Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 4
Lecture No 22
Longitudinal Modes: Short Period and Phugoid

Good morning. We will be now continuing our lecture on dynamic stability for longitudinal case.

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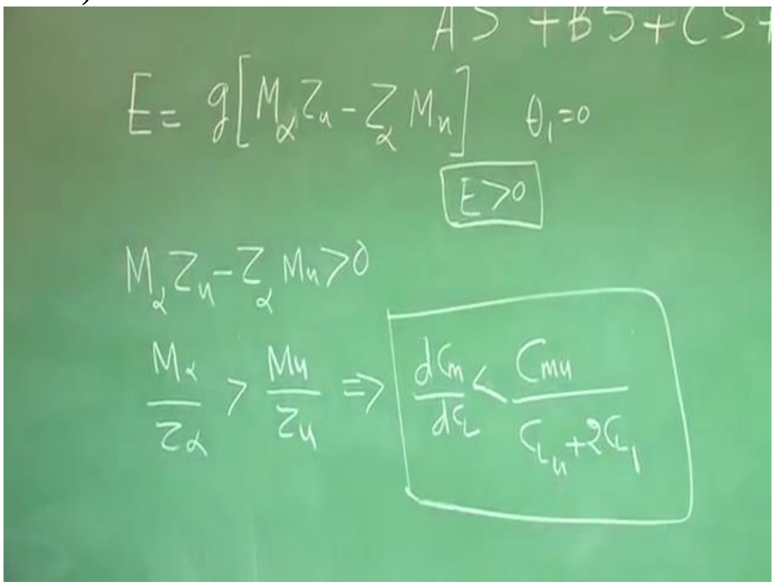

I was mentioning about this E and E was given as G into M alpha ZU - Z alpha MU. Remember, for Theta 1 equal to 0. Now let us try to extract some more information out of this E. What is the condition for dynamic stability? That A should be greater than 0. That is all, A, B, C, D, E all should be greater than 0.

But we are only studying the case E greater than 0 and see whether I am getting some meaningful, additional information or not. That means M alpha ZU - Z alpha MU is greater than 0. That means M alpha by Z alpha is greater than MU by ZU. Now we substitute the expression of M alpha, Z alpha, MU, and ZU.

Then we will get this expression as DCM by DCL is less than CMU by CLU + 2CL1. I am sure you should be able to do it. You have to simply mechanically substitute the expression of M alpha, Z alpha, MU and ZU. And from there you can derive this. Please derive yourself. If you have some difficulty, you can go back to my last course on static stability of the aircraft. There also I have derived this.

But I am sure, it does not the extra efforts to do all these things. Now let us see. Once this is done, what is the meaning of this?

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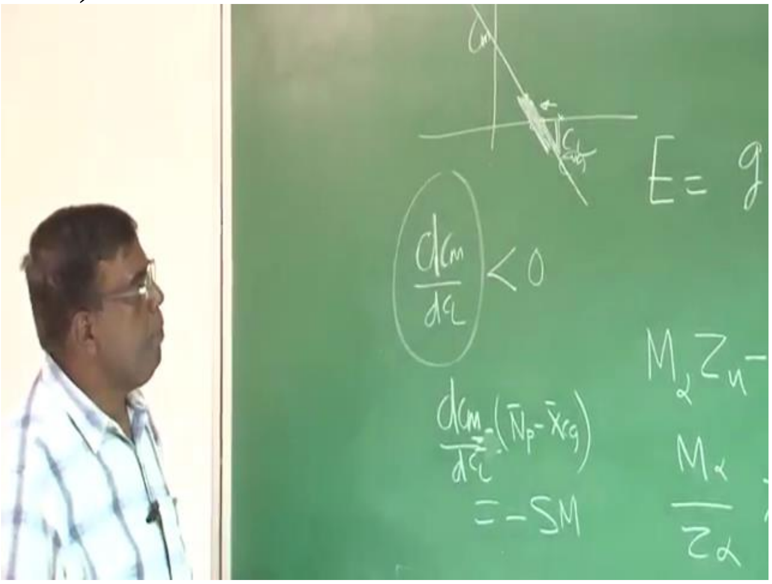

What is DCM by DCL? Remember DCM by DCL. The moment somebody addresses DCM by DCL, we say DCM by DCL less than 0 is the condition for static stability. If you see here, again this is CM, pitching moment coefficient. This is CL. And this is the line at trim. If this slope is negative, we say the aircraft is statically stable because we know that if there is an increase in CL because of disturbance, it will immediately generate negative pitching moment and alpha will be changed.

So it will go down. So that CL will again come back. It has initial tendency to come back. So suppose CL has increased, the aircraft will automatically generate negative pitching moment. So the angle of attack will be reduced and the CL will have a tendency to go back to the equilibrium CL. so we say, DCM by DCL should be less than 0.

Also we have seen earlier DCM by DCL is approximately I can write as the distance between the neutral point and the CG of the airplane with a - sign. And this is nothing but I can write - as static margin, stability margin. This we know. But what does this say?

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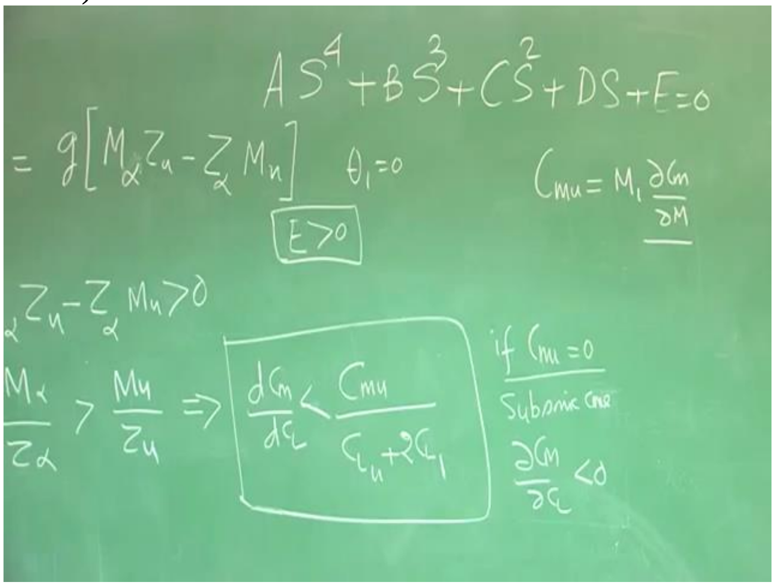

This says, if CMU is 0 meaning thereby what? What do you mean by CMU is 0? That means, wherefrom I got this CMU? Let us again go back. CMU was M1 DCM by DM. And as we are going from a transonic to supersonic, around that zone, the aerodynamic tail of the Centre will be moving far-off. Even for the wing also it moves but till having a large tail arm. So that will give a nose down moment.

So that will make the aircraft to pitch down like this. And that is why it is called a tuck under. But for some sonic case, that value, CMU is 0. There is no shift in aerodynamic centre of the wing or even the tail. So for subsonic case the condition still remains, DCM by DCL less than 0 which will satisfy both static and dynamic stability. But if it is on a transonic, around a subsonic, transonic region where CMU is nonzero, what happens then? What is the information you get out of it?

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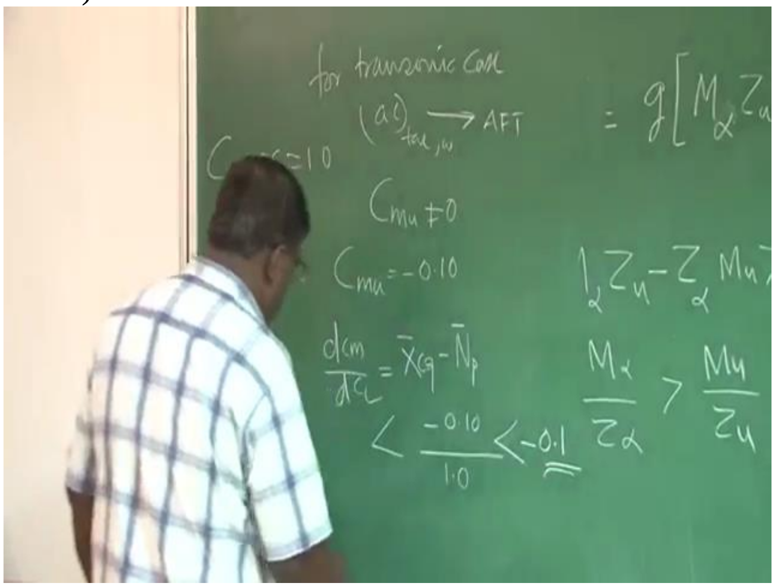

For transonic case where AC of the tail or wing moves aft. So tail arm is there so nose down moment comes, tuck under arms. We call tuck under phenomena. A lot of aircraft we used to lose because we could not identify this phenomena. So CMU, such case is not equal to 0. Then what happens? Let us take an example. Let us say, CMU is - 0.10, then your DCM by TCL which is nothing but XCG - neutral point, this should be less than - 0.1.

And CLU, what is the value of CLU we take? Let us say we take CLU + 2CL1 is typically 1.0. Correct? You can see that if I am flying with CL 0.5, then 2 into 0.5 becomes 1. CLU is very small. So it is not a bad approximation to take this value as 1.0. So I put it as 1.0. So this becomes, it should be less than - 0.1. So what is the message we are getting now?

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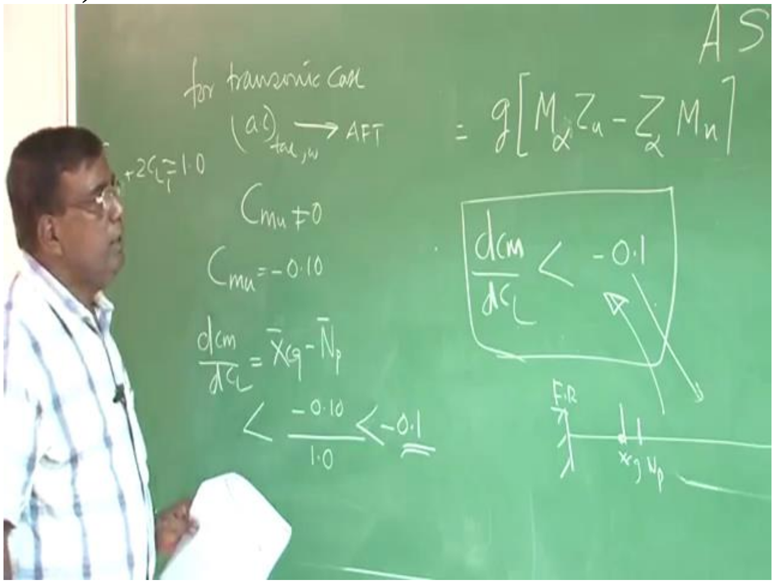

It is saying if you want to ensure both static and dynamic stability for such case where it is going for transonic, then DCM by DCL should be less than - 0.1. Just being less than 0 is not sufficient. Right? For a specific case. For a case where I have assumed these values. So, important thing is it is not sufficient just to have DCM by DCL less than 0. That means, if you are not aware of this condition and you think you are neutral point is here, CG is here and this is the airplane fuselage reference.

And you think neutral point is behind CG. So the aircraft would be dynamically stable. But this says, no man, that is not true. You have to ensure that this separation is validated through this condition because DCM by DCL is nothing but distance between X CG - NP. That is the understanding. That connects static stability to dynamic stability. Right, clear?

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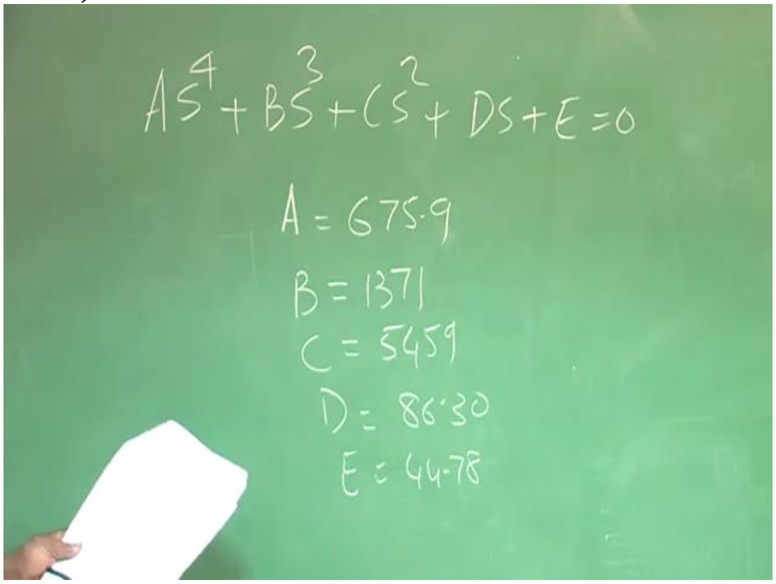

Let us try to handle this characteristic equation. Let us say for an aircraft, the A, B, C, values are like this, A is equal to 675.9, B equal to 1371, C equal to 5459, D equal to 86.30 and C equal to 44.78. You can immediately check, applying Routh's criteria that both the conditions are satisfied. One is that all the coefficients are greater than 0. Then the condition of D into BC - AD - B Square E greater than 0. You can check both these conditions are satisfied.

But what we want to find out now is the roots of this a question. And if we apply a numerical method, we will find, it will generate roots like this.

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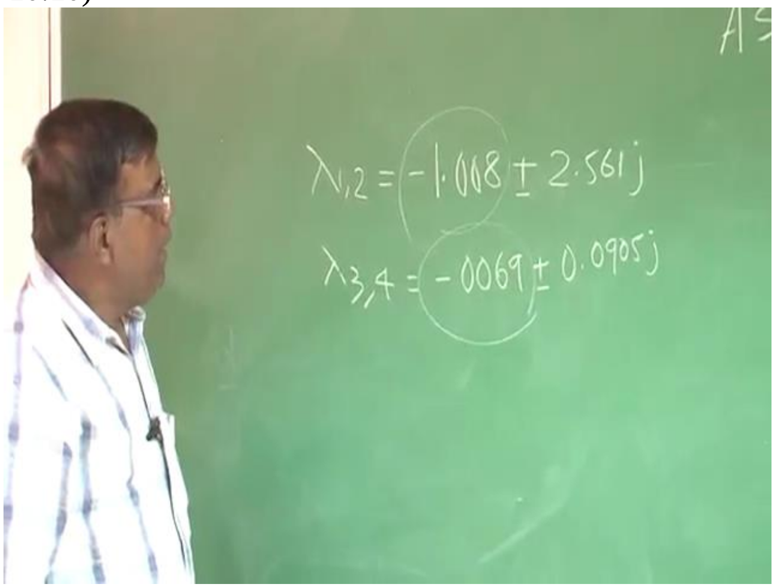

Lambda 12 as - 1.008 + - 2.561J and Lambda 34 as - 0.0069 + - 0.0905J. Now let us again come back here. We are expecting 4 roots. This is 2 + 2, 4 roots are there. Now we have seen, A, B, C, D, E greater than 0 and also second condition is also satisfied. So it will be having roots which will lead to dynamic stability. No instability. That means no real part of this root will be positive. That comes from the conditions.

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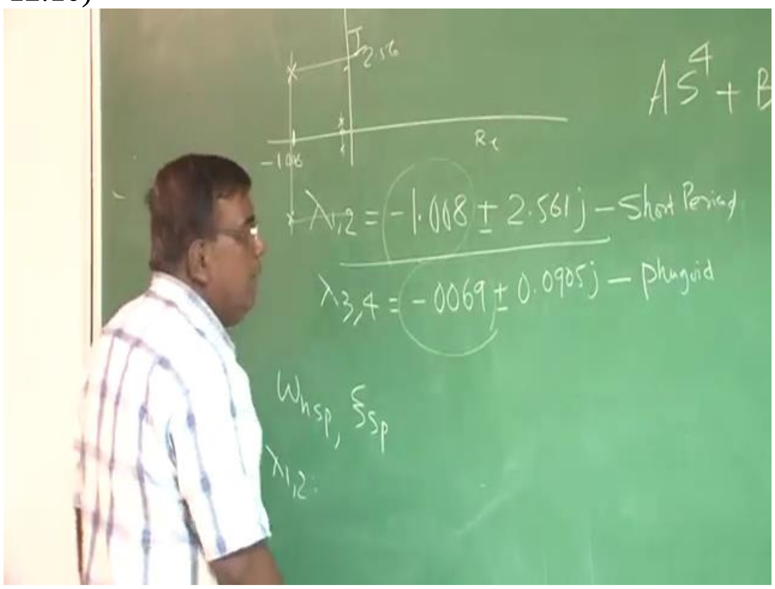

And now what is important to see here is that this pair of roots, the real part if I plot. This is imaginary, this is real. Let us say - is somewhere here. - 1.008 and this value: 2.56. But the other one is somewhere here. That is - 0.0069. Now can you tell me which one will belong to short period? That means what is short period?

That if slight disturbance is given, then the airplane will excite like this and come back to equilibrium. So there is a very short time, so short period. And within that short period, we can fairly assume that the U remains constant. There will be change in the speed. So that is short period. Another is Phugoid, like this.

For short period, the real root has to be relatively large, negative. So this belongs to short period and this belongs to Phugoid. Now you are export. If I try to ask you what will be omega N short period, Zeta short period, you should be able to find out? Yes or no? You have done it so many times. Now it is only second order equator you are handling. If I take this Lambda12, the method will be very simple.

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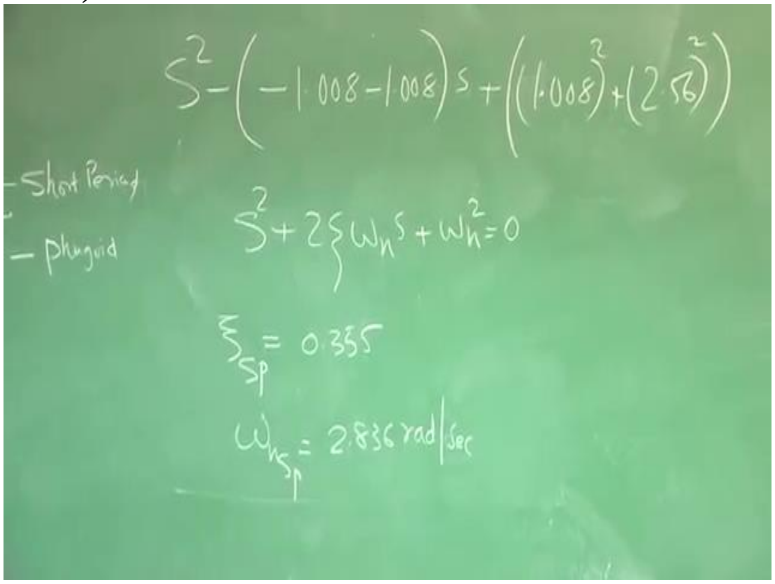

I will consult the equation as S square - sum of the roots. That is - 1.008 - 1.008. Sum of the roots into S + product of the roots will be 1.008 square + 2.56 square. Now I will compare this with S Square + 2 zeta Omega N S + Omega N square equal to 0. And I can find the value of zeta and Omega N as Zeta SP means short period will come out to be 0.355, Omega N short period will come out to be 2.833 radian per second.

Now you could see that although AS4, BS cube, CS square what not equal to 0, but finally whatever we learnt in the second order system, we are applying that as far as longitudinal dynamics is concerned. That is why we are spending so much time on second order system.

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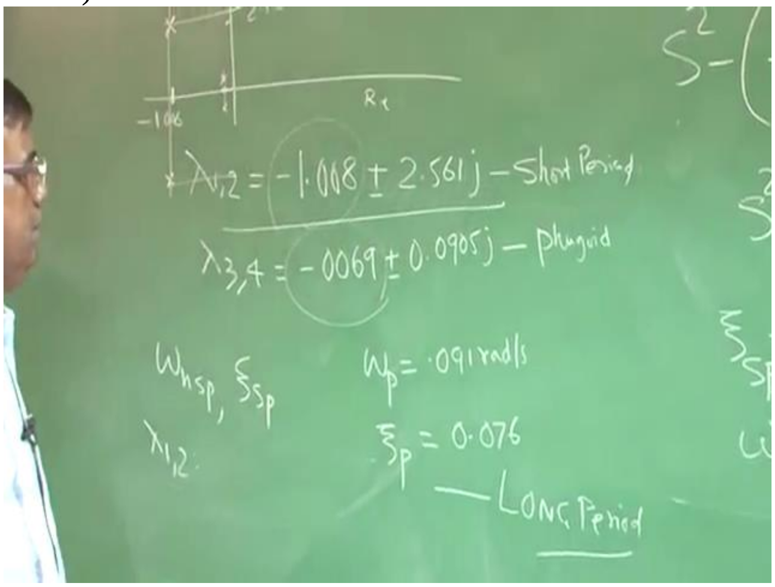

Now if we take the second pair of roots, Lambda 3 and 4, if you do similar exercise, you will find Omega N Phugoid as 0.91 rad per second and Zeta Phugoid you will get as 0.076. So you could see physically also, short period is this. So larger Omega short period, 2.836. And Phugoid is having motion like this. So Omega N is very very small.

The damping for short period is large. Means immediately it comes out, short period, within very short period, it comes back to equilibrium. So it is 0.355. Phugoid takes time. Long period. So it is also called Phugoid or long period. So it takes time to come back to the initial equilibrium or steady-state. So now you could see that although we wrote so many equations, finally we are handling this such a simplistic way.

Because please understand, finally we want to design an aircraft. If you have to solve so many big big equations, when is the time for designing the aircraft? So smart man or smart generation will evolve such approximate way of handling things. And you can quickly see how to find out the natural frequency damping ratio, so many other characteristics you will find. And accordingly you can design an aircraft.

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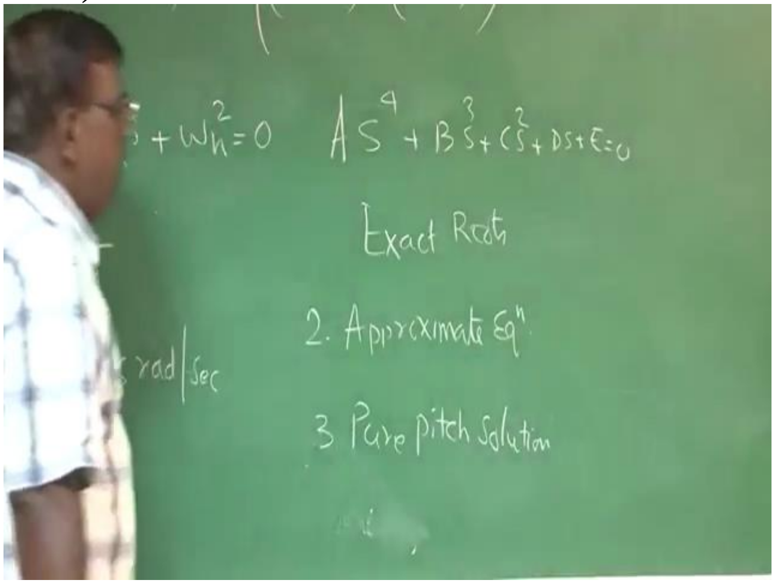

Whatever roots we getting here through AS4, BS cube, CS square + DS + E equal to 0. We call them exactly roots because they have been obtained by solving this exact equation. Next step, we will come to approximate equations. And from there we will get again the meaning of short period and Phugoid through approximation. That will be simpler.

And finally we will compare them with the pure pitch solution. Remember these 3 things because as we will be evolving this longitudinal dynamics, at the end, I must try to tell the designer okay hold on for solving this exact equation because we do not know exact values of A, B, C, D, E because your configuration is not finalised.

Do not worry. You start trying the approximate method to find zeta and Omega N. And still if you find that no, you are not ready, use the pure pitch solution. Once pure pitch, then upgrade yourself like this.

So how to handle this, that will my next part of lecture which you know will have more of design influence. We will now jettison out all these expressions, etc big big equations and all. We have to come back to the aircraft because finally we are proving ourselves to make sure that we have the correct physical feel to design an aircraft rather than lost in big big equations. Thank you very much.