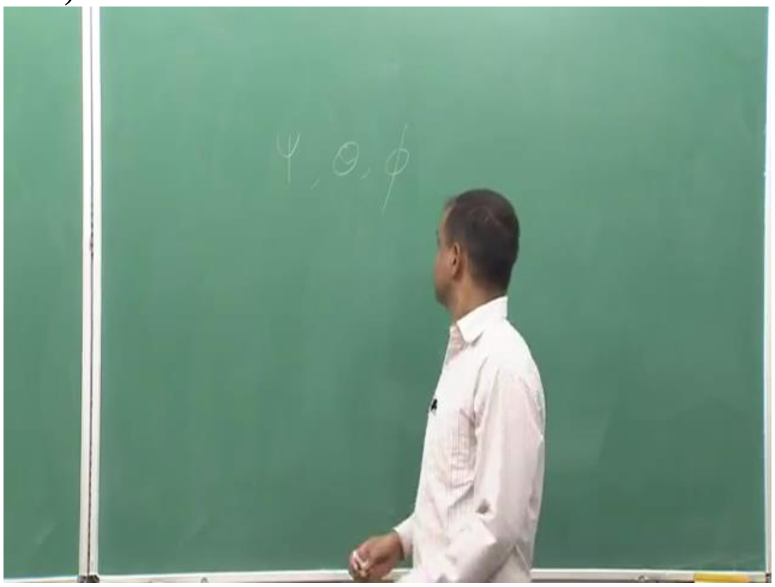

If you know the orientation of the aircraft, Psi, Theta and Phi, you can find all 3 gravity forces. We will see it later.

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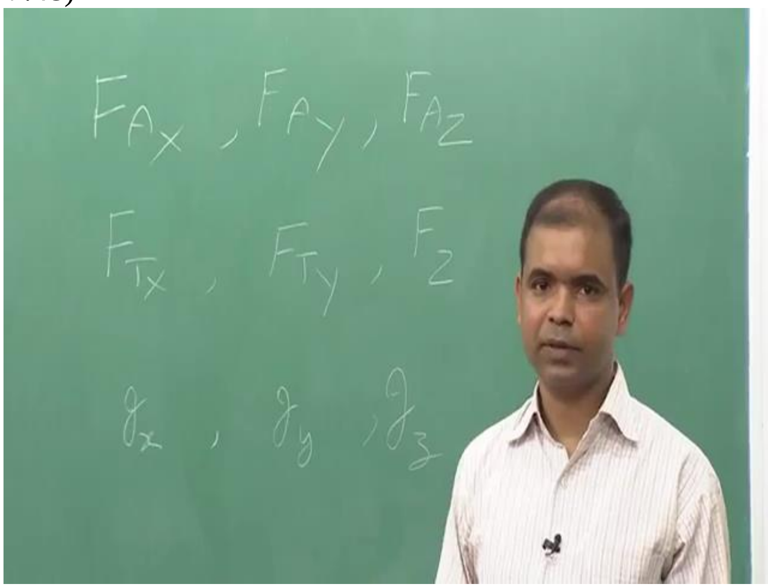

So finally you have all the 3 aerodynamic forces, F aerodynamic X, F aerodynamic Y and F aerodynamic Z. And the thrust forces, you can decompose it into X, Y, and Z directions. You know the orientation and mass. So using GX, GY and GZ, the gravity components in X, Y and Z direction, you can calculate the gravitational force acting on the aircraft in all 3 axises. Now you will be using these forces and the equations of motion that we have derived in the previous classes.

And you will see how you can find the trajectory of the aircraft. In previous classes, we have derived degree of freedom equations of motion and we have 2 set of equations. Each set has 3 equations, the first is the conservation of linear momentum equation and second set, conservation of angular momentum equations and they were…

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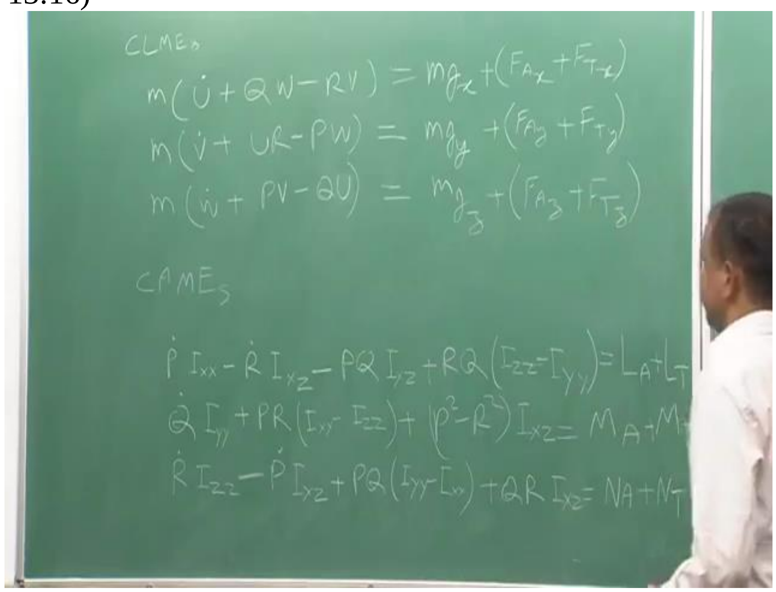

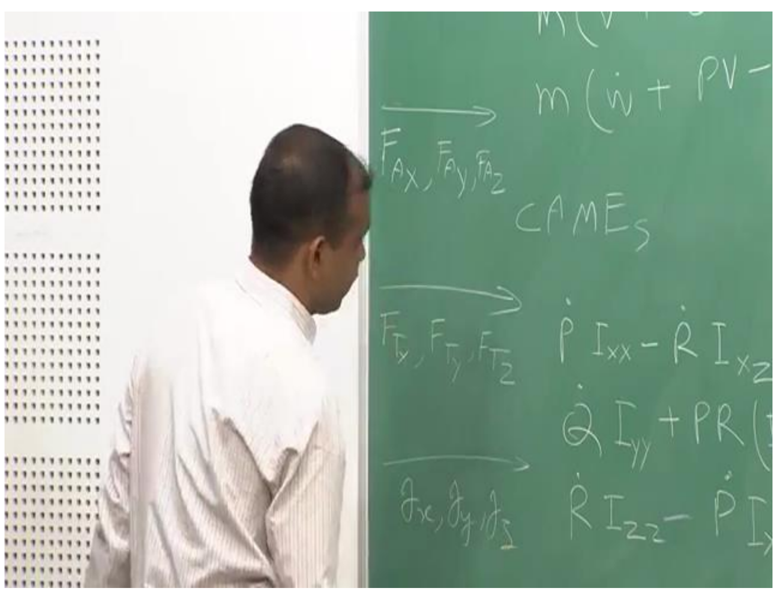

… if you talk about the conservation of linear moment equations, you can write MU dot + QW - RV equal to MGX, the gravitational force + F aerodynamic in X direction and thrust force in X direction. The second equation MV dot UR - PW equal to the gravity force in Y direction + aerodynamic force in Y direction + thrust force in Y direction. And the third equation, MW dot…thrust force in Z direction.

And the second set of equations, the conservation of angular momentum equations. And these were, P dot IXX - R dot IXZ - PQIXZ + RQIZZ - IYY. The rolling moment due to the returning force and rolling moment due to the thrust, LA + FT. The second equation, Q dot IYY + PR IXX - IZZ + P square - R square IXZ is equal to aerodynamic M thrust.

And the third, the yawing moment due to the aerodynamic forces and thrust. We had derived this 6 set of equations. We have the information of aerodynamic forces, all 3 aerodynamic forces. We know the thrust forces. And we know all the gravity components.

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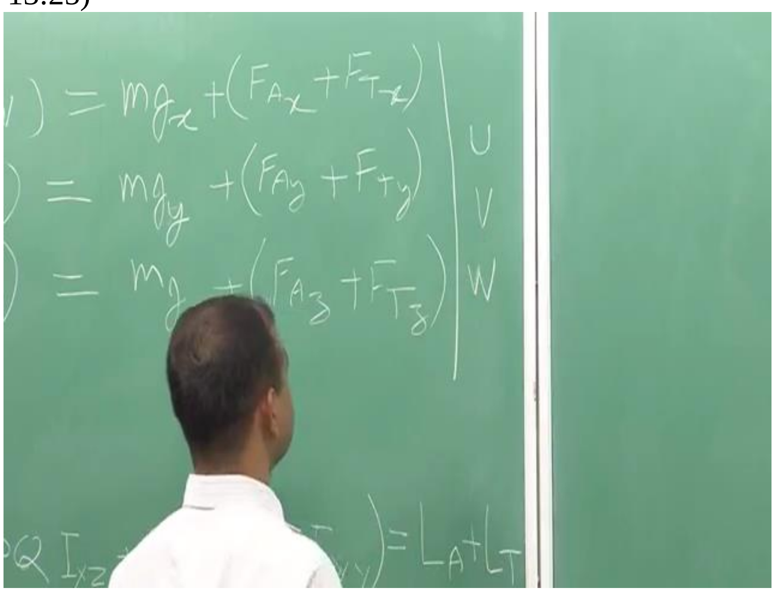

And we have the initial condition of the aircraft, then using these set of conditions, we can find the 3 velocity components, U, V and W. The solution of these equations will give us the 3 velocity components, U, V and W. You can solve these set of equations using any numerical method. There are a lot of tools. You can find some solver in the Math lab also or I personally like to solve these kind of equations with the help of (())(13:56) methods because these equations are coupled.

You see. The U dot is here and U zeta is here. Similarly with W also and V also. And these set of equations also. So you can solve using any numerical method and finally, you will get the U, V, W and from here, solving these set of equations, you will get the P, Q and R. Now the question is, how we can find the trajectory of the aircraft.

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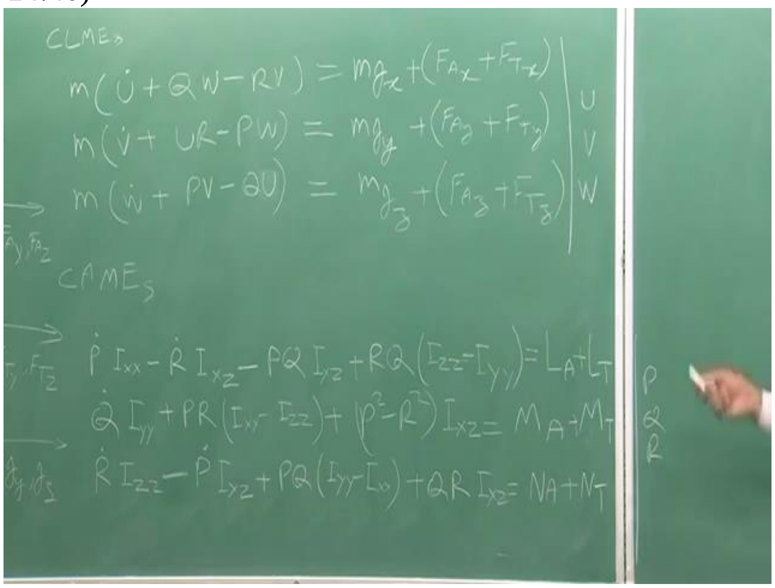

We have 3 velocity components and these velocity components are in the body frame of the aircraft. And if we want to determine the trajectory of the aircraft, we will require these velocity components in the inertial frame. Because whenever we talk about the trajectory obviously we are talking in terms of inertial frame. So we need any methodology to convert these velocity components, these body axis velocity components into the inertial frame.

And if you remember, we had studied the Euler angle, Euler rotation. With the set of those equations, we can convert. Let us see. I will give you a quick review of the Euler angle.

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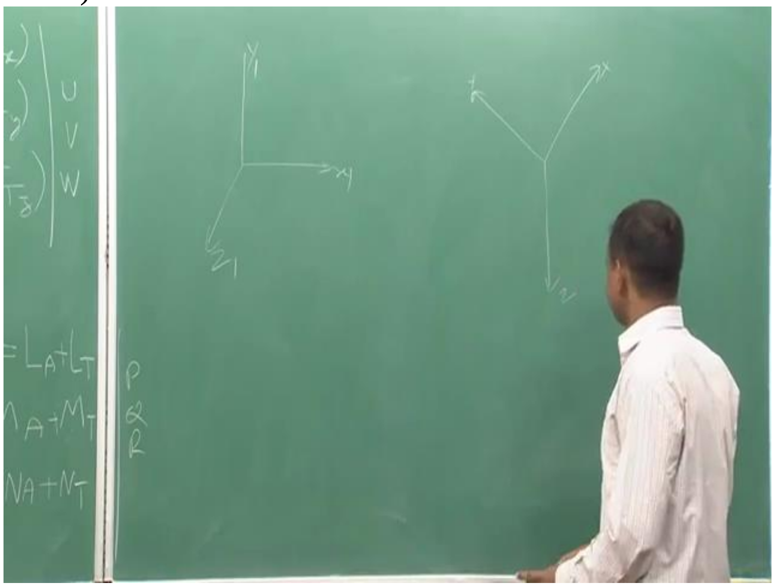

Suppose this is our inertial frame. These are 3 axis, X, Y and Z. And we have to, this is in the inertial frame. I can write it, X1, Y1 and Z1. We have one another thing. This is our aircraft body frame. So how we can transform our equation from this frame to this frame or this frame to this frame? We can reach from one frame to another frame with the help of 3 rotations and there are many types of rotations used.

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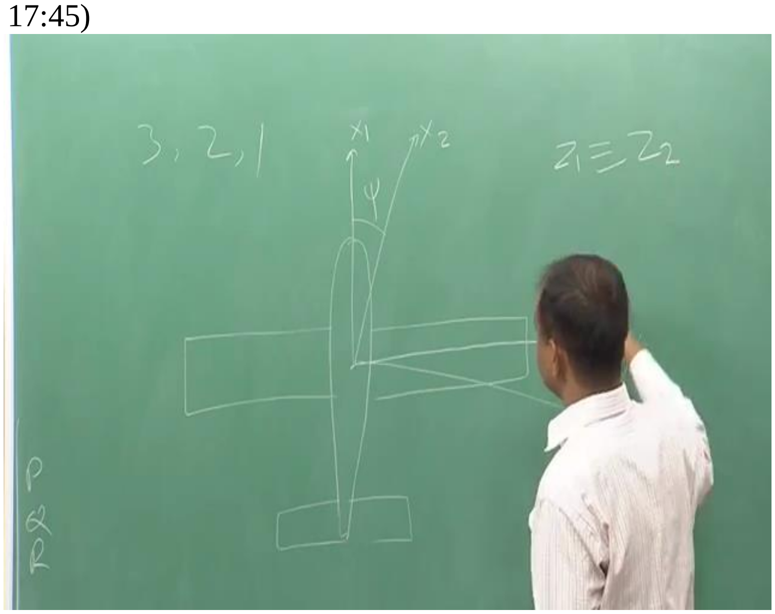

But in Euler, we used the sequence of rotation, 3, 2, 1. First, we rotate our frame about Z axis. Then, the new Y axis which we got after the rotation in the Z axis. And lastly, we rotate our frame about the new X axis which we got after the second rotation. Let us see it. Let us say I have top view of my aircraft.

This is the positive Y, this is the X. And if we rotate it like this, this is the Psi. This is the Psi rotation. Initially we are in the inertial frame. Suppose this was our X1 and this is the Y1. We are representing inertial frame with X1 and Y1 and finally we are trying to come into the body frame and that will be our X, Y and Z And in intermediate, we have got this frame called X2 and Y2. And for this rotation, Z will remain common. Z1 equivalent to Z2 because we are getting about this Z axis.

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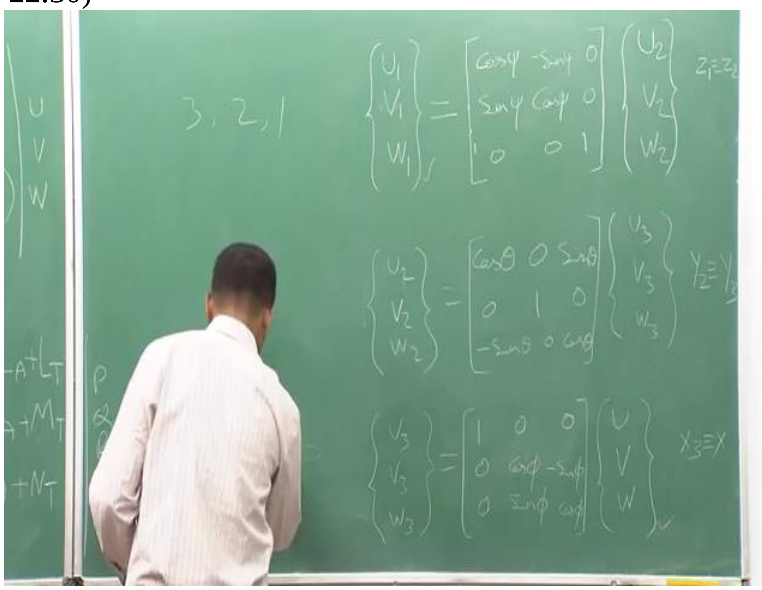

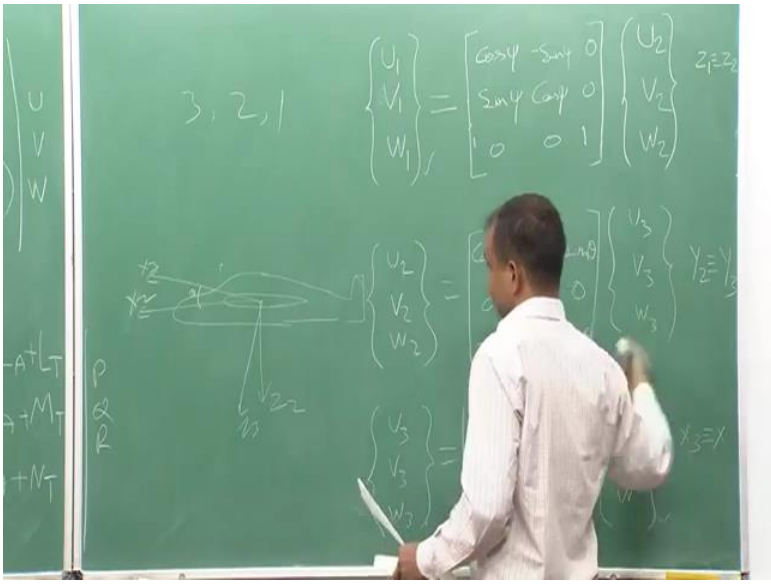

We can write in matrix form, this rotation. Suppose our velocity component in inertial frame, U1, V1 and W1. And after the Psi rotation, it becomes U2, V2 and W2. This U2, V2 and V2 in intermediate frame and this we got after the rotation, Psi. We can write this matrix, cos Psi - sin Psi 0. Sin Psi, cos Psi 0. 0 0 1. And second rotation, we rotate about the new Y axis and using that rotation, you can get U2, V2 and W2, U3, V3 and W3.

This is if the second intermediate frame and for this you get prematics, this we get due to the Theta rotation. Cos Theta 0 Sin Theta. 0 1 0 and - Sin Theta 0 and cos Theta. And finally the third rotation, in this rotation, we have Z1 and Z2. Same way in this rotation, we have Y2 and Y3. Same because we are rotating about this Y2 and after the rotation also, it will remain the same. In third rotation, we can get U3, V3 and W3.

And finally we reach in body frame and this is U, V and W. So after these 3 rotations, finally we have reached from this inertial frame to the body frame and in this case, X3 is nothing but the X because we are rotating about this X3. If you see this rotation, suppose this was my aircraft and this is X and this is Z. This is X2 and this is Z2. After the Theta rotation, we will get X3 and Z3. The Y2 and Y3 will remain same.

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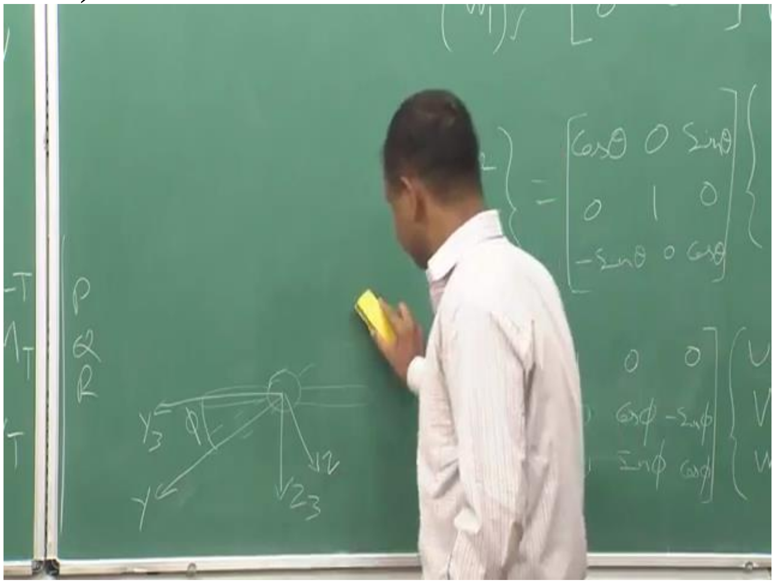

In this third rotation, you will get the matrix 1 0 0, 0 cos Phi - Sin Phi, 0 Sin Phi and cos Phi. In third rotation, we have front view of the aircraft. If this is my front view, suppose this is our Y3 and this is Z3, for this rotation, for the positive Phi, the right wing down is the positive Phi. So, right wing down means if this is my front view, it will be like, it will become like this and this is my final Y and this is Z. And this angle is Phi.

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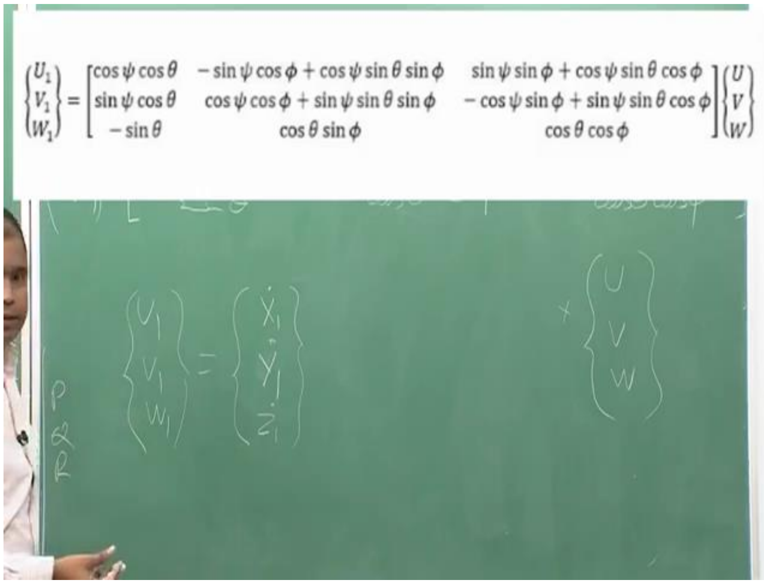

So using these 3 rotations, we can convert our frame from initial frame to the body frame or body frame to inertial frame. We can combine all these rotations in one equation. We will get in inertial frame, all 3 velocity components and this can be given by the multiplication of 3 matrices. This U, V, W is in body frame and by multiplying this matrix, we are able to get U1, V1 and W1. This is now in inertial frame and this is our desire because we want to find the trajectory.

You can write, U1, V1, W1, X1 dot, Y1 dot and Z1 dot. So from here we can find X1, Y1 and Z1. And this will ultimately give us the trajectory of the aircraft in the inertial frame.

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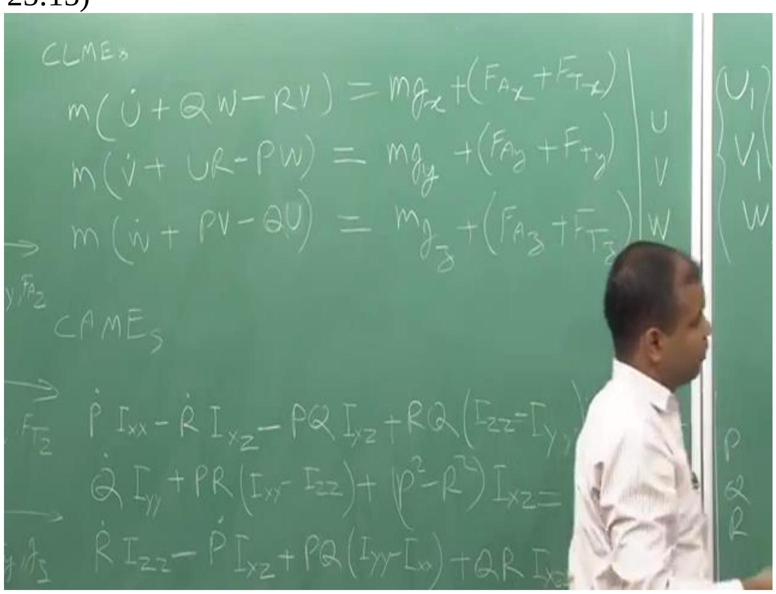

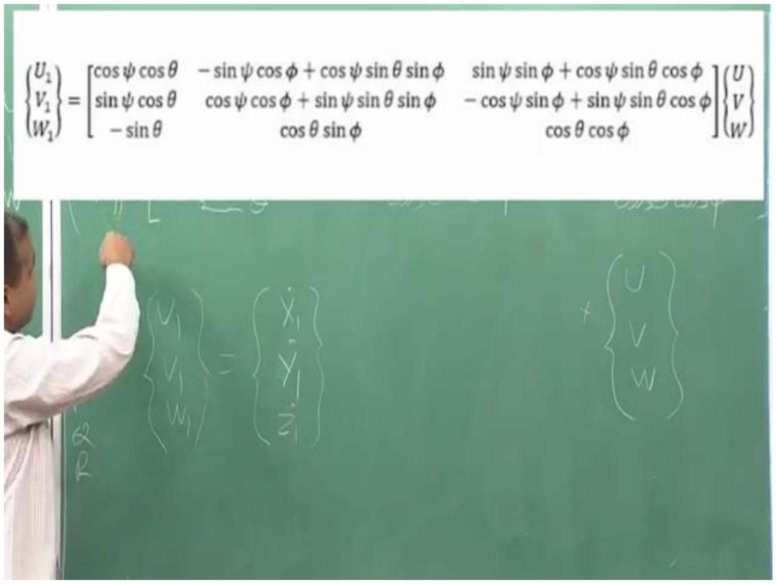

From these 3 set of equations, we already have got the U, V, W. This U, V, W. But to find U1, V1, W1, we require Psi, Theta and Phi also. And from where will we find this? So to get this, we will do a small manipulation. The metrics that we have derived to convert body frame to inertial frame or inertial frame to body frame, this we call as the flight path equations. So now, our target is to find the 3 Euler angles, Psi, Theta and Phi.

And what information we already have? We know the P, Q, R. So we have to find the Psi, Theta, Phi from P, Q, R because this is the rotation and if you have the rotation and if we know the initial condition of rotation, obviously we can find the orientation. And that we will try to find here.

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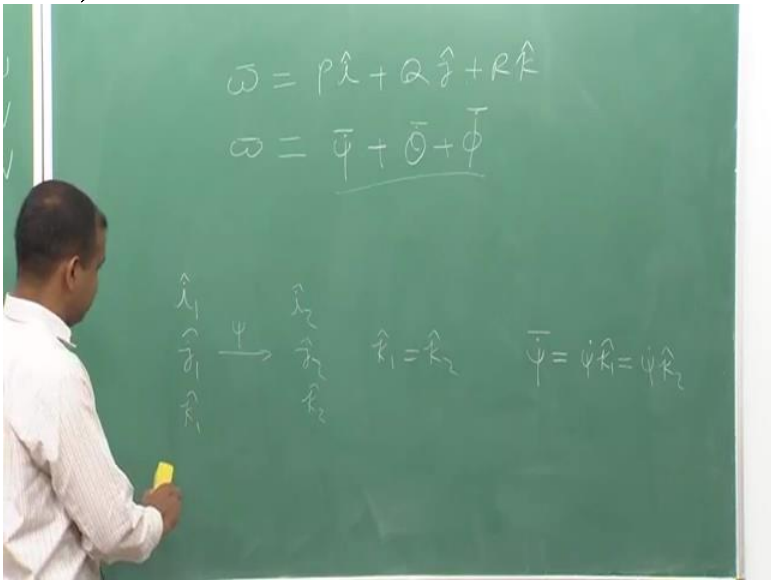

If we have the angular velocity of the aircraft, we can write it, this is P, Q, R are the rotations, the angular speeds in the body frame of the aircraft. So we can write PI + QJ + RK. And if we know the rate of Euler rotation, we can write it, the angular speed of the aircraft is Psi dot + Theta dot + Phi dot.

Somehow if I am able to determine the expression for this in terms of I, J and K and if I compare with this equation, I will get the Psi dot, Theta dot and Phi dot in terms of P, Q and R. How we can do this? The 3 rotations that we performed to determine the Euler angle, Psi, Theta and Phi, the first was Psi and this was about the Z axis. And for this, suppose we transform our coordinate system from body axis and in the body axis, we had the unit vectors, I1, J1, and K1.

And from here we got the I2, J2 and K2 and the rotation that we performed was the Psi. Here, K1 will be equal to K2 because we are doing this rotation about the Z axis. So, K1 is equal to K2. And I can write, the Psi dot vector is equal to Psi dot Q1 and this is equal to Psi dot K2.

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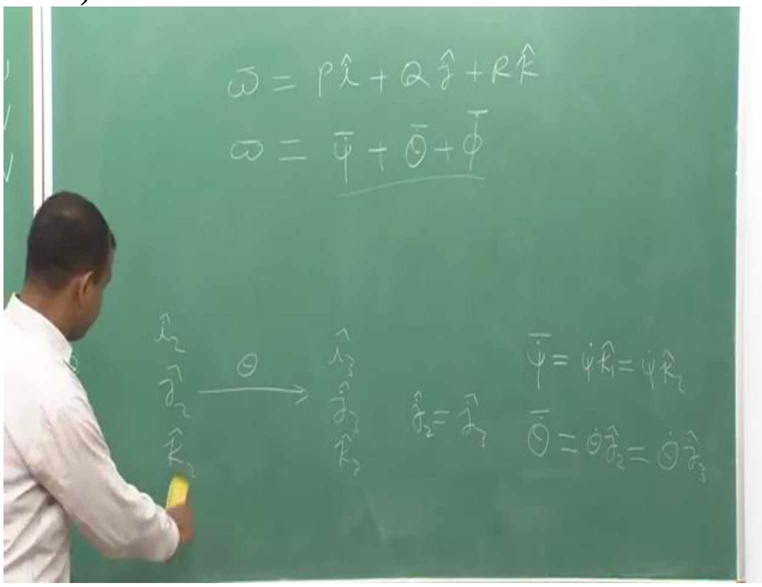

Similarly when would not the second rotation that was from I2 and K2 I3, J3 and K3. From this rotation, we got the Theta angle and this we are doing about the Y axis. Means, this J2 and J3 are the same and we can write the Theta dot vector is equal to Theta dot J2 or Theta dot J3. In the third rotation that was the Phi that we performed about the X axis that was the X3 or I3.

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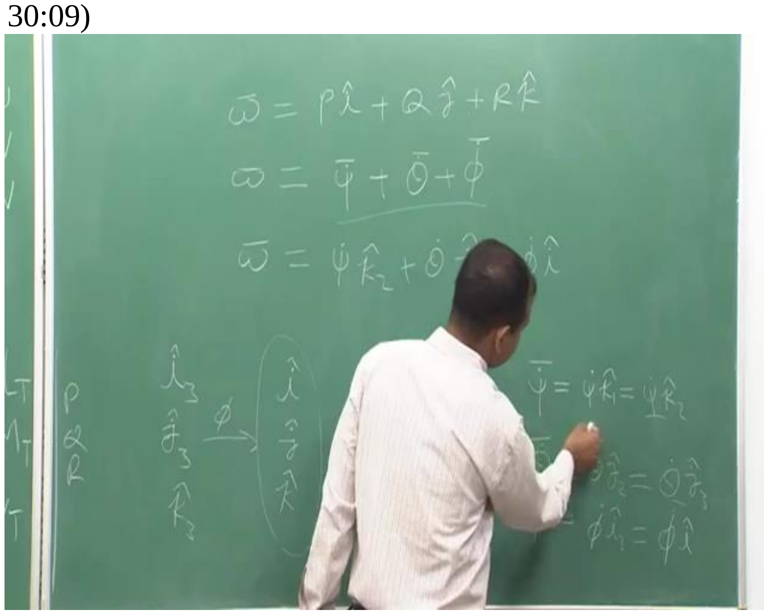

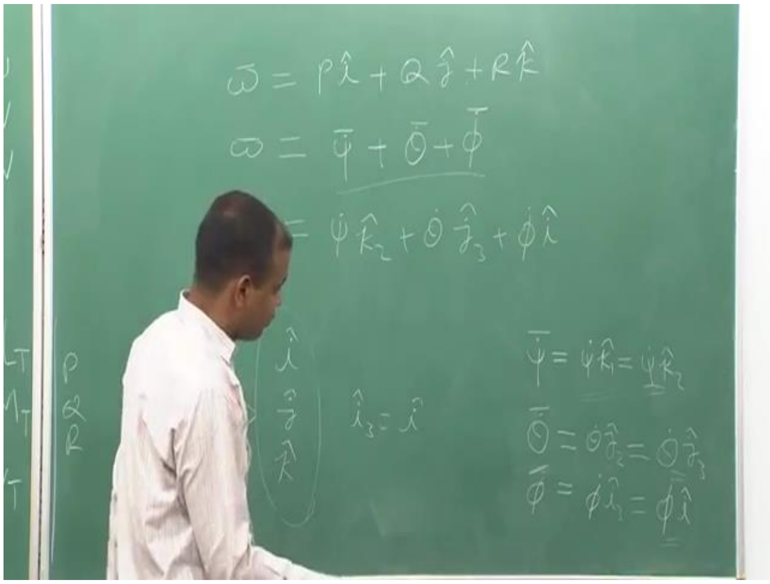

And we got, we reached from I3, J3 and K3, the rotation was Phi and we reached this body frame. That was I, J and K. And this rotation we are performing about the X3 axis or I3. So I3 will be I and we can read this Phi dot vector is equal to Phi dot I3 and this is equal to Phi dot I. And we can write the omega using these equations. Omega is equal to Psi dot K2 + Theta dot J3 + Phi dot I.

Why I am using this and this is only because our ultimate aim is to reach in I, J and K frame in this body frame and if I start from, if I take this Psi K1, I will have to transform my frame 3 times, first from K1 to K2, then K2 to K3 and then K3 to K. And this will take 3 rotations. If I start from K2 directly, I can reach I, J, K frame only after 2 rotations. Similarly, if I start from Theta dot J3, I will require only one rotation and I will reach in the I, J, K frame.

If I start from here, I will require 2 rotations. That is why, and picking this one. And this is already in I form. No requirement of any rotation. Now how we can find this K2 in terms of I3, J3 and K3.

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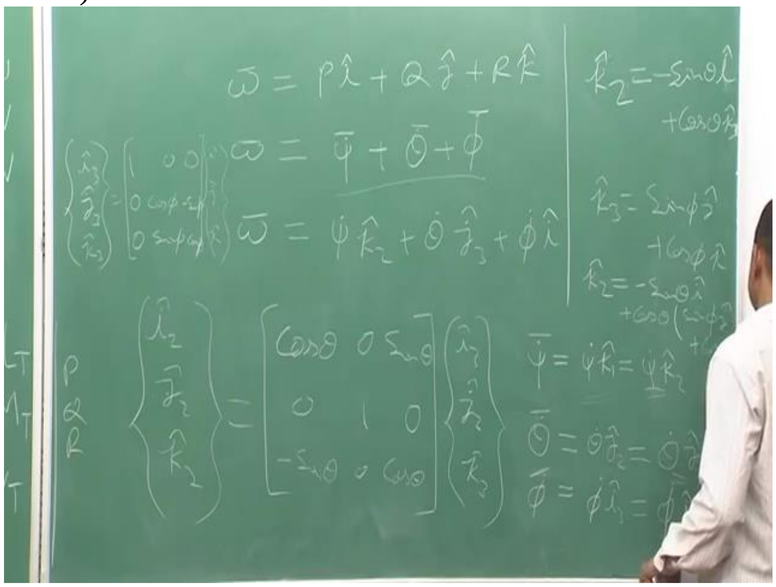

If you remember our second rotation that was Theta, and we were getting the I2, J2 and K2 and the rotation matrix was, suppose this is I3, G3 and K3. This was the second rotation that is the Theta and this was given by cos Theta 0 Sin Theta, 0 1 0, - Sin Theta 0 cos Theta. So from here, I can get K2 in terms of I3, J3 and K3 and I can write, K2 is equal to - Sin Theta into I3 + cos Theta into K3. And I have to find this, this I3 already I know.

It is I. So I can eliminate it and I can write it I. Now, I am left with K3 and for this I can use the third rotation and that was I3, J3 and K3 and the matrix we had 1 0 0 and here we had I, J and K. The matrix was 0 cos Phi - Sin Phi, 0 Sin Phi and cos Phi. And from here I can get K3 in terms of I, J, K and K3 is Sin Phi J + Cos Phi K. And if I substitute it here, I will get K2 and this is unit vector. Sin - Sin Theta I + Cos Theta and K3 is Sin Phi J + Cos Phi K. So now I have K2 in terms of I, J and K. So my this task is complete and I have to find J3 also in terms of I, J and K.

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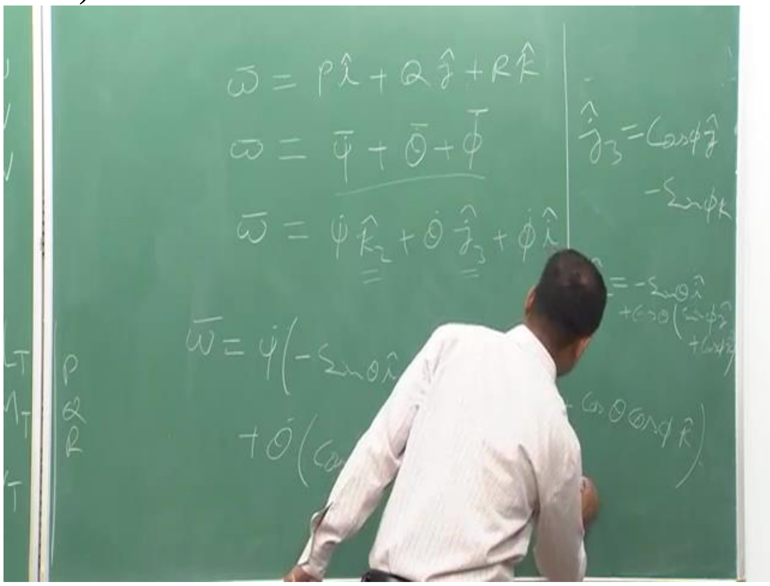

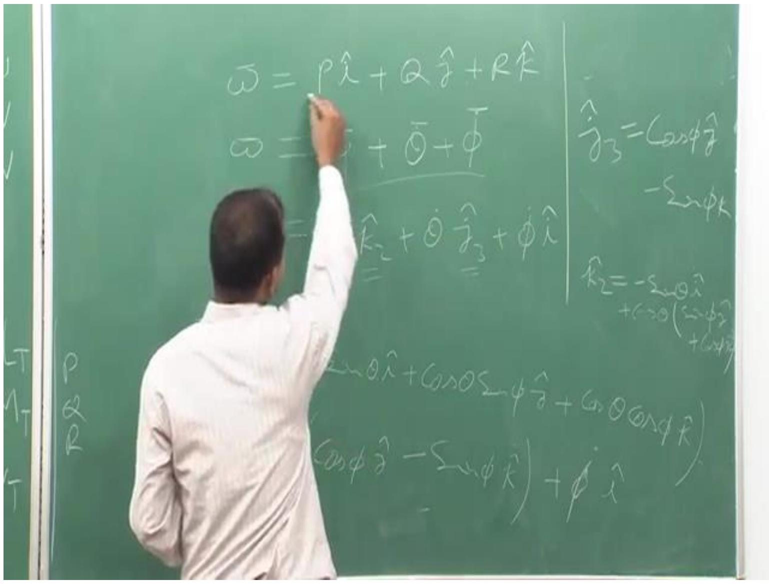

And from here I can write J3 is equal to Cos Phi J - Sin Phi K. So finally I have omega Psi dot and K2 is what? - Sin Theta I + Cos Theta Sin Phi J + Cos Theta Cos Phi K + Theta dot J3 and this I can write Cos Phi J - Sin Phi K. And this is already, we have + Phi dot I.

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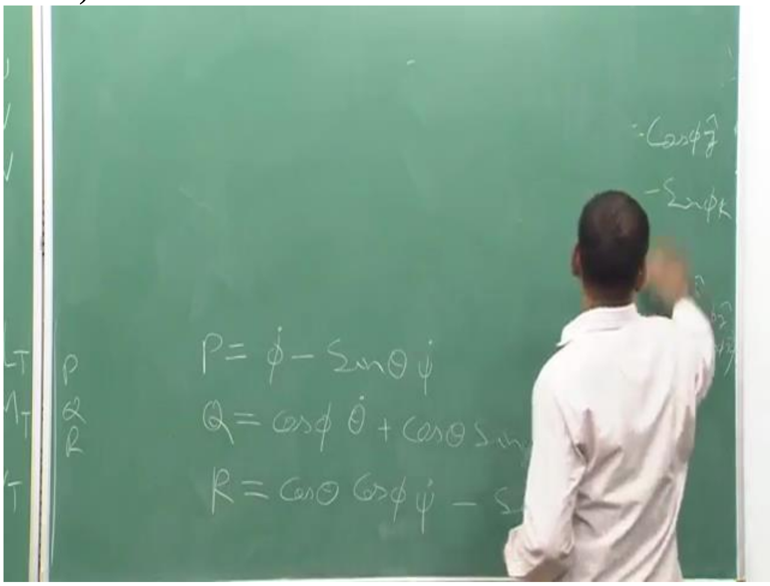

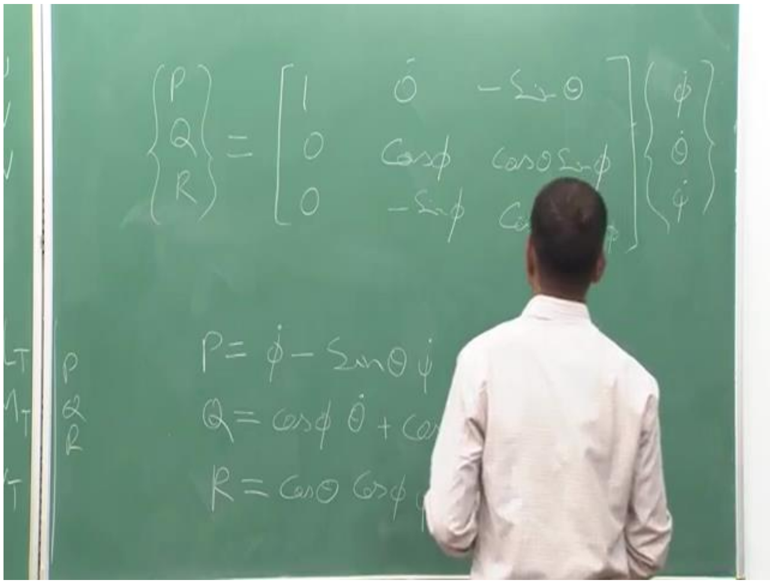

And now if I rearrange these terms and if I compare this was the question, I will get P is equal to Phi dot - Sin Theta Psi dot. Q is equal to Cos Phi Theta dot + Cos Theta Sin Phi Psi dot and R is equal to Cos Theta Cos Phi Psi dot - Sin Phi Theta dot. And if I write it in a matrix form, I can write, P, Q, R is equal to 1 0 - Sin Theta, 0 Cos Phi Cos Theta Sin Phi. 0 - Sin Phi Cos Theta Cos Phi. And this in multiplication with Phi dot, Theta dot and Psi dot. And this set of equations we call the kinematic equations.

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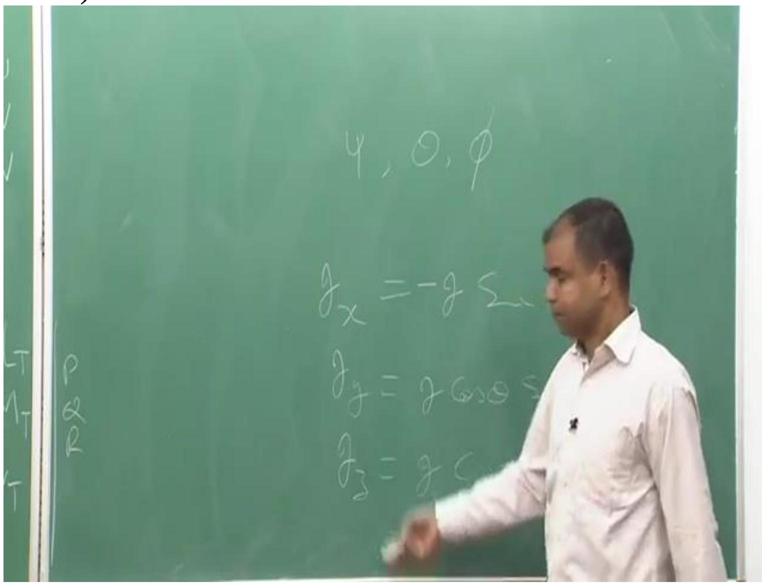

Once we have the Psi, Theta and Phi, we know the orientation of the aircraft. If you remember, we had derived the gravity equations using this Psi, Theta and Phi and those equations were given by, let me rewrite here, GX is equal to - G Sin Theta. G1 is equal to G Cos Theta Sin Phi and GZ is equal to G Cos Theta Cos Phi. Now we have the gravity components. You can solve these set of equations in iterations using some numerical methods.

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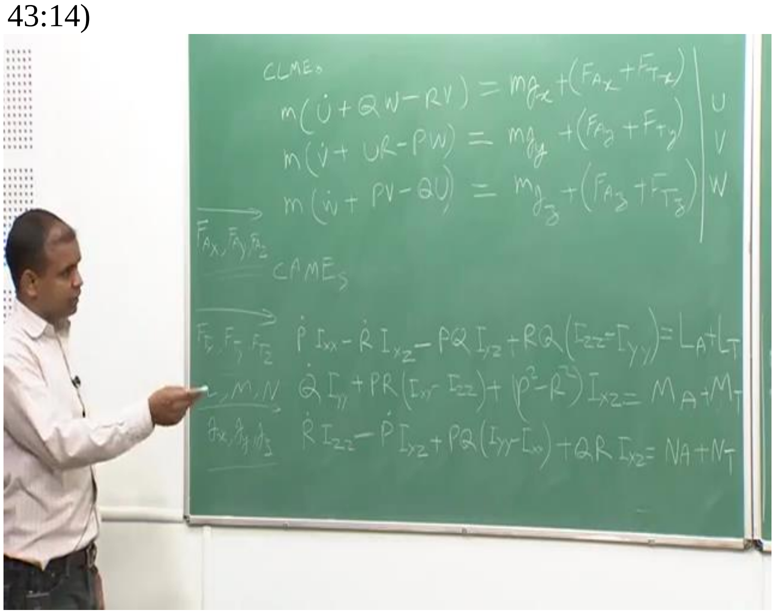

And if you are solving them for small a time delta T, whatever you get after this, suppose it is 0.001 seconds or 0.01 seconds, usually we keep it very small. Once you solve it for delta T time, you will get the new values of these forces, this U, V, W and P, Q, R. Means these forces, of course you will get L, M, and N also, these new moment values and if you keep iterating, for the next iteration whatever you have got the new values, that will work as the initial condition and you will get the values after 2 delta T and similarly if you keep iterating, you will get the continuous path of the aircraft, that is o trajectory.

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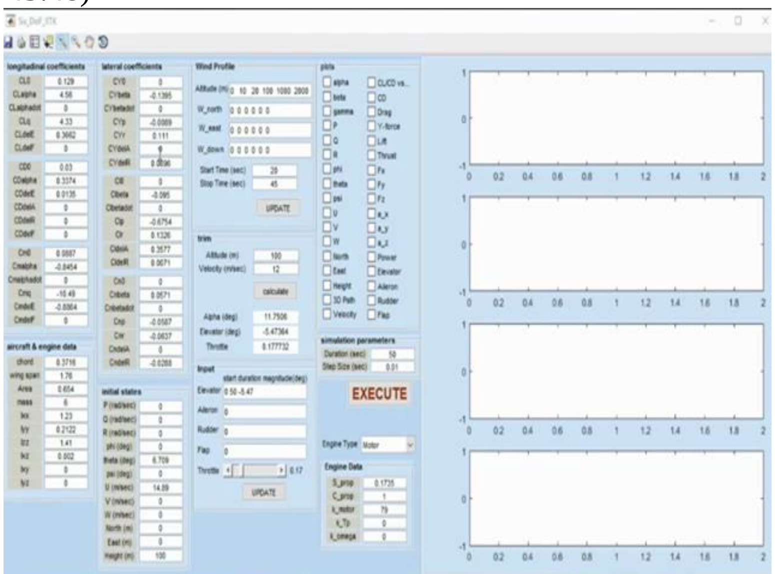

We have developed a crystal user interface for 6 degrees of freedom simulation. In this, here you can feed all the aerodynamic coefficients associated with the longitudinal motion, the lift coefficient, drag coefficients and the pitching moment coefficients. In this window, all the aerodynamic coefficients associated with the lateral and directional motion of the craft, coefficient for the Y force, all the rolling coefficients and plain coefficients.

Here parameters associated with the dimensions of the aircraft and all the inertial properties, all the initial state of the aircraft and in this window, if there is any wind, you can put here.

Altitude is divided in different slots. In each slot, whatever the component of velocity is, can be given here. So in (())(44:46), the number of elements in all the boxes will be same. The start time of the wind, the stop time of the wind. This window has nothing to do with the simulation but you can get an approximation for the trim.

Suppose I want to fly at 100 m altitude at 15m/s, for this my elevator input is this much and the throttle required is this. This is the input window. I can give inputs for different control surfaces. Suppose I want to trim at 100 m altitude at 15 m/s, for this my elevator input will be this much.

So I give. If I want to give more inputs, I can add here. Like if I give elevator input starting from 5 for 10 seconds and suppose 5 degree, so similarly I can add as many inputs I want to give.
And similarly for the (())(45:57) also. And this is my throttle and this is the simulation parameter for the duration I want to simulate and the step size for the internal calculations. And here is the engine parameters. I can select the type of engine. We execute it.

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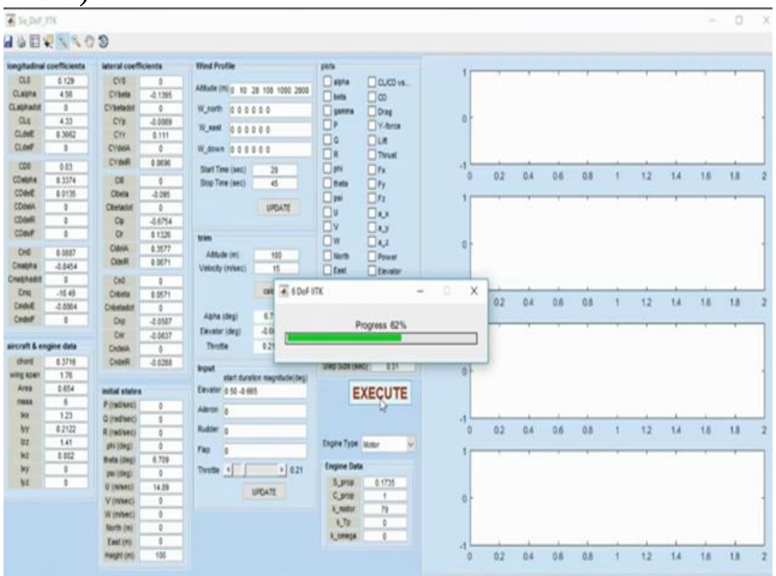

Aerodynamic coefficients that we have entered here, you can find them analytically. For this you can refer the book, aircraft dynamics by Marcelo Ark Napolitano or an aircraft design by Daniel Rammer. If you have any other book, you can follow that. You can calculate the CA2 simulation also. If you want to get more accurate values, you can perform an internal test. Then you can calculate all the aerodynamic coefficients.

Now if I want to see the angle of attack, it could be plotted here, pitching angle. If you want to see this path in 3-D space, the velocity and whatever I will select here, the 1st 4 will appear here. Suppose I want a tailwind of 2 m/s and flying at approximately 100 m altitude. At 10 to 1 km altitude, 2 m/s wind and I am sure that my aircraft will not go beyond that altitude. Starting from 20 seconds till 45 seconds, now we execute it.

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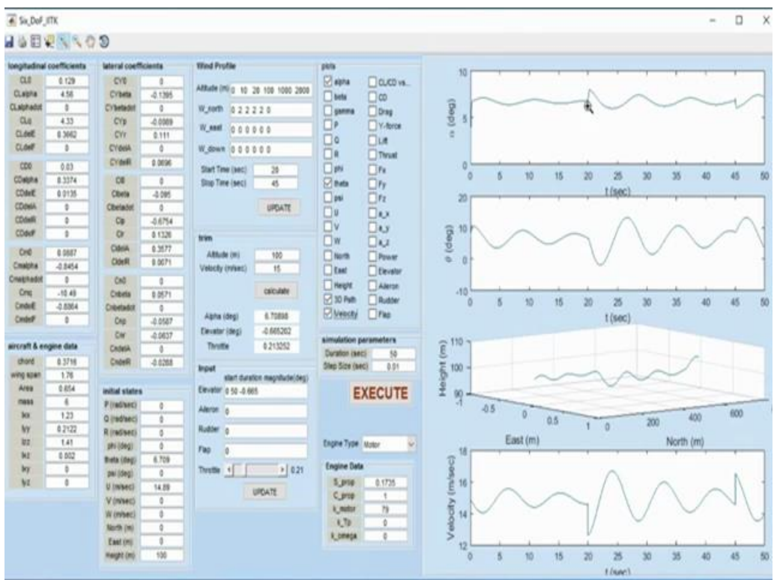

Here you can see, when there is wind, the angle of attack has changed. Again when there is (())(48:36), then again there is a change, there is a transition. The altitude is losing. Then again it is gaining when the wind is not there. Because this is tailwind, that is why, it is losing its altitude. Here are some tools. You can use them. Suppose I want to see this area. I can zoom it.

We have put this software on the Right laboratory, IIT Kanpur websites. It is open source. You can download. If you have any idea, you can add into this. If you have any doubt, you can ask me. My email ID is, vijayd@iitk.ec.in. You can ask the questions on the forum also. We will be happy to answer. Thank you very much.