Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 4
Lecture No 19
Dimension Stability Derivatives

Good morning friends. We are working towards building longitudinal perturbed equations of motion.

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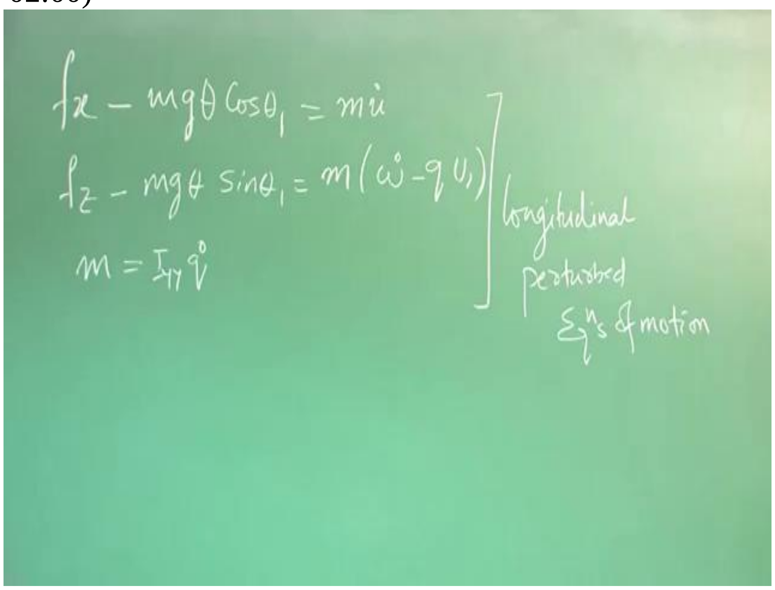

And in that we have seen that we only consider motion along X, our FX - MG Cos Theta1 equal to MU dot. our 1 equation. And there was FZ - Theta sin Theta1 is equal to MW dot - QU1 and M equal to IYY Q dot. Sometimes I may be using FAX. As long as they are small, letters FX, FZ and small M, we all understand these are perturbed aerodynamic forces along X direction, body X direction.

FZ along body Z direction and M is about body Y axis but we are also clear that we are is instability axis system as the body fixed axis system which is designed in such a way, the orientation of X is towards the velocity vector in the vertical plane. We are talking only about the longitudinal perturbed equations of motion. Why we are doing all these things? Why I am taking taking so much of pain? Because we know that this U, this U dot here, Q, then W.

That is W dot here, Q dot here. These are all perturbed quantities. So we will be tracking those perturbed quantities to comment on whether the aircraft is dynamically stable or not. Right? That is our purpose. To solve these equations, I need to know what is the dependence of perturbed aerodynamic forces Fx, FZ and moment on motion variables, their rays or control variables. So for that, what we did?

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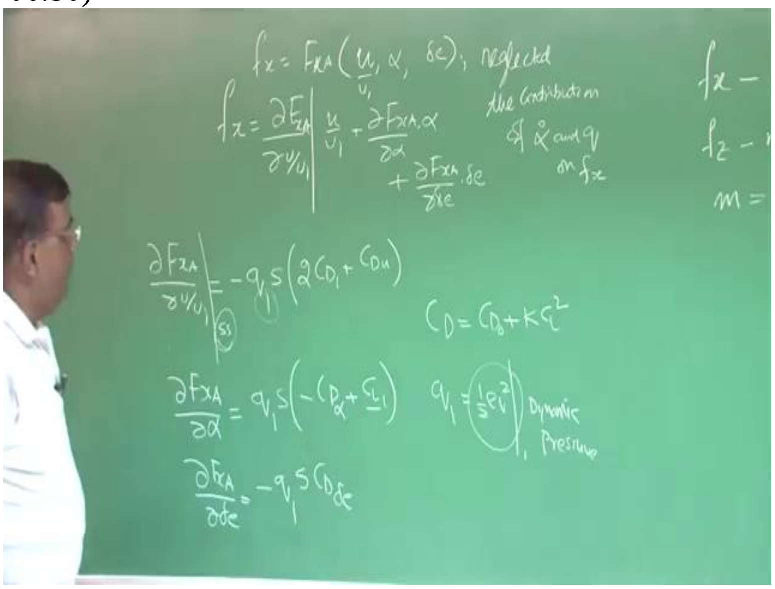

We started writing FX, the function of, we have assumed that it is a function of U by U1 alpha and Delta E. That means, we have neglected the contribution of alpha dot and Q on FX. So what is the message here is we have neglected the contribution of alpha dot and Q on perturbed aerodynamic force, FX. Right? This should be very very clear.

Now from there, what we did? Now we can write it like this D alpha into alpha + DFXA by D Delta E into Delta E. Also my apology. Sometimes I may be using FAX instead of FXA. So both are same things. You have to be patient with me. Sometimes I will be using notations which I often forget. But as long as you understand what is the meaning of DFXA by DU by U1, this is the partial derivative as to how much the force along X is changing per-unit change in U by U1.

Similarly for alpha and Delta E. And we have also found out DFXA by DU by U1 and the expression was - Q1S 2 CD 1 + CDU. Similarly, of course at steady state. Because of that steady-state, I am using the term, 1 here. And what is the steady-state for our case? It is the cruise. Similarly DFXA by D alpha, we found it to be Q1S - CD alpha + CL1.

Similarly DFXA by D Delta E was found to be - Q1S CD Delta E. CD alpha you know. When I say CD, I say drag pull I represent, CD not + KCL square. So I can easily find out DCD by D alpha for small angle but we will assume that CD not does not depend upon alpha. And I can take a derivative and we have shown what will be the approximate expression for CD alpha.

And also you know what is CL1? CL1 is the CL to maintain cruise, level cruise at a given altitude. And given altitude and given speed at steady state is the Q1. That is Q1 is at cruise, what is the dynamic pressure? Once I know this, it is so straightforward for us. What I have to do?

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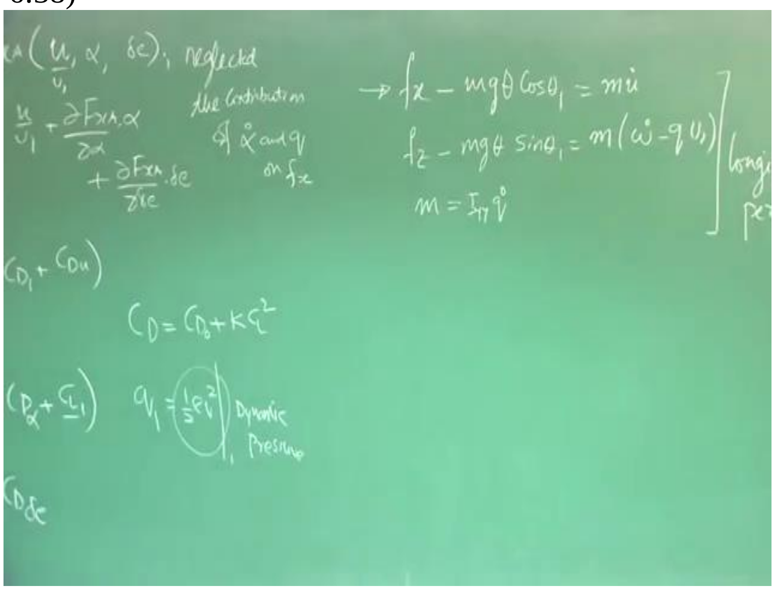

We will take the first equation FX and see how we can further simplify it in a manner where we can use it, use our normal understanding of Laplace transforms, etc and etc.

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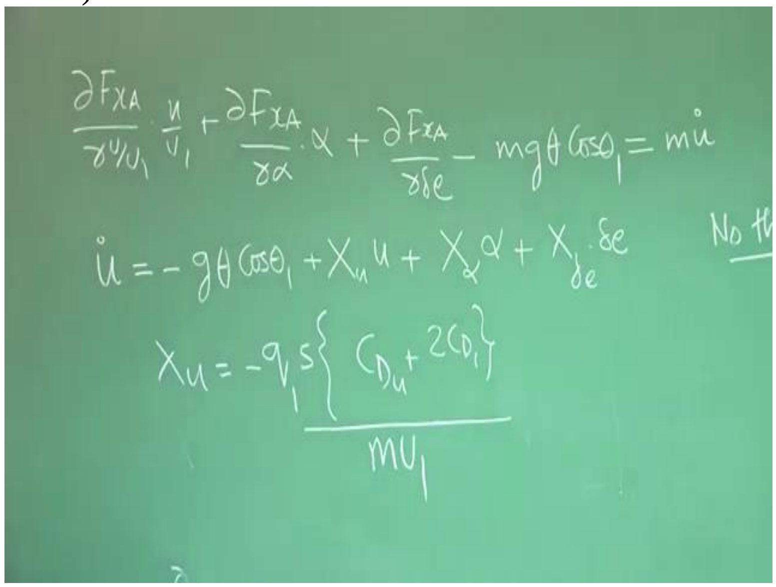

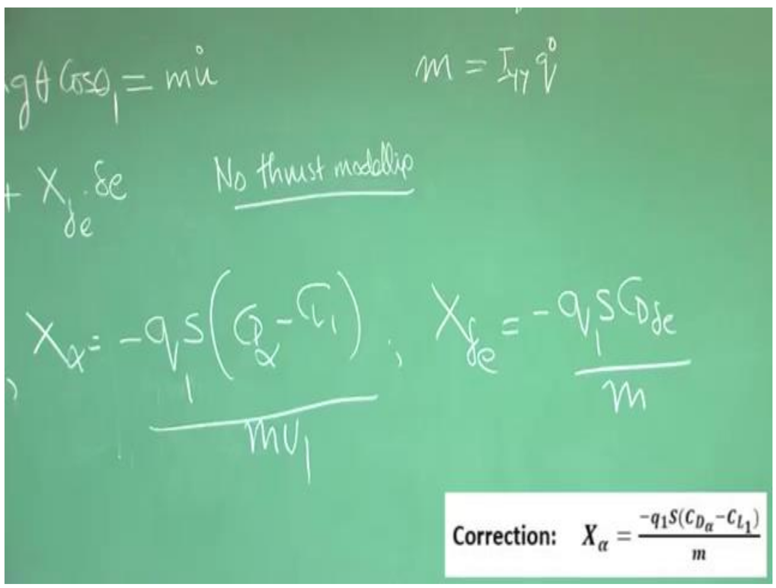

So if I substitute these things there, then I can write for aspects, if I have to write DFXA by DU by U1 into U by U1 + DFXA by D alpha into alpha + DFXA by D Delta E into Delta E. That is for FX. Then I write - MG Theta Cos Theta1. What was Theta 1? Theta 1 is the altitude of airplane at cruise. It is not the flight path angle. Okay, please understand. This is equal to MU dot.

And now if I substitute the equation of DFXA by DU by U1 and DFXA by D alpha and DFXA by Delta E, whatever expressions I have given you, then I can straightforward write U dot equal to - G Theta Cos Theta 1 + XU into U + X alpha into alpha + X Delta E into Delta E. Please be reminded that we have assumed that no thrust modelling which can be easily done in the manner which we are doing for drag.

And what will be the expression of XU? XU will be, you can yourself do it. Once we substitute that expression and once I write it, you will understand that XU is nothing but - Q1S CDU + 2 CD 1 divided by MU1. If you see that, we substitute for DFXA by U1. - Q1 is CDU + 2 CD 1. And then when I write U dot, so naturally U1 is absorbed here. So simply you can find out this once you know XU.

So similarly X alpha will be equal to - Q1S CD alpha - CL1 divided by MU1. And X Delta E will be equal to - Q1S CD delta E by M. That is all. Okay? What is CD delta E? Do not get lost into this expression. These are mechanical. It does not require rocket science to derive these expressions. I have explained enough but you should not lose the insight. What is CD delta E?

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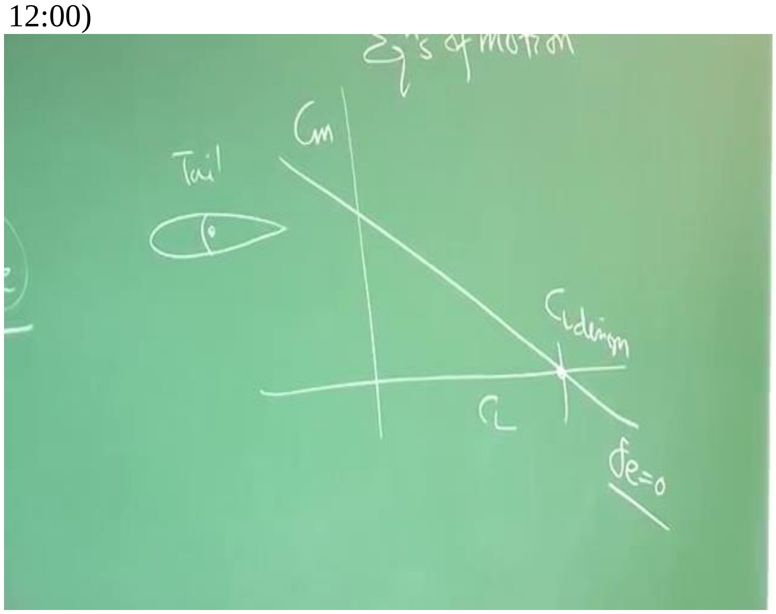

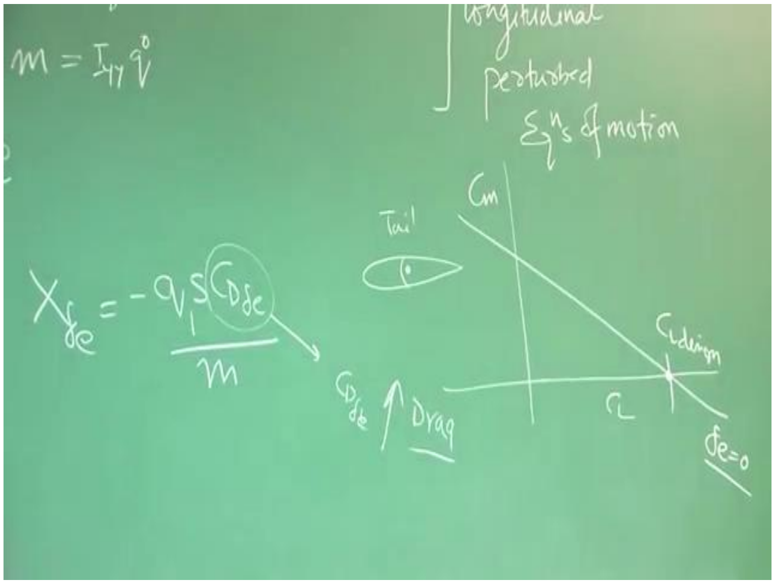

CD delta E is see this is the airplane tail and this is the elevator. If I am flying a machine, I will always prefer to, suppose this is my CL design. So I will always prefer a configuration of wing, the location, the tail location, the tail moment arm, all in such a way that CM versus CL should follow trim such that here DCM by DCL is negative restricted by the amount of static margin you are going to design that you already know.

Also I will prefer, the average should automatically get this configuration at Delta E equal to 0. That is for example, your main mission is to cruise. Then I should be able to get a trim CL design where I need not put any Delta E. Why? Because the moment I put Delta E, this CD Delta E tells me that there will be an increase in drag. Increase in drag because you are trimming it.

So a good designer will ensure that this trim drag, this CD Delta E which attributes the trim drag should be carefully handled. The best way to do it is that for most of the operation which you want to fly the airplane, make sure that it is trimmable with Delta E equal to 0 or very very low. That is why CD delta E is also a very important parameter.

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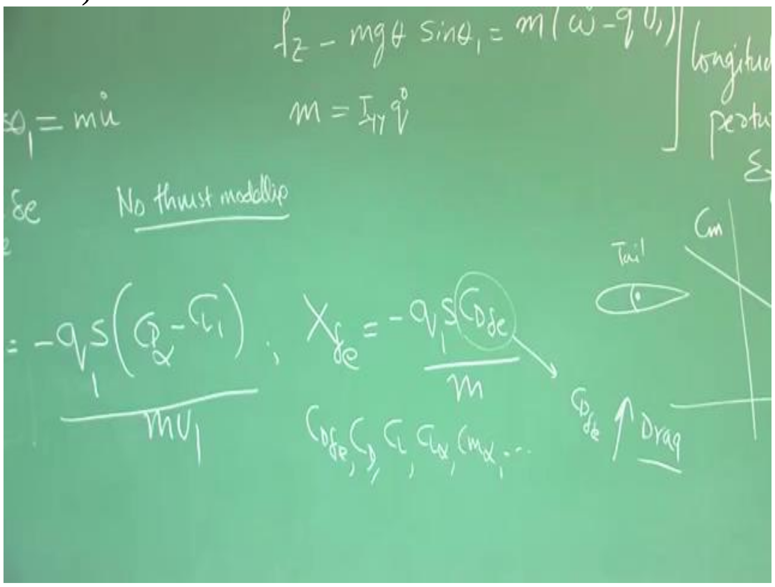

And then another important thing you understand, this CD Delta E, CD, CL. then you will find CL alpha, you will find CM alpha will be coming, etc. These are non dimensional derivatives. They do not have any dimensions. But if you see all these, XU, X alpha, X Delta E, they all have dimensions. For example, the dimension of XU, you will find it is second inverse .

Dimension of X alpha you will find ft second. Forgive me, I am using ft here. Similarly X Delta E, you will find it is having a dimension.

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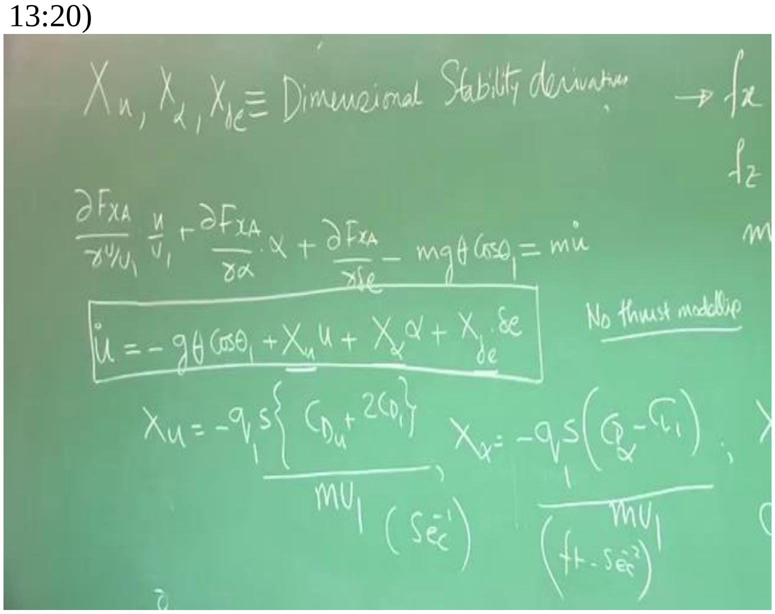

So this XU, X alpha, X Delta E, they are termed as dimensional stability derivatives. You will soon realise, these dimensional derivatives, XU, X alpha, X Delta E, similar like MU, M alpha, M Delta E, ZU, Z alpha, Z Delta E, all will play an important role in deciding the dynamic stability of the airplane. So these are termed as the minstrel stabilit derivatives.

So this is the example of how to simplify the equation in a convenient form for our further analysis. I have demonstrated it for 1st equation, FX which is U dot equal to this, this, this + X Delta into Delta E. Similarly, a similar exercise I will do for FZ.

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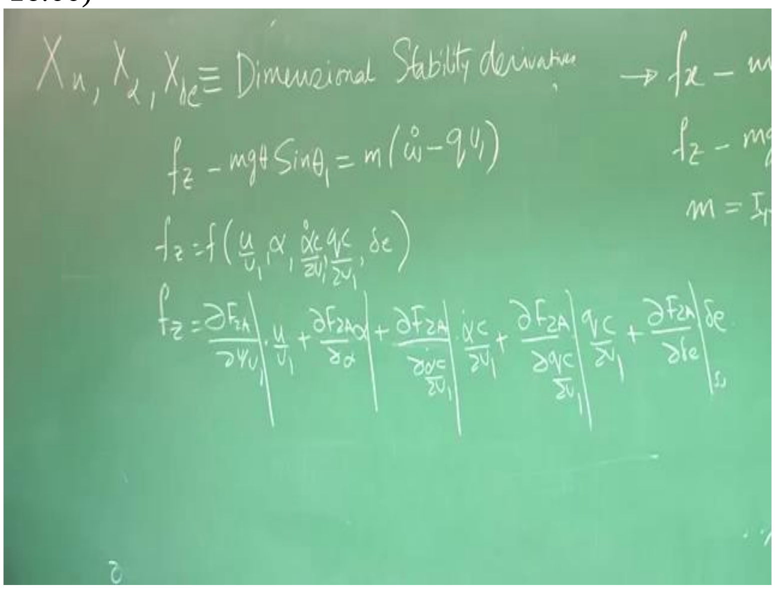

Then I can show that once I try to pick FZ - MG Theta sin Theta 1 equal to MW dot - QU1. If I pick this second equation and I know again, FZ will be function of U by U1, alpha, alpha dot, Q C by 2U1 and Delta E. Then I can write FZ as DFZA by DU by U1 into U by U1 + DFZA by D alpha into alpha + DFZA by D alpha. Alpha dot, we will non-dimensionalise this. So we say, D alpha dot C by 2U1 into alpha dot C by 2U1 + DFZA + DQ C by 2U1 into QC by 2U1 + DFZA + D Delta E into Delta E. Okay?

So what is the next step? We have already learnt when we handled the FX equation. We have to substitute the expression derived for this, this, this and this ad steady-state. Already those expressions, we have identified and derived. We will substitute those here and put it here and complete this equation and once you do that, you will get…

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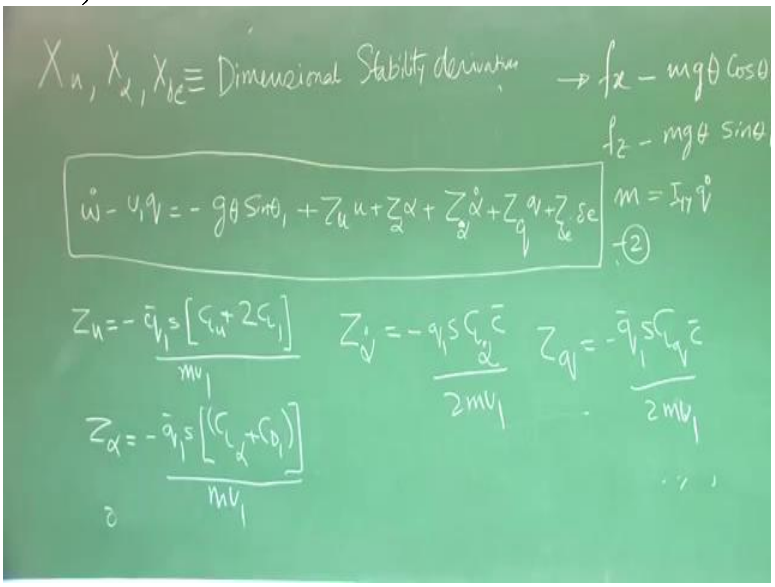

W dot - U1Q is equal to - G Theta sin Theta 1 + ZU into U + Z Alpha into alpha + Z alpha dot into alpha dot + ZQ into Q + Z Delta E into Delta E. And what will be the expression for ZU? - Q1S CLU + 2CL1 divided by MU1. Then Z alpha will be equal to - Q1S CL alpha + CD 1 divided by MU1. Z alpha dot will be given as - Q1S CL alpha dot C by by 2MU1. And ZQ will be equal to - Q1S CLQ C bar by 2MU1.

Please understand that we have already derived the expression for CL alpha dot, CLQ. So these things are just algebraic manipulation. What we have done? We have expanded FZ, substituted those expressions and then divided by MU1 and we got these expressions. This is what we are looking for. So this is my second equation.

And also, please understand that as XU, X alpha is Delta E, this ZU, Z alpha, Z alpha dot, ZQ, Z Delta E, they are all dimensional stability derivatives. Okay? You can check their derivatives. They have a finite dimension.

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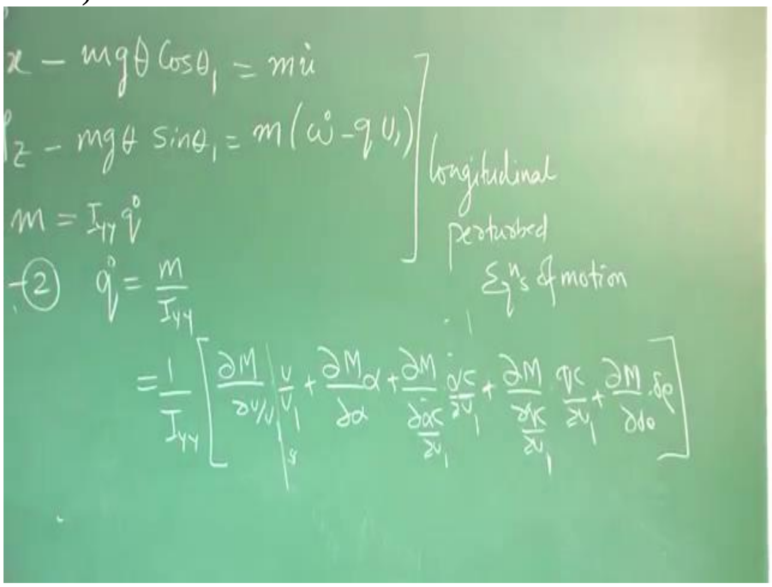

Similarly for M. When I come for M, we will get an equation of the form, for M we will write, we know that M is equal to IYY Q dot. So I can write Q dot is equal to M divided by IYY. And again for M I will write DM by DU by U1 into U by U1 + DM by D alpha into alpha + DM by D alpha dot C by 2U1 into alpha dot C by 2U1 + DM by DQ C by 2U1 into QC by 2U1 + DM by D Delta E into Delta E. Of course, by now you are experts that these derivatives, partial derivatives are to be evaluated at steady state. And we have all done this. We know this expression.

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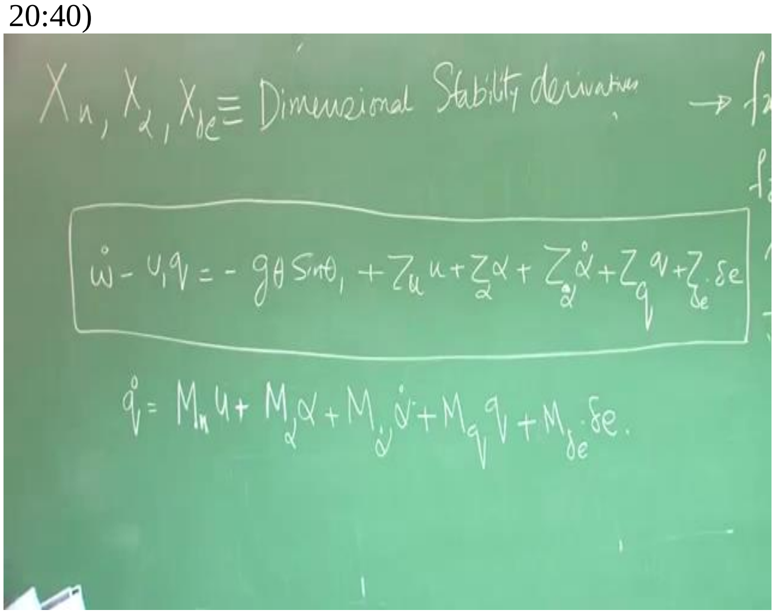

And if I again substitute it here, for M I put this divided by IYY, so I will get equation of the form Q dot equal to MU into U + M alpha into alpha is M alpha dot into alpha dot is MQ into Q + M Delta E into Delta E.

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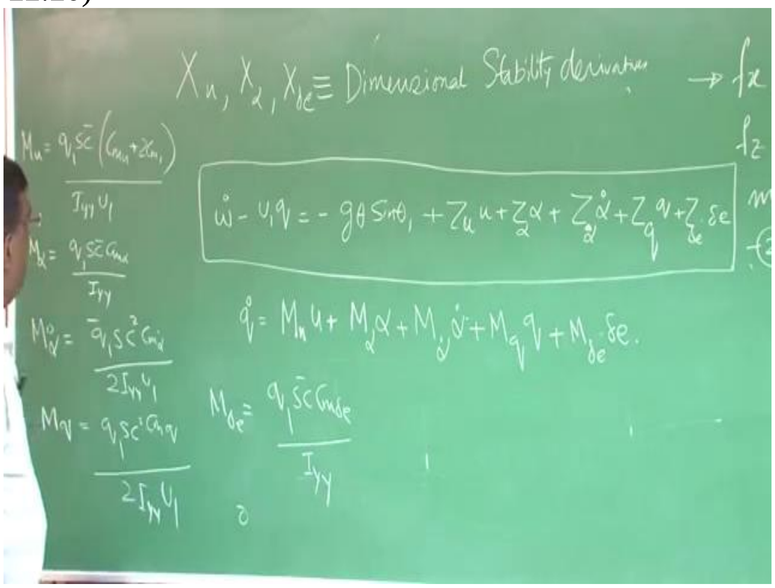

And you could check yourselves, the expression for MU would be, if I write it here, the expression of MU would be given as Q1SC bar CMU + 2CM1 by IYY U1. Then M alpha would be equal to Q1SC bar CM alpha by IYY. Then comes what? M alpha dot. M alpha dot would be equal to Q1SC Square CM alpha dot by IYY U1. MQ will be given as Q1SC Square CM Q by 2 IYY U1. And then M Delta E would be equal to Q1SC bar CM Delta E by IYY. Okay?

Now let us see here, try to understand this dimensional stability derivatives. If I pick M alpha, you could see, it has CM alpha. What was CM alpha? CM alpha was DCM by D alpha. And we have realised that CM alpha has to be native for ensuring study stability of the airplane. And what is the dimension of CM alpha? It is dimension less. So these are dimensionless parameters, derivatives. However, what happens to M Alpha? You could see, it is no more dimensionless. It has a dimension.

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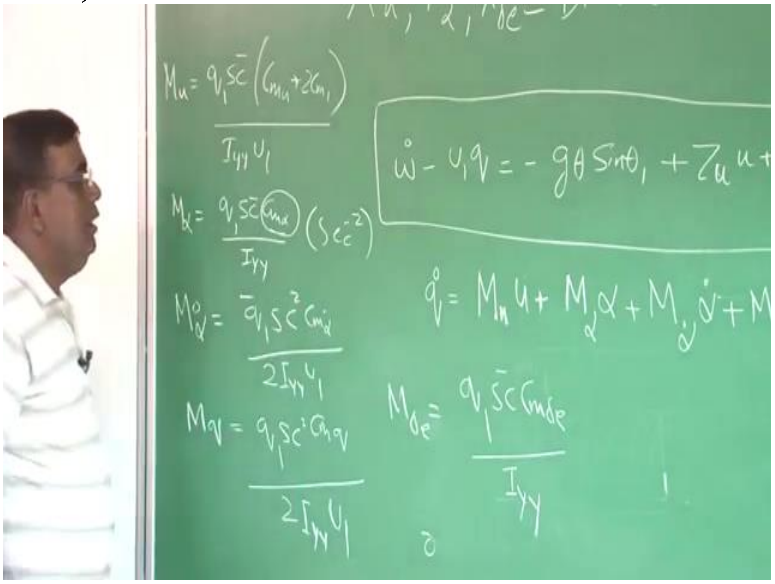

To be more precise, if you want to know the dimension, this will be, you can check yourselves. Of this dimension. Again you could see this, M alpha, M alpha dot, MQ, MU, all are dimensional stability derivatives. This was nont dimensional stability derivative because they attribute to stability.

So this should be very clear in your mind. Before we do some jugglery here and there, our life needs some rest because so much of expressions we are deriving and we need to know what we are going to do with all these. A big question will come whether this approach is helping us or not.