Aircraft Dynamic Stability & Design of Stability Augmentation System
Professor A.K. Ghosh
Department of Aerospace Engineering
Indian Institute of Technology Kanpur
Module 4
Lecture No 23
Short Period Mode Approximation

Good morning friends.

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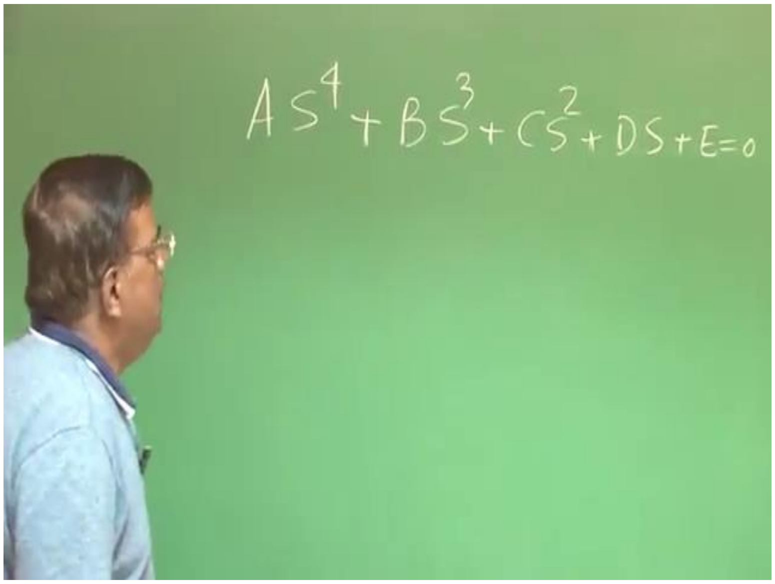

We are continuing our discussion on dynamic stability and in this phase we are talking about longitudinal dynamic stability and you recall that we developed equation, characteristic equation of the form AS4 + BS cube + CS square + DS + E equal to 0.

And we know, this A, B, C, D, E, these coefficients can be evaluated once I know the aerodynamic characteristics, inertial characteristics of the airplane using the expressions which you know how to find it out. We will now try to visualise this through the examples which I was discussing yesterday.

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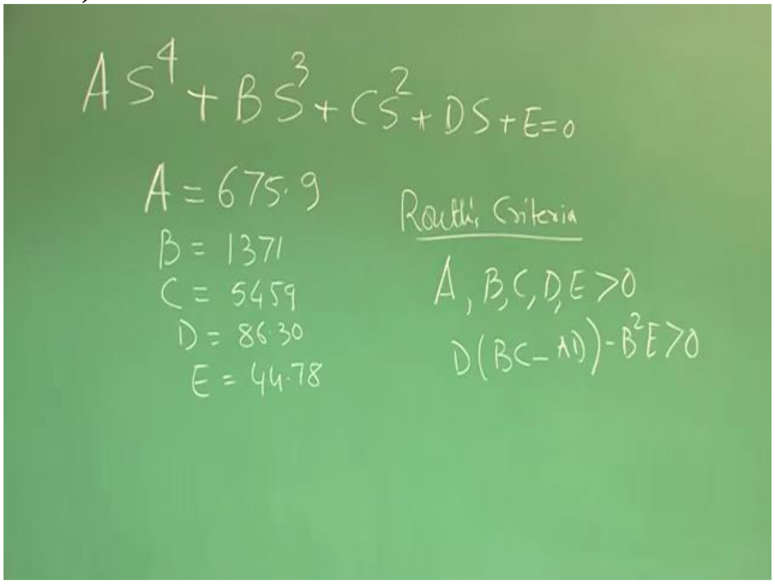

That is for a business jet airplane, the values of A, B, C, D, E are like this which I have given you yesterday. A is equal to 675.9, B is equal to 1371, C is equal to 5459, D as 86.30, and E as 44.78. To quickly check whether this longitudinal dynamics has all the stable roots or not, before we try to get that understanding, we must also understand that because of S4 here, we expect 4 roots.

And the conditions with Routh's criteria, I repeat again, there will be one session where my TA will be talking more about Routh's criteria from implementation point of view. But we know that what we have been using is A, B, C, D, E, they should be greater than 0. And D into BC - AD - B Square E greater than 0. If these 2 conditions are satisfied, then I understand, this longitudinal dynamics has got all the stable roots.

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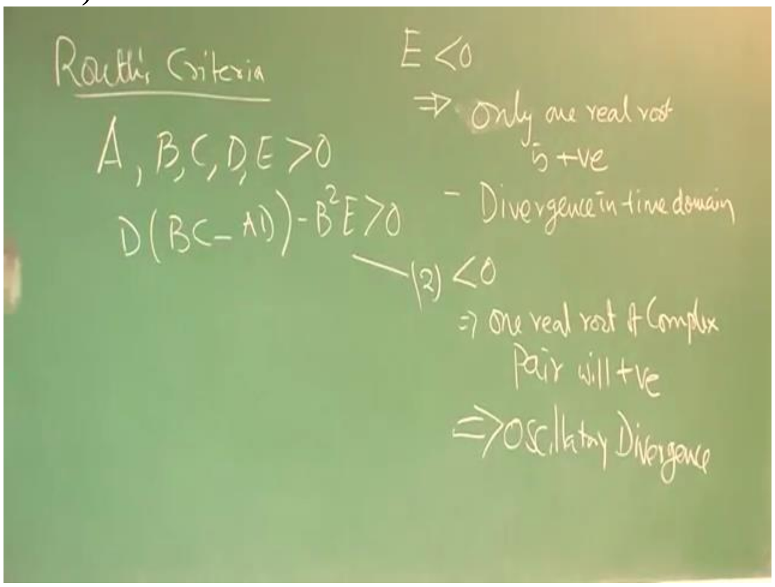

Just to complete the discussion here, if E becomes less than 0, this will only give an indication or it will suggest that one of the real roots, only one real root is positive. That is a change from negative to positive. So it is suggesting divergence in time domain. And then if, second condition, if this becomes less than 0, this will suggest one real root of complex pair will be positive. So this will lead to oscillatory divergences.

So it will oscillate and the amplitude will go on increasing if the second condition is not satisfied. All this detail you will see once you do numerical problems which I have told my TA to do it for you. Now we will try to use these values which are generally representative values for business jet type of aircraft.

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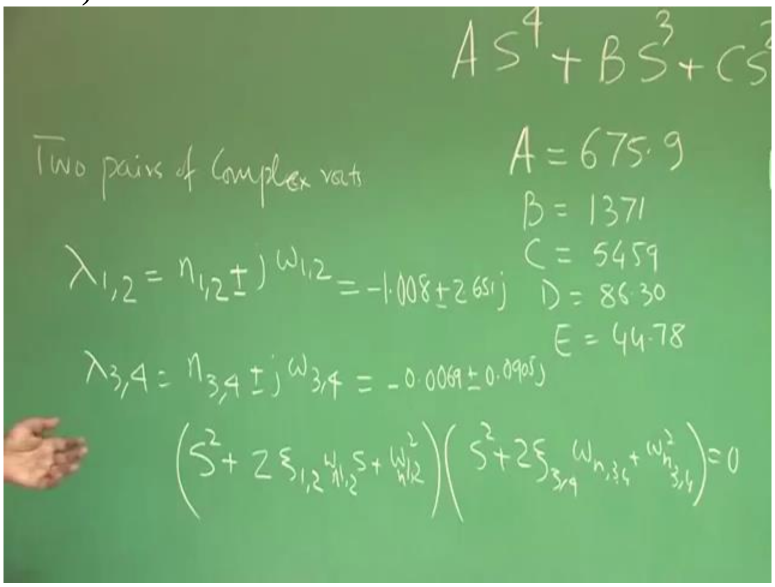

And you will find that this AS4 + BS cube + CS square + DS + E venue try to solve it by numerical methods, you will get 2 pairs of complex roots. Generally you will find this. And for this case if I write, lambda 12 is let us say N12 + JW12 + - complex conjugate and lambda 34 I write N 3,4 + - JW 3, 4.

And for this case, the result shows this, this is - 1.008 + - 2.651 J. And here it is - 0.0069 + - 0.0905 J. This is typically you will find the trend will be like this. What is that we need if I try to see among these roots? The first pair you see, the real part is large negative as compared to the second pair of roots. And the moment the real part of a complex conjugate is negative, we know it is having a damping.

A large negative means it will be damped very fast. And once there are conjugates, I know there will be oscillations. So complex conjugate with real part negative means it will oscillate and damped very fast. Similar thing, if this was positive, again it is a complex conjugate. Let us say instead of - 1.088, suppose it was + 1.008 + - 2.65 J, because it is a complex conjugate, it is oscillatory in nature. That we know.

However since real part is positive for this example case, it will be divergent oscillations. So it will go on doing like this and the amplitude will go on increasing. So divergent in oscillation. But as long as the real part is negative, it is a damped oscillation. What is indirectly also we are getting? That this quartic S4 equation we can for our convenience, for this type of aircraft I can read it as product of two second order equation, characteristic equation.

That is S square + 2 Zeta12 into Omega N 12 S + Omega N 12 square + S square + 2 Zeta 34 + Omega N 3, 4 square, this is equal to 0. This is very very illustrative of some physical meaning of this equation. What does it say? It says, I can approximate, I can break, represent this equation by product of two second order equation. And now if I see these values, one is large negative and one is small negative, I can always think, go back and try to understand how does an airplane behave when disturbances are given in longitudinal mode?

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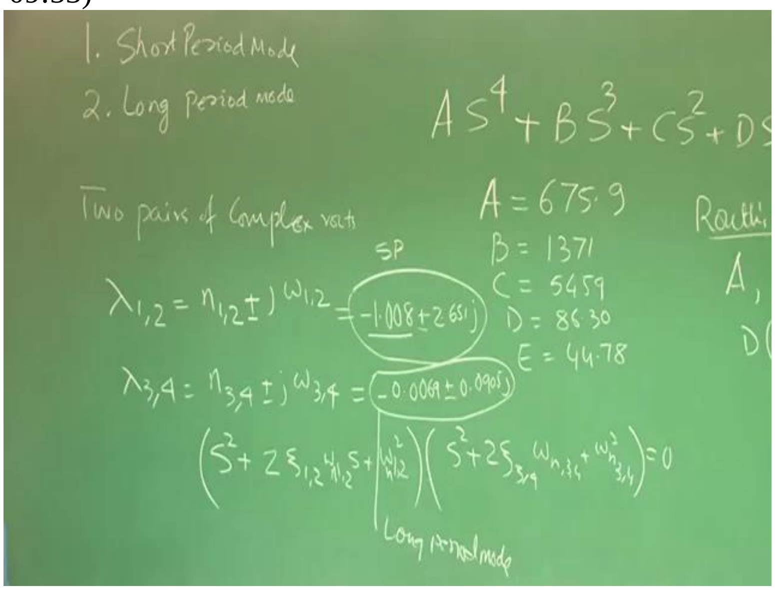

So, one will be short period mode. Another will be long period mode. You could quickly check from this. Since this pair of root has got large negative value compared to this pair, so this root will correspond to short period mode and this will correspond to long period mode. Because real part is very small as compared to the other pair of roots. Right?

Now if I try to build a physics via this understanding of this example, I know the airplane is in longitudinal mode. If there is a disturbance given, it can just get excited like this and come back to equilibrium in very short time. So I say short period mode. This is this one. And second thing, second one, it could be, it goes like this, long period and then it comes to equilibrium.

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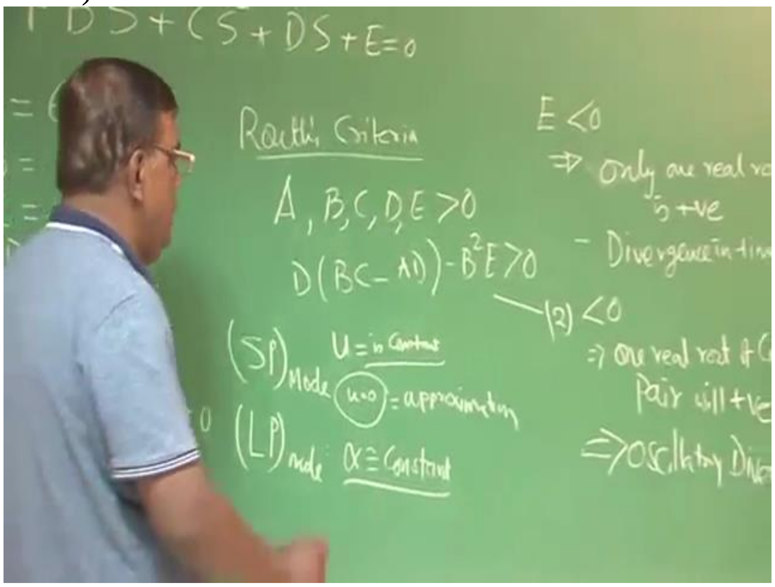

And it is also seen which we can also justify, as far as short period mode is concerned, the time is so short it is fair enough to approximate, U is constant. That is, time is so short that we can always neglect change in the U. So U is constant. That means, when I say U is constant, that no change in velocity or the speed, that means small U, we are talking about 0. Perturbation in U is 0.

That is, time is so short that it comes back to equilibrium but there is hardly any change in the perturbed U. So as far as we understand, there is hardly any change in the velocity, U. So perturbed U is 0. So this is an approximation. For long period mode, we will see that we will say alpha remains constant. Please understand, it is not a very good approximation or in a sense we are telling the perturbed alpha is 0.

For a short period mode, the U remains constant, so the perturbed U is 0. For a long period mode, the alpha remains constant. So the perturbed alpha is 0.

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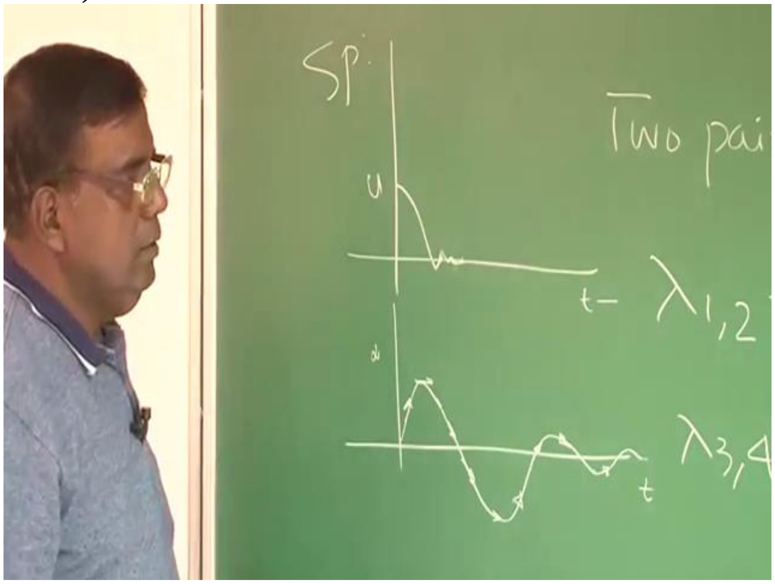

For a short period, if I try to visualise, it is something like this. Very, disturbance, immediately it comes like this. For a Phugoid mode, it will be something like this. And the airplane, it always goes like this. This is an approximation which is not very very accurate, but we understand it is not a very good approximation but fair enough to get some initial understanding from design aspect.

So now, since are always discussing, we need some neat expression which I can readily use for designing an aircraft. We will see how this approximation, short period approximation and Phugoid approximation is going to help us to get some feel for some design numbers. That will be our now, attempt.

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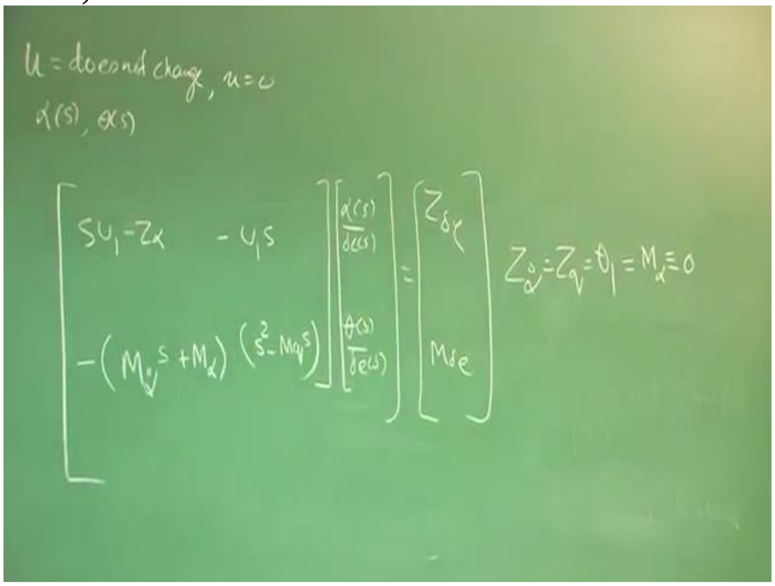

So we are talking about short period approximation. What we said? Perturbed U is 0. That means this dynamics is more governed by Theta and alpha. Alpha is changing, Theta is changing. Let us do this short period approximation. We have written the general equation for longitudinal dynamics which was developed using perturbed equation of motion.

Now, for short period we know does not change. That is, U is, small U is 0. So this U is perturbed, is supercilious because it is automatically taken care. So we are more concerned about alpha of S and Theta of S. Right? Short period. So, no change in U but change in Theta, change in alpha. So we retain those 2 equations. And then once we do that, we can simplify this as SU1 - Z alpha and - U1 S, - M alpha dot S.

Let me write this. I will explain. S square - MQS. Then this is alpha of S by Delta E of S. Theta of S by Delta E of S equal to Z Delta E, M Delta E. Of course we will be assuming Z alpha dot is equal to ZQ is equal to Theta 1 is equal to MT alpha, etc to be identically 0. Let us see how does it come?

So now we are simplifying our, using approximation that U does not change or perturbed U is 0. We are more bothered about alpha S and Theta of S.

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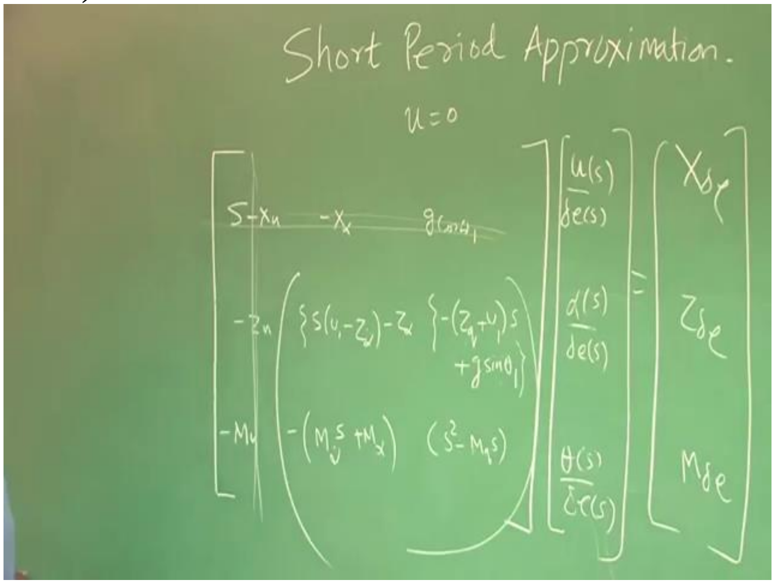

So we are dropping these 2 terms. This row and column. We are only left with this which is written here. For simple case, I am assuming ZQ as 0, Theta 1 as 0, Z alpha dot as 0. And you could see what is this first equation? This into U + X alpha into this + G cos Theta into this equal to X Delta E. But we are assuming, there is no change in U. Perturbed U is 0.

So that equation becomes supercilious. Now similarly if you see, if I come here, ZU into U + this term into this, this term into this becomes Z Delta E. Similarly MU into U + this into this + this into this becomes M Delta E. So what we have dropped? We have dropped first equation and also this one I have dropped because this is with you, ZU into U, MU into U. So those have been dropped.

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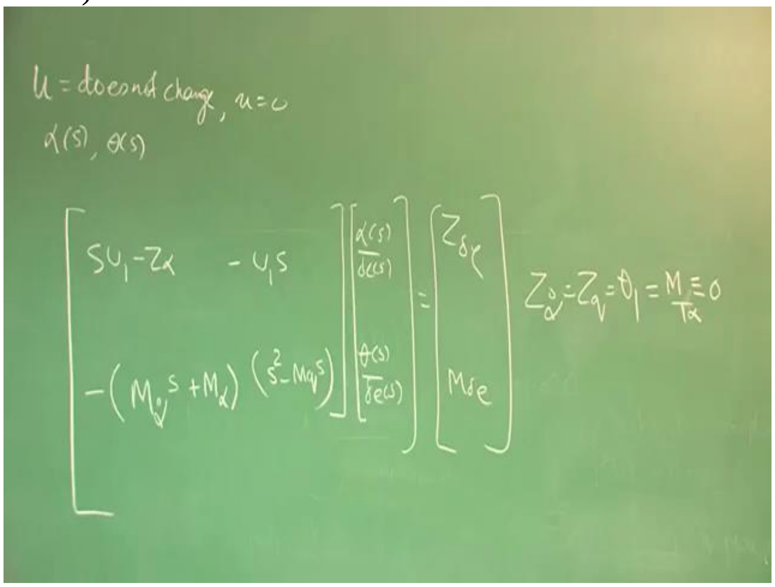

So we are only having this which is here. Please understand, these are approximations and approximation is, U does not change, total U does not change. So perturbed U is 0 and U equation is automatically taken care. So I am dropping that. So I am only talking about alpha and Theta. And accordingly, this is simplified.

For a case where Z alpha dot, ZQ Theta 1, M alpha dot what will be? It will be MT alpha, not M alpha. MT alpha equal to 0. If I put M alpha equal to 0, you understand it becomes statically neutrally stable. So it is MT alpha. Please correct that. So now so what happens?

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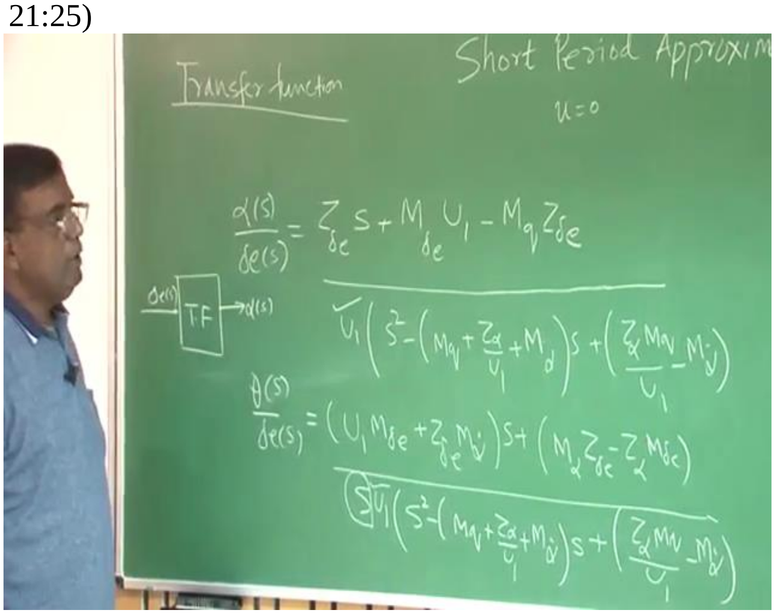

So we get, if I take the determinant of it and I try to solve it, then I get alpha S by Delta E of S will be equal to, let me write this Z Delta E into S + M Delta E into U1 - MQ into Z Delta E, this is divided by U1 into S square - MQ + Z alpha by U1 + M alpha dot into S + Z alpha MQ by U1 - M alpha dot.

You know how to find this expression. You have to simply apply Kramer's rule and you get alpha S by Delta E of S like this. Similarly I get Theta of S by Delta E of S as U1 M Delta E + Z Delta E M alpha dot S + M alpha Z Delta E - Z alpha M Delta E divided by S and then same term, U1 into S square - MQ + Z alpha by U1 + M alpha dot S + Z alpha MQ by U1 - M alpha dot.

What is this alpha S by Delta E of S or Theta S by Delta E of S? These are called transfer functions. Very very important. Transfer functions. So directly, this is telling, if I know Delta E of S and if I know all these derivatives, I can find alpha of S and once I know alpha of S, I can take a inverse and get alpha in time domain.

Those part, I am not talking about over here. This is very very important. This transfer function that is output to input that is if I have a transfer function here, and if I give input, Delta E of S and if I operate this transfer function, it will give me alpha of S. This is extremely important you will find when you will be designing controller or also for stability augmentation system. Okay, this is fine.

And let us have a closer look into this. Why we are doing all this? Why we are tolerating all these big big expressions? As I have told you earlier, this you have to do only once. Once you do it, understand it, forget it. Then use it as a reference.

If you see the denominator for alpha S by Delta E of S and the nominators of Theta S and Delta E of S, you find, this whole term is here from here to here but there is an additional S here.

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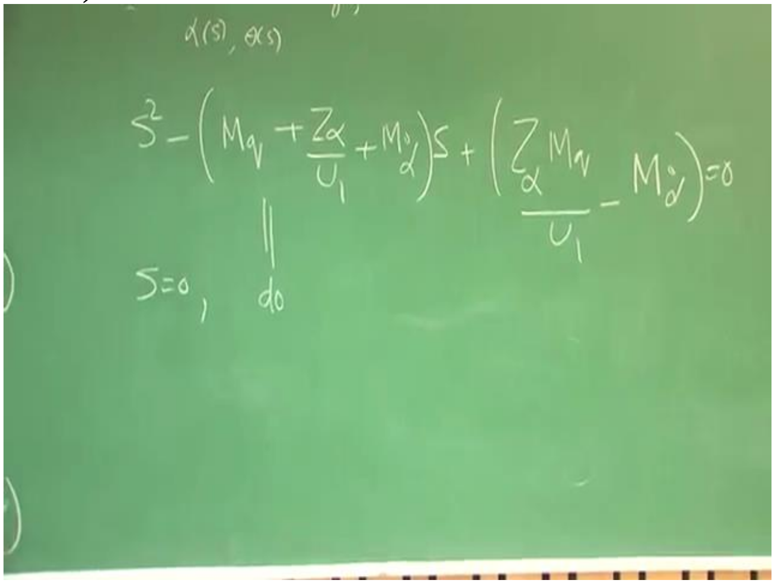

Now when I talk about characteristic equation, you know by now this denominator should be equal to 0. That is S square - MQ + Z alpha by U1 + M alpha dot S + Z alpha MQ by U1 - M alpha dot equal to 0. This becomes the characteristic equation for our alpha S by Delta E of S but when I come to Theta S by Delta E of S, I get in addition, S equal to 0, another is same equation.

Right? You could see from here. So what is this S equal to 0? Please understand, we are writing transfer function of Theta S. Theta is the pitch angle. As far as stability is concerned, there is a neutral stability in terms of Theta. That is the airplane flies like this, flies like this, flies like this. Aerodynamically, they are identical. It is neutral. It does not change as far as Theta is concerned.

You may ask one thing, what about moment? It does not change but the Theta is 0, it is like this, like this or like this. As far as aerodynamics is concerned, whether I am moving like this, I am moving like this or I am moving like this, it all depends on the relative airspeed. As my relative airspeed, they are same, they are identical. So it does not distribute between Theta 0 or Theta 10 degrees, Theta 15 degrees in normal case.

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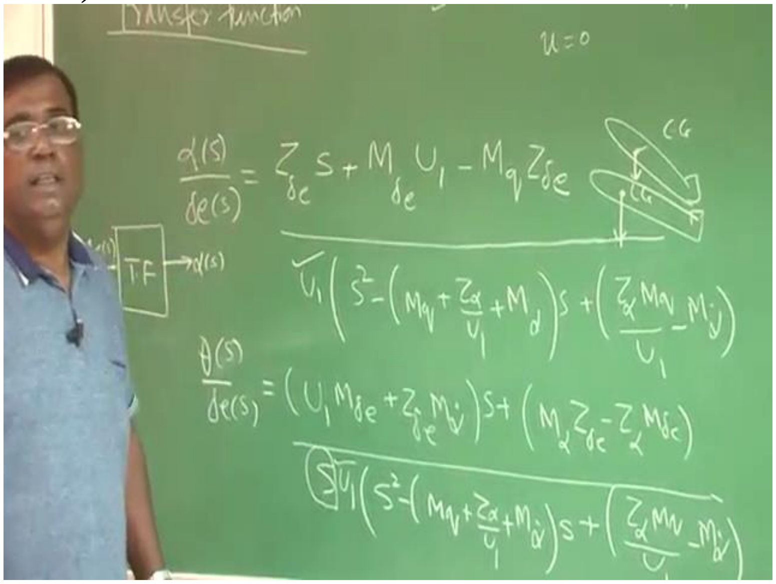

As far as gravity is concerned, what comes to our mind always is that if Theta is like this, another Theta is something like this. What about gravity? But we know that all these moments are about centre of gravity. When it passes through centre of gravity, it does not make any change. Aerodynamic does not make any change because it depends on the relative airspeed.

So Theta 0 or Theta 10 degree or Theta 15 degree or Theta 30 degree. As long as relative air is same, it is aerodynamically identical. And since weight passes through CG X does not matter. So that is why S equal to 0 corresponds to neutral stability in terms of Theta and with this understanding this should be clear. That cannot be extended for alpha.

That is why, there are no S equal to 0 in this case because with alpha, things will change. So once we understand that, we will delete it and we will work with this equation.

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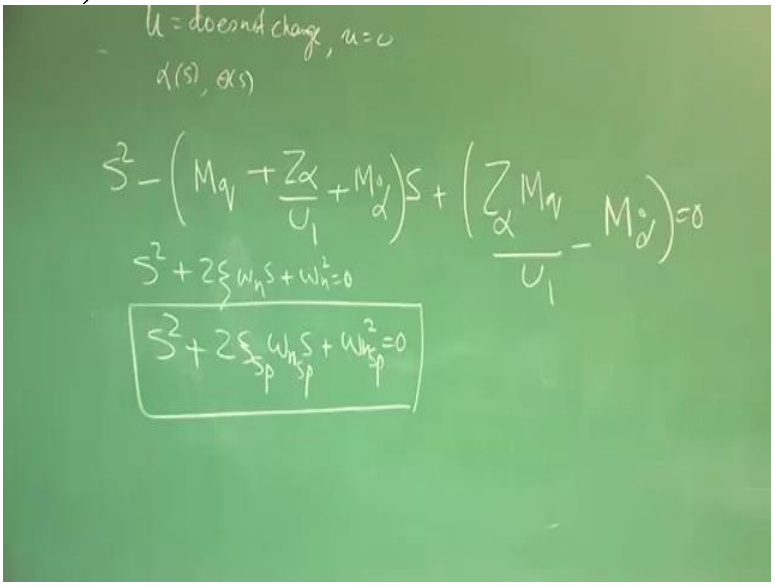

Now see, you are already experts. I have to compare this with S square + 2 Zeta Omega N S + Omega N square equal to 0. But I know, now I am talking about short period approximation. So I write S square + 2 Zeta short period Omega N short period S + Omega N short period square equal to 0. This becomes my characteristic equation and now I compare this with this and to find out the expression for Omega N short period by approximation, let us see what do we get?

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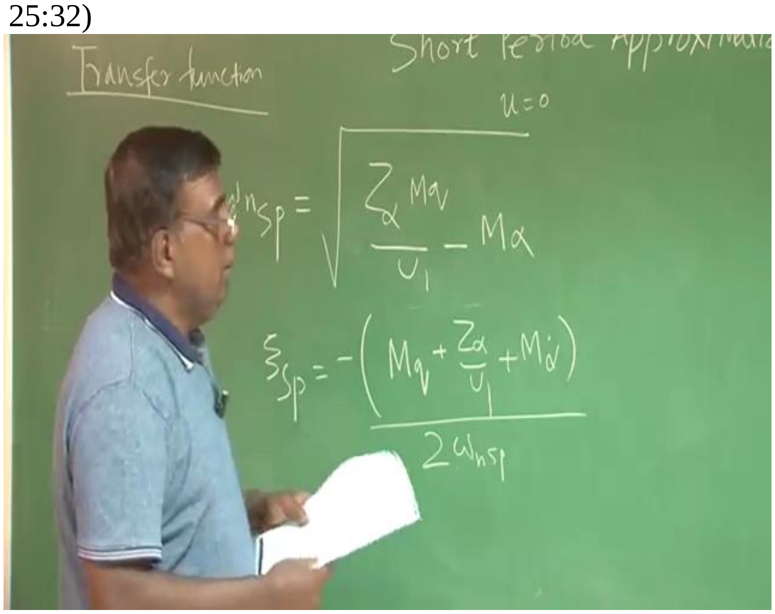

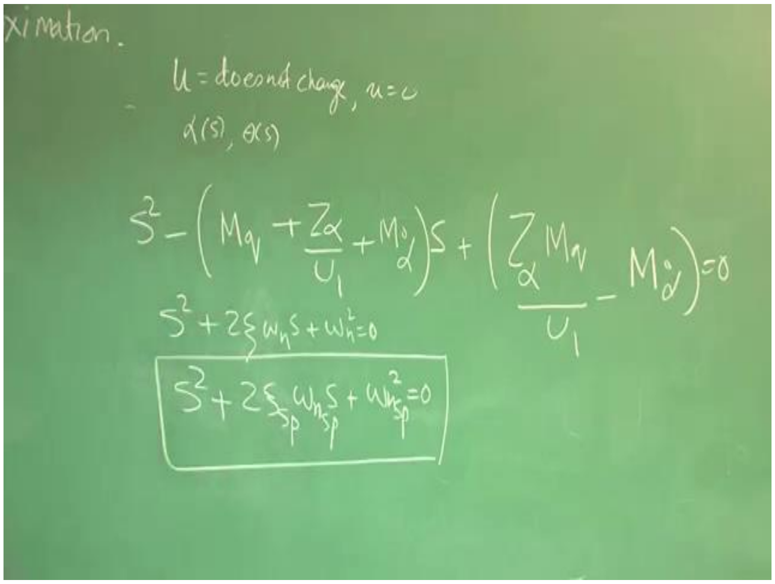

If we do this comparison, we will get Omega N short period is equal to Z alpha MQ by U1 - M alpha and Zeta short period we will get as - MQ + Z alpha by U1 + M alpha dot divided by 2 Omega N short period. It should not be difficult for you. 2 Zeta short period Omega short period should be equal to this term, MQ + Z alpha by U1 + M alpha dot and Omega N short period square will be total this term.

So that is how this thing has come. This should not be M alpha dot. This should be M alpha. Sorry. I would to draw attention here. This characteristic equation, S square - MQ + Z alpha by U1 + M alpha dot S + Z alpha MQ by U1 - M alpha, not M alpha dot. That was wrongly written earlier. I have tried to correct it.

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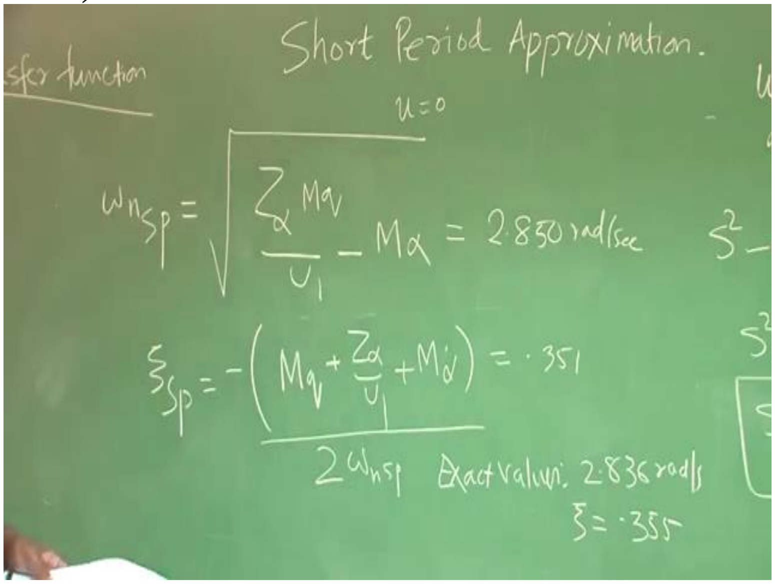

And now if I compare this, they get an expression, Omega N short period is like this and Zeta short period is like this. And if we put all those exact numbers which we have used to solve AS4 + BS cube + CS square + DS + E equal to 0, if I put this aerodynamic characteristics, inertial characteristics, then we get this as 2.850 radian per second and this we get around 0.351 and if you recall, the exact values were 2.836 radian per second and Zeta was 0.355.

And if I compare Omega N short period which I have developed through approximation then I find, Omega N and Zeta are fairly close. So it is a general trend. For most of the cases, we will find, this approximation as far as short period is concerned, is a very good approximation.

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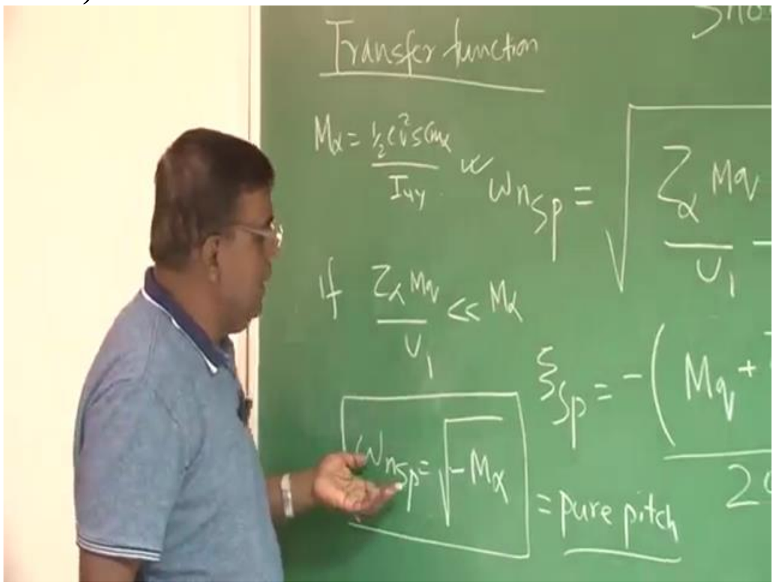

This is another thing you try to understand. If Z alpha MQ by U1 is less than, very less than compared to M alpha, then I can neglect this term. Then what will happen? Then Omega N short period will become under root of - M alpha. And what is this expression? If you recall, this we opted for pure pitch.

This approximation, how realistic it is, you can see that it is divided by U1. It is a large number. So mostly you will find that this is not a bad approximation. So for a designer, what he does? Designer if it decides the natural frequency for short period will be of a particular order, let us say 3 rad per second or 4 rad per second, he can immediately check what will be the static margin.

How does he get that idea? Because M alpha is nothing but half Rho V square S CM alpha by IYY. And static margin is linked with CM alpha. This is dynamic pressure. So designer knows what altitude he is going to fly, primarily. He has a rough idea about what is the moment of inertia for same class of airplane.

So he can easily decide what sort of static margin he can design for or if he is designing for a particular static margin, then what sort of natural frequency short period he will get. Same understanding he can use it for finding damping ratio as well. So this is the beauty of this approximation.

And before I complete this short period part, please also understand that as far as longitudinal dynamics is concerned, the short period approximation is a fairly good approximation. And you will see, similar approximation may not hold true for Phugoid type of approximation.

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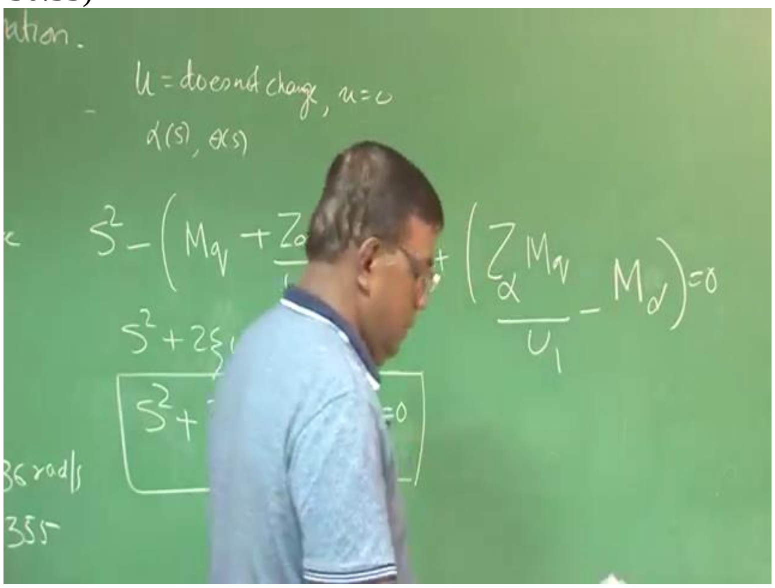

I repeat again. Please check this. Here, it should be M alpha, not M alpha dot. One request I have got to all of you, since we are writing so many equations, do not get demotivated. You should do it yourself. It does not require a huge knowledge of mathematics. Do it yourself once. You have to do once and try to understand the physics of this expression which I am trying to stress when I am comparing pure pitch to the exact and to the approximate values.

These are extremely important. You will realise soon. From the handling qualities of the pilot, this Omega N, Zeta they separately as well as together, they play a huge role in making the pilot in command, very very comfortable. That is why we need to understand, if I want to tweak Omega N short period, what is the meaning of the thing I will change from this expression, from this expression and finally validate that through exact solution.

That is why we are giving so much of time in doing all this analysis to understand few important parameters so that finally we can make the pilot comfortable because dynamic stability talks about the transient and the pilot is the person, the passengers are the person who will experience this transient.

So we have to make them comfortable. We can make them comfortable through correctly choosing the values Omega N, Zeta or the combination. Thank you very much.