Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 5
Lecture No 26
Lateral, Directional Stability Derivatives Continued…

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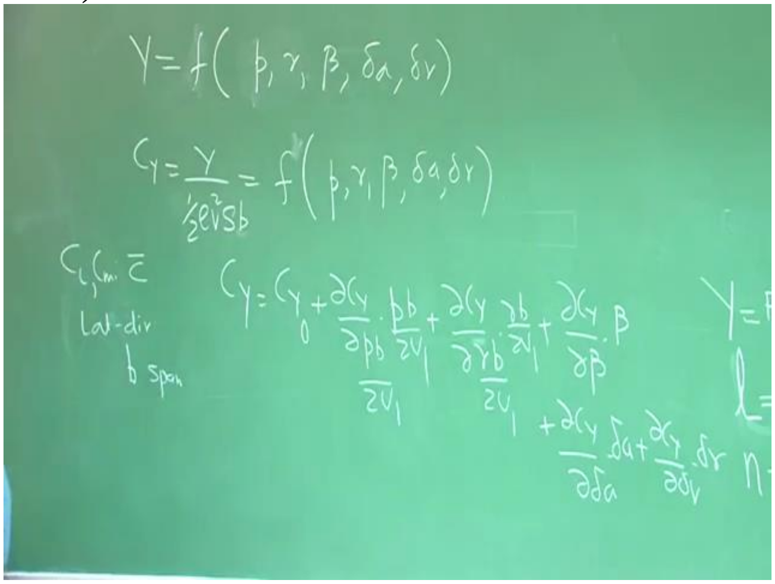

If I write Y is function of P, R, beta, Delta A, Delta R, then you know that CY which I can define as Y by half Rho V Square S into B. This is to be noted. For CL, for CL and CM, the characteristic length was mini aerodynamic chord. But for our lateral directional coefficient, this characteristic length is B, the span. That is why B has come.

So this is again, I can write function of P, R, beta, Delta A, Delta R. So that you do not get confused, I write extra R or let me delete this. So CY is function of this. So I can expand CY as CY not + DCY by DPB by 2U1 into PB by 2U1. You understand why we are using PB by 2U1? Because we want to non-dimensionalise roll rate, right?

+ DCY by again R is rate, so DRB by 2U1 into RB by 2U1 + DCY by D beta into beta. There is no need to multiply by B and divide by 2U1 because beta itself is dimensionless. Similarly, DCY by D Delta A into Delta A + DCY by D Delta R into Delta R.

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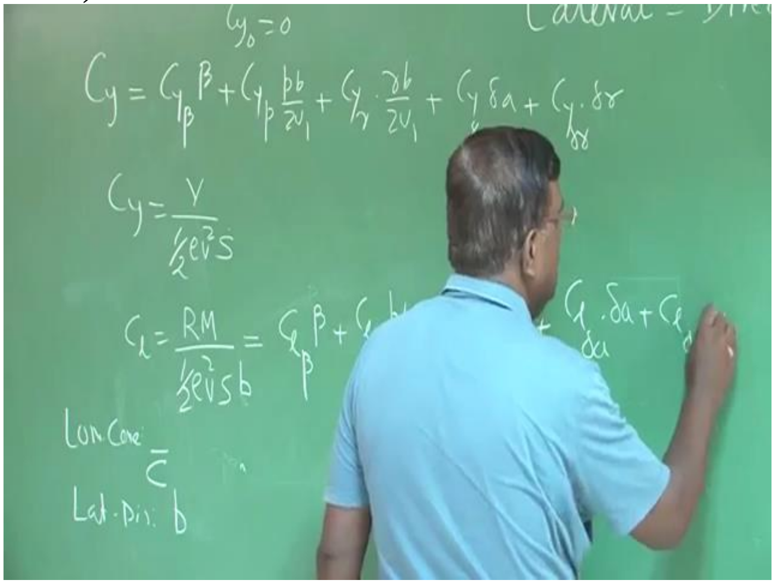

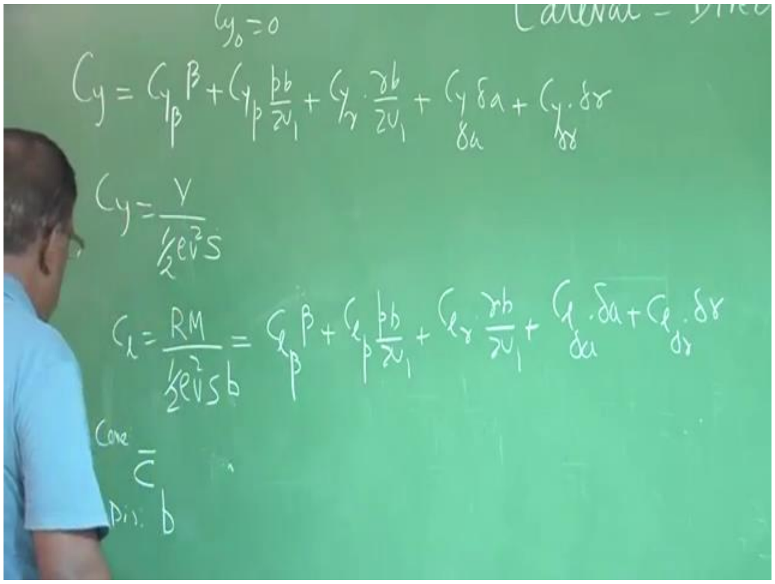

In a short notation, we can write this as CY equal to CY beta into beta + CYP into PB by 2U1. And CYR, CY Delta A and CY Delta R where CY is side force Y non-dimensionalised with dynamic pressure half Rho V square into S. If I know this, I can easily write the expression for CL and CN. What is CL? CL is rolling moment non-dimensionalised with half Rho V square S. But it is rolling moment, so length term is required. So it is B.

Please note that for longitudinal case, the characteristic length was C bar. But for lateral directional case, the characteristic length will be the span, B. B is used here. So this I can easily write again, CL beta into beta + CL P into PB by 2U1 + CL R into RB by 2U1 + CL Delta A into Delta A + CL Delta R into Delta R.

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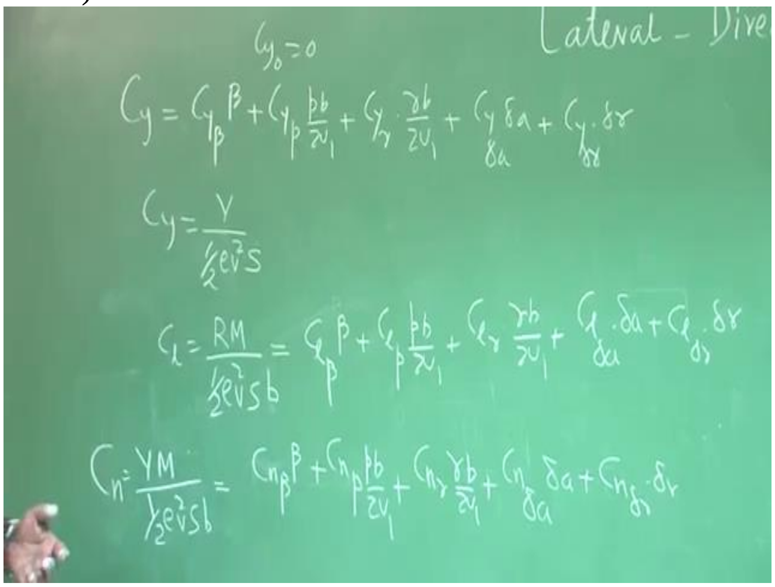

Similarly, I can write, for your yawing moment coefficients CN which is nothing but join moment non-dimensionalised with half Rho V square S. What? B or C? It is directional case. So it is B. This is equal to, I write again, CN beta into beta + CN P into PB by 2U1 + CN R into RB by 2U1 + CN Delta A into Delta A + CN Delta R into Delta R. Please understand, this sort of an expansion strictly true for low speed and moderate performance as plain where we are assuming everything to be linear.

Today with modern aircrafts, the airplanes are very high performance with high rates, high angle of attack, so there, writing this corresponding this in terms of linearised concept may not be true unless and until you cleverly, judiciously, locally linearise it. But there are different ways of handling it. So that should be kept in your mind all the time. So now if I erase this, we make the board clear.

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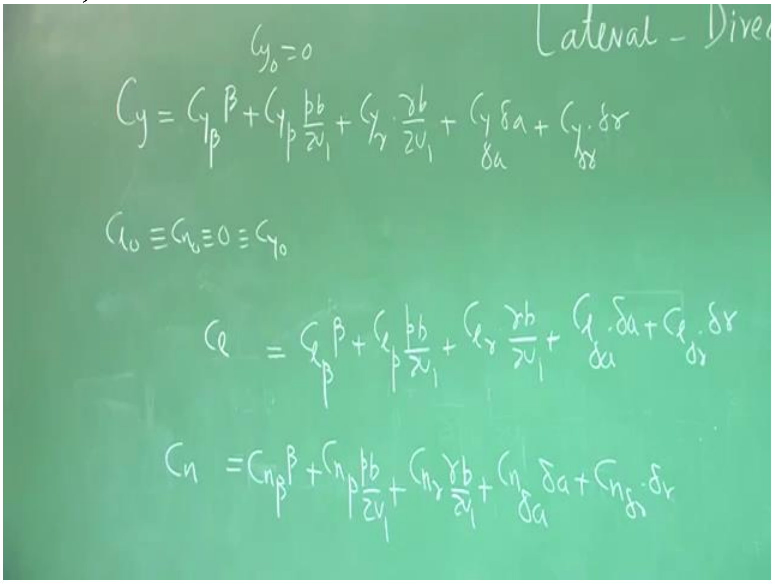

So this is CL, this is CN. It goes without saying that he put CL not and CN not identically equal to 0. Now see here. Before you do anything, before you play any game with this derivative, let us be sure we have understood this in terms of their sign. In stability many of the times, perhaps most of the times, one should be very very careful about the sign. The sign decide its more importantly than the magnitude.

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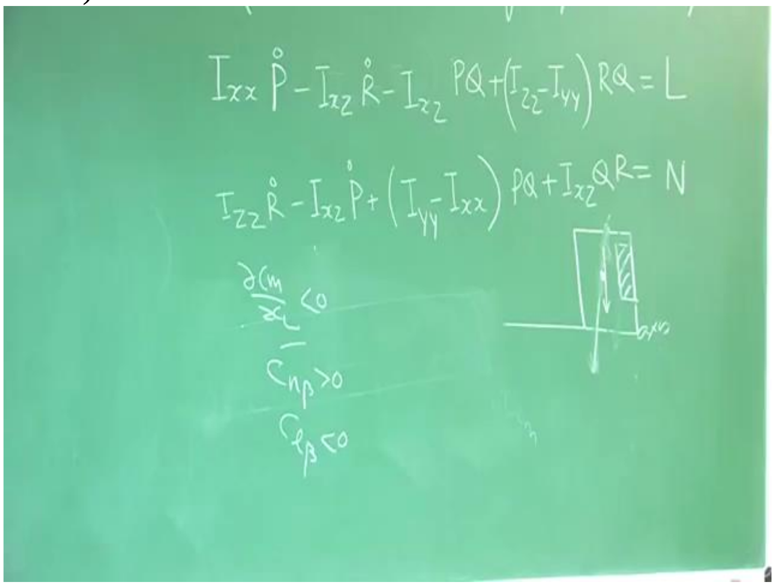

For example, DCM by DCL should be less than 0 decides longitudinal static stability. CN beta greater than 0 decides directional static stability. CL beta less than 0 decides lateral static stability. It is a sign. The magnitude decides the control part of it. If this is very large negative means highly statically stable. So if we want to change from one trim to another trim, lot of efforts you have to put. Similar thing, if CN beta is large, that means it will become very sensitive to cross wind. Similarly CL beta, you understand that.

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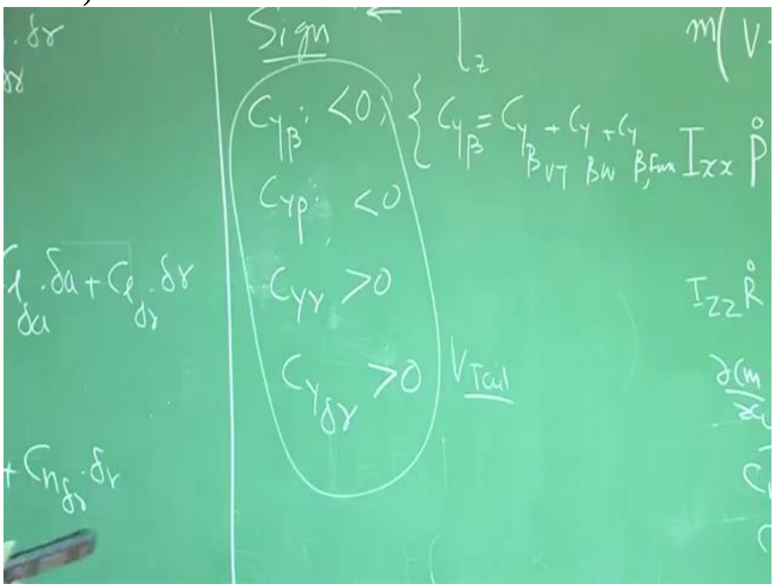

So if I want to crosscheck whether I understood the signs or not, let us take one by one. CY beta. What will be its sign? We will draw a tail like this and positive beta means wind is coming from the right-hand side. I am flying like this. This is the relative air speed. So if this is positive beta and you know that your X, Y is here, Z is here. The force of the vertical tail will be opposite of Y. Right?

So the sign of CY beta is less than 0. Is this clear? I want to demonstrate the sign of CY beta, suppose this is the vertical tail. I am moving like this and let us say this is the relative air speed. Then the side force on the vertical tail will be in this direction. But Y is my Y direction, X is this. Y is this direction and force is in this direction for positive beta.

So CY beta becomes negative . Next is CYP. If we take only vertical tail, please understand the signs of the whole derivative should be seen as a combination of all the components. We are discussing about the sign of CY beta and we have seen that CY beta because of vertical tail will be negative. But at this point, please understand, CY beta will have a comparable because of CY beta of vertical tail, CY beta of wind, CY beta of fuselage.

All this, right? So we have to check each and every component, how they are going to add. I am only giving an illustrative example. I am using the vertical tail for most of the time because that contributes the maximum for most of the cases. So CY beta less than 0 I have understood. Now CY P. If I again try to see what is the vertical tail contribution? You could see that if this is the vertical tail and P positive means right-wing going down like this

So this is pushing the air downwards and this fin will experience force in this opposite direction. So Y is this way so force is in this direction. So again CY P because of vertical tail is less than 0. But with wind and sweep, it may change the sign. I am talking about the vertical tail. Then comes CYR. Again CYR you see, this is the airplane, this is the vertical tail. Positive R is this.

So this tail is pushing the air this way. It is experiencing a force in this direction and Y is also in this direction. So CYR is greater than 0. I repeat, as it takes positive R, this vertical tail is pushing the air this way and it is experiencing a force in this direction and we know Y is in this direction. So CYR is positive. And then CY Delta R.

We know what is Delta R positive. Delta R positive is towards left like this, deflected towards left. I am moving like this. So the force will be coming in this direction. This vertical tail will experience the force in this direction which is along the positive Y direction. So CY Delta R will be positive. Please use pen and one paper like this and try to crosscheck these signs.

What I am talking here let us not forget, all this I am taking the contribution of vertical tail. However we know, the overall contribution has to be seen.

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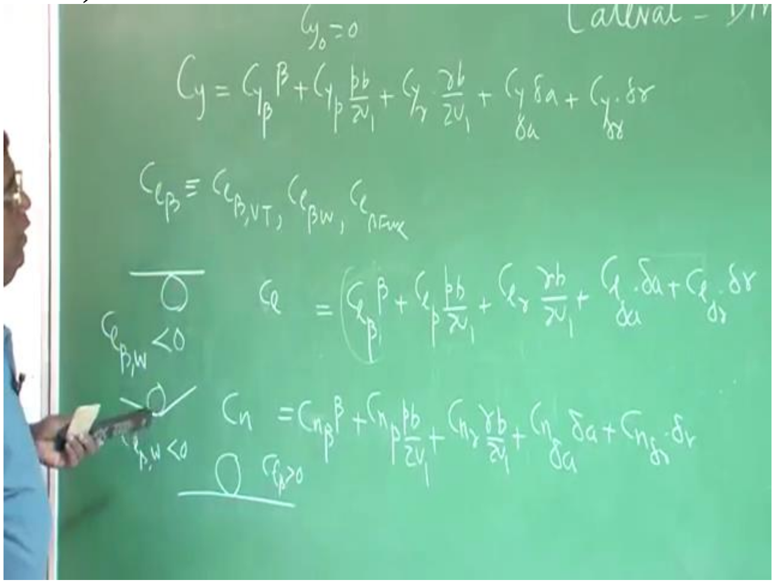

Now coming back to CL and CL you see is CL beta. And the moment CL beta comes, if I want to check the sign, now CL contribution will be because of CL beta vertical tail, CL beta wing, CL beta fuselage. At least here we know that if it is a high wing, CL beta because of going will be less than 0. That is contribution of wing if it has a dihedral like this, again CL beta wing contribution will be less than 0.

And if it is a low wing like this, then of course CL beta is greater than 0. This much we know. For vertical tail, let us see what happens. If this is the vertical tail, CL beta means it has banked, started side slipping. So as it side slips, the force will act in this direction and that will give rolling moment about centre line whereas the left-wing will go down. So again that is a negative sign.

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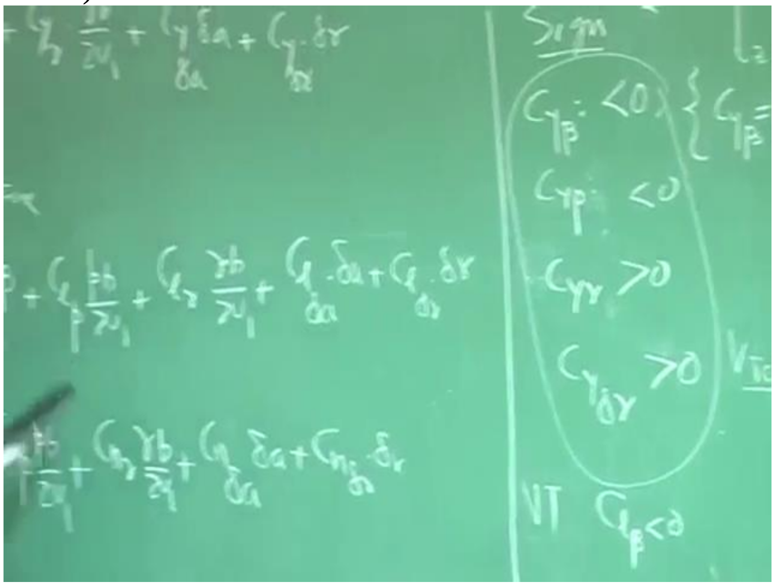

So CL less than 0 when we are talking about vertical tail. Then comes CLP. What is CLP? Extremely important.

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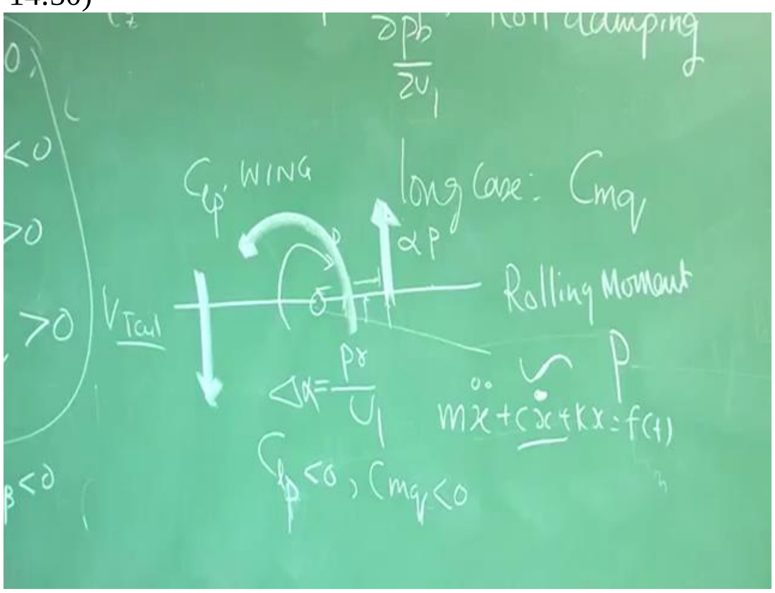

CLP. This is DCL by DPB by 2U1. This is also called roll damping. Remember, for longitudinal case, similar was CMQ which was pitch damping derivative. So let us see what is CLP? Who are the prime contributors? Here, the wing is the prime contributor. For CLP, the prime contributor is wing. What happens? Suppose this is positive P.

I am flying into the board and positive roll rate means right-wing going down. What is, what do you see here? At each point, at a distance R, this will see a vertical component of relative air which is P into R. Right? Omega R. This divided by U1, let us say moving with forward speed, that will give me an approximate angle of attack at each section.

So this will, in cumulative way, generate a force in this direction which will be proportional to alpha, Delta Alpha here which has P. So this will be proportional to P, roll rate. Similar thing will happen here in reverse direction. So this will in turn give a rolling moment. It will give a rolling moment which will be proportional to P. That is more important.

And you know, the reaction moment, proportional to P, proportional to the rate, characteristics proportional to the rate, we attribute it to damping derivative, right. Or it causes damp into the dynamics. As we have seen for second-order system, CX dot + KX equal to F of T. You see this one, CX dot. So here, we see that the moment it tries to bank like this, immediately there is a moment to oppose it.

And that opposing moment is proportional to the rate and that is why this is called roll damping. And primary contributor is wing and of history, the sign CLP is negative. For a positive P, right-wing going down, moment is this way, left going down. So CLP is negative. As CMQ for longitudinal case was negative and both are damping derivatives. This one for lateral case, this is for longitudinal case. So CLP is, what about CLR? Let us see what is CLR? Once CLP is over, now look for CLR.

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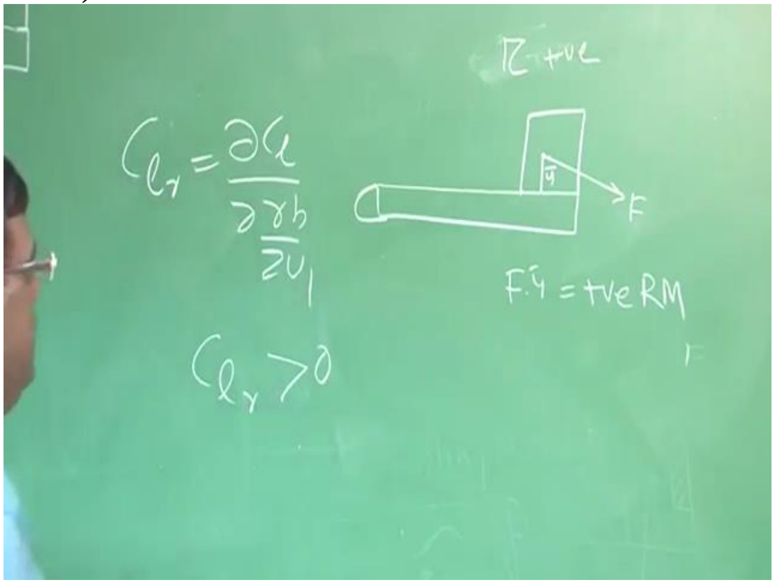

What is CLR? CLR is DCL by DRB by 2U1. That means, the question is, if this body is having a yaw rate like this which is a positive yaw rate, right-wing going back, will it create a rolling moment as a reaction or not? You see, very simple, as it rotates like this, this vertical tail is pushing the air this way. So it will experience a force in this direction.

And that force will be somewhere at the centre of this fin or vertical tail. So that force into the distance between the force and the centreline will give me a positive rolling moment. That is, if this is the fuselage, this is the vertical tail, as it goes for right-wing positive, that is right-wing going back, so there will be a force experienced in this direction and this force into this Y bar will give me a positive rolling moment.

So CLR is greater than 0. Sign wise, CLR is greater than 0. I repeat again here, if this is the vertical tail, if it is having a positive R, yaw rate like this as it moves like this, this tail will push the air this way. So it will experience a force in this direction. Now let me show it. I repeat. As this body is having a positive R, right-wing going back, this tail is pushing the air that way.

So the tail will experience a force in this direction. And this force into this distance top set from the centreline will give a positive rolling moment. And that is what positive CL. that is why CLR is greater than 0. Okay?

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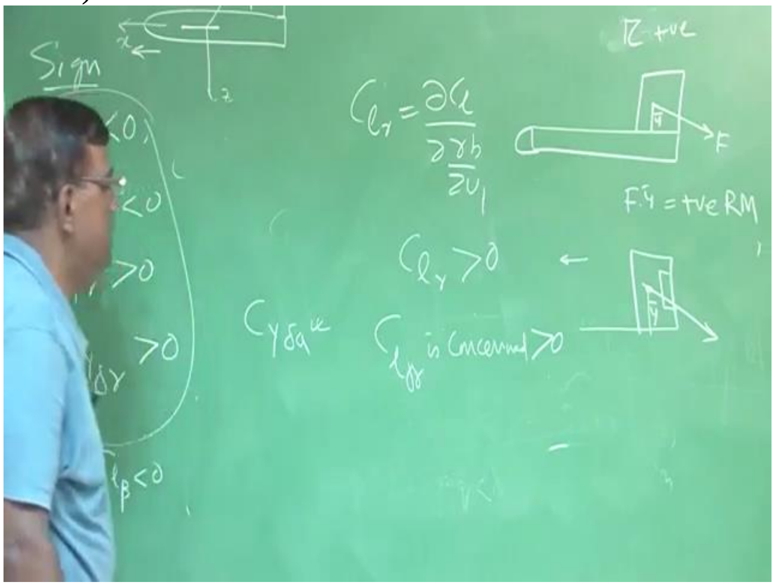

Once CLR is over, then CY Delta A and CY Delta R, that is straightforward depending upon which way you are deflecting the aileron. You know, if this is my wing, if I put this aileron down, and this aileron up, then I will bank like this. So if I define this combination as positive Delta A, then CL Delta is negative. If I define otherwise, then CL Delta will become positive.

So depending upon Delta A sign, we can pick positive or negative. But as far as CL Delta R is concerned, you know that Delta R, if this is the rudder, Delta R positive is this, towards left, looking from the top and I am moving forward. As I deflect like this, force will be in this direction. So this force into the distance from the axis will give me a positive rolling moment. So CL Delta R is greater than 0.

This is very simple. If I deflect towards left, force will be in this direction. This force into this distance, Y bar will give me a positive role when I am moving in this direction. Right? So CL Delta R is positive. So I repeat here again with this small example, if this is the fuselage, this is the rudder, I deflect it like this. This is positive by convention which everybody agrees.

Then as it moves forward, the force is in this direction. And this force into this distance from centre line will give a positive rolling moment. That is why CL Delta R is positive. Now let us talk about CN. If I come to CN, let us see what is the sign of CN beta?

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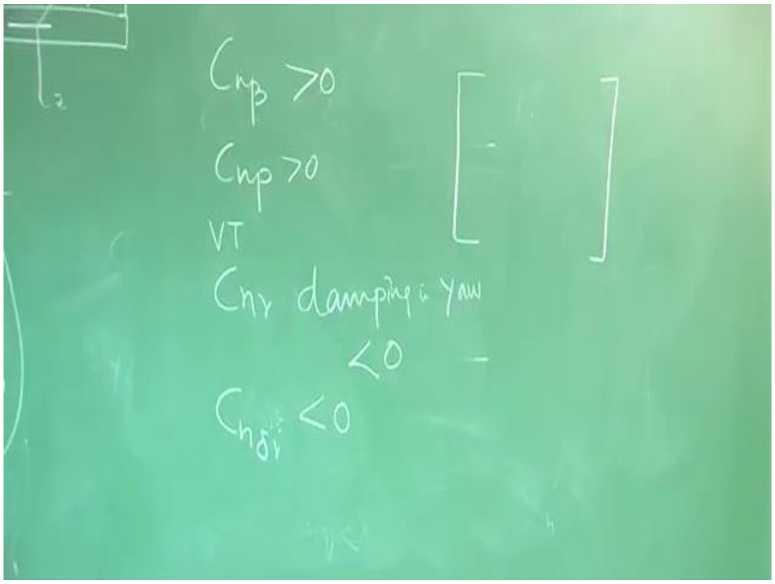

CN beta, we have no confusion. It has to be greater than 0 if it is statically stable in directional case. What about CN P? Again if it copout contribution of vertical tail, let us see what happens? This is the body and this is the vertical tail. CNP, that is if I give a P like this, roll rate, how the yawing moment will generate. As I roll it like this, you could see that the vertical tail will get a force in the direction and that force and CG somewhere here, will give a positive yawing moment.

Am I correct? CNP. P is this. As it is going P, vertical tail will get a force in this direction which will give a yawing moment positive. So CNP is greater than 0. CNR, what is CNR? Do you know CMQ? Damping in pitch. CNP is damping in roll. So CNR is damping in yaw. So this is again for statically stable as plain, CNR is negative, less than 0. And then if I talk about CN Delta R, you can check yourself.

If this is the fuselage and this is the rudder and this is the positive towards left, as I move forward, force will be like this in this direction and this will give a yawing moment about CG in the negative way. The force is there, the point is here. So it will give a moment like this. Yawing moment negative. So, C and Delta R will be less than 0.

This understanding of the signs are extremely important. And please understand, most of the cases, I have shown the contribution due to vertical tail and wherever being played an important role, dominating role, I have mentioned it. But in practice, you have to see the contribution because of all the 3, 4, 5 components. That may have propeller also. Okay?

So that becomes an involved thing but you as a flight dynamics man should understand that I need to be bothered about this derivative because why? To understand that finally when you are going for dynamic stability studies, there are stability matrix. A huge one, a lot of time to write and after decking laplace transform and all, this matrix is constructed with these derivatives.

And if they are not correct in sign and magnitude, your stability matrix will be altered. And then you will get some fictitious result. So that is why before you go to that, I thought a revision on this will be extremely important. In the next class, we will go for developing short period equation of motion pertaining to lateral directional case. Thank you very much.