Aircraft Dynamic Stability & Design of Stability Augmentation System
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Module 7
Lecture No 38
Mode Shape: Longitudinal Case

Good morning friends. I thought that we must also discuss something on mode shapes.

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Before we talk about mode shapes, let us understand what we are talking about. Let us say if I take a longitudinal case then if I define, this is to be, this is the horizontal, this angle is Theta. And if this is the relative velocity, then this is U and this angle is alpha. If I give a disturbance to an airplane, I know the alpha will change, Theta also will change.

It is not only that how is their magnitude is changing relatively, that is important. It is also important whether their change is in phase or out of phase. That is when alpha is increasing, Theta is also increasing or when alpha is increasing, Theta is decreasing which means they are out of phase.

So we need to know that information from the dynamics or we need to extract that information from the dynamics. And that will help us in designing the control. In a very restricted manner but a very useful manner, we are talking about linear system. Now, control has gone much beyond using a linear approach or using transfer function approach.

We talk about state space modelling, so many things but a good control designer knows, he has to be very clear about the transfer function approach a lot of information you get. You understand, you could see physically. Close your eyes, you could see yes, this is happening, Alpha is increasing, Theta is increasing, what is their magnitude? What is their phase difference?

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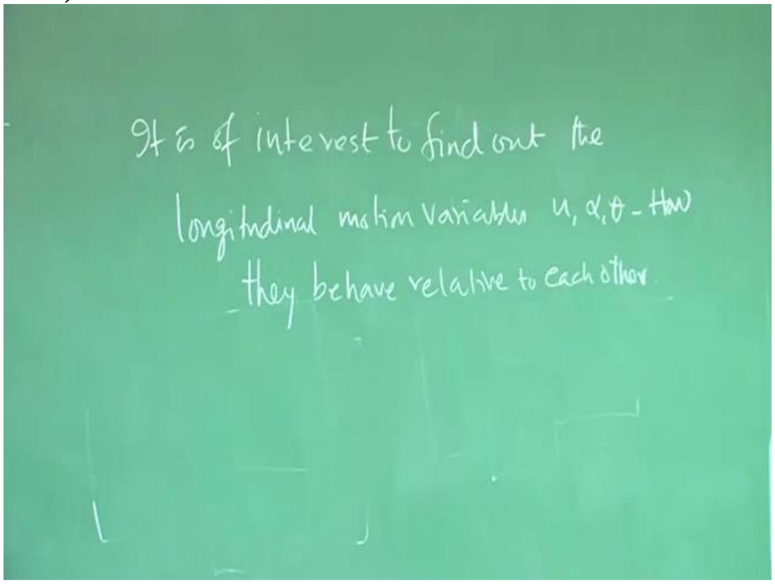

So in a nutshell, we say when we talk about mode shapes, we talk about it is of interest and we know why the interest is the interest to find out the longitudinal motion variables. So U alpha Theta, how they behave relative to each other? That is important. These are the motion variables.

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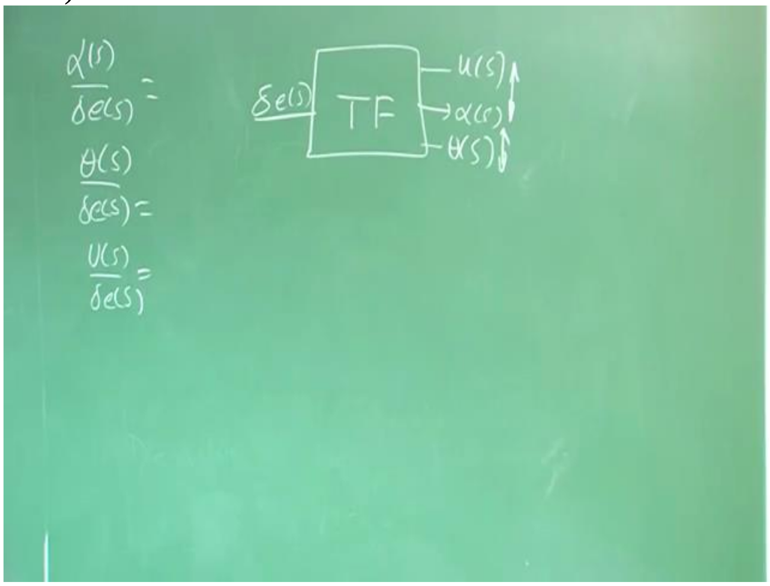

Remember, when we initially did the dynamic stability postulation, we have seen, alpha S by Delta E of S, Theta of S by Delta E of S, U of S by Delta E of S and we have got expressions in frequency domain and we refer these, address these ratios as transfer function.

And what does it tell you that if I have an aircraft system which has transfer function if I give a Delta E of S, I know what will be the output alpha of S. Right? This is input output. Similarly for this Delta E of S, what will be Theta of S? Similarly what will be U of S. Right? This is typically longitudinal motion.

But if we talk mode shape, we are now trying to relate the relative changes between any 2 variables, U, alpha or alpha, Theta or U, Theta in terms of magnitude and in terms of phase and that is what we refer as mode shapes. Okay? This part is clear? How do I find out more shapes numerically? That is the question. Let us do that.

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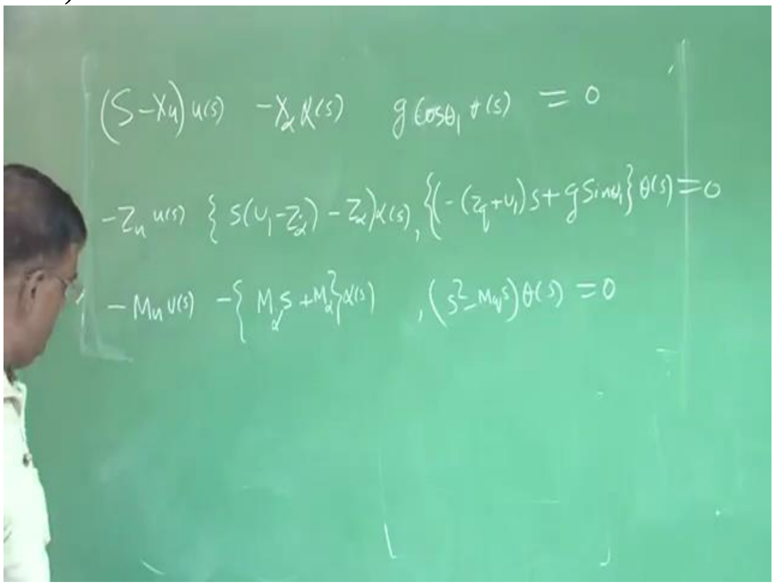

When we try find out the mode shapes, we again go back to this equation, that matrix, remember, S - XU into U of S. I have developed this, - X alpha into alpha of S. Then G Cos Theta 1 Theta of S. Then here it is - ZU U of S. Then this is a huge term, SU1 - Z Alpha dot - Z Alpha into alpha of S. Then we have - ZQ + U1 S + G. You have all these things. So do not blame me if I write something wrong. G Sin Theta 1 Theta of S.

And the 3rd one is, - MU into U of S, - M alpha dot S + M Alpha into alpha of S. Then this term is S square - MQ of S into Theta of S. This is it. And we say when we tried to find mode shapes, the easiest way to do it is use free response. That is no external force. So this is equal to 0, this is equal to 0 and this is equal to 0.

No forcing function. Now you know, you are smart. What you have to find out is the relative ratios. You can easily see from here. You can write, although I say easily you can write but you have seen that I take help of this paper because somebody has done this for us. You should do. At your age, I also used to derive this expression but now I do not.

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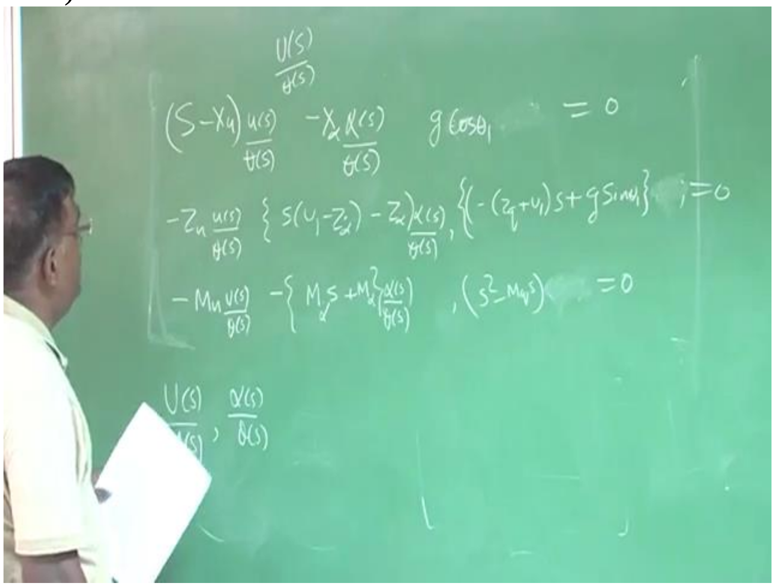

If you are understanding, you can do yourself. So it will be something like this, S - XU, - MU. Now our aim is to find out U of S by alpha of S. So it is very simple. I can again write this in terms of S - XU and divide it by Theta of S if I am looking for Theta of S. This will again divide by Theta of S. Here Theta of S will go. So I am looking for U of S by Theta of S.

Then 2nd equation, what will happen is ZU Theta of S here. Alpha of S by Theta of S and here Theta of S will go. And here U of S by Theta of S, here alpha S by Theta of S and this will go. So what I am looking for? This relative values in terms of magnitude and phase for U of S by Theta of S, alpha of S by Theta of S. And this you know how to solve. These are linear algebraic equations.

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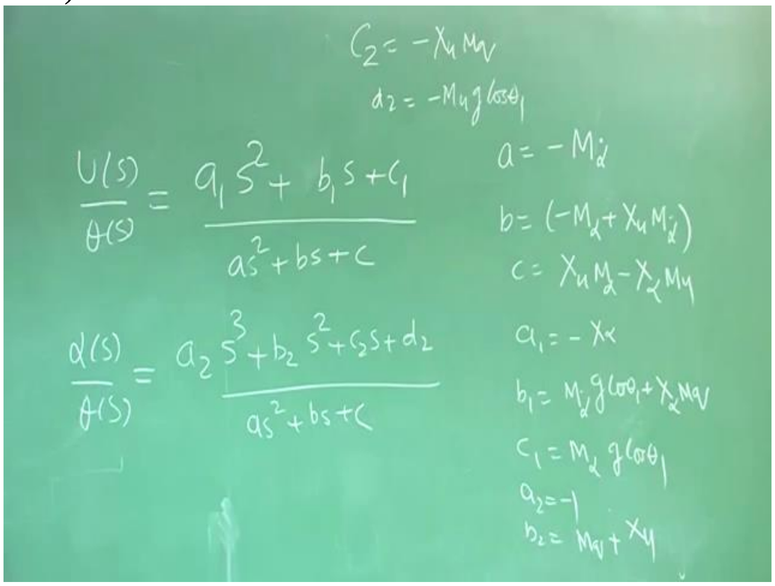

And if you try to find out the expression, we can write U of S by Theta of S like this. In some general form I am writing A1S square + B1S + C1 by AS square + BS + C. Right? These are algebraic equations, you have to solve these equations to find out U of S by Theta of S. You can find alpha of S by Theta of S. Okay?

And let us say, the generic form as this, where A1, B1, C1 will have some expression. Similarly we can solve these 3 equations and find out alpha S by Theta of S and lecture that is given in terms of A2 S cube + B2 S square + C2S + D2 by AS square + BS + C. And you could yourself check the expression here, A will be given as - M alpha dot, B will be given as - M alpha + XU M alpha dot, C will be given as XU M alpha or XU M alpha - X alpha MU.

And A1 will be - X alpha, B1 will be equal to M alpha dot G Cos Theta 1 + X alpha MQ. Then C1 equal to M alpha G Cos Theta 1. Similarly, A2 is equal to - 1, B2 equal to MQ + XU, C2 equal to - XUMQ and D2 equal to - MUG Cos Theta 1. These I have taken from Roshcom. If you want to further study, you can refer that. Wonderfully, it has explained.

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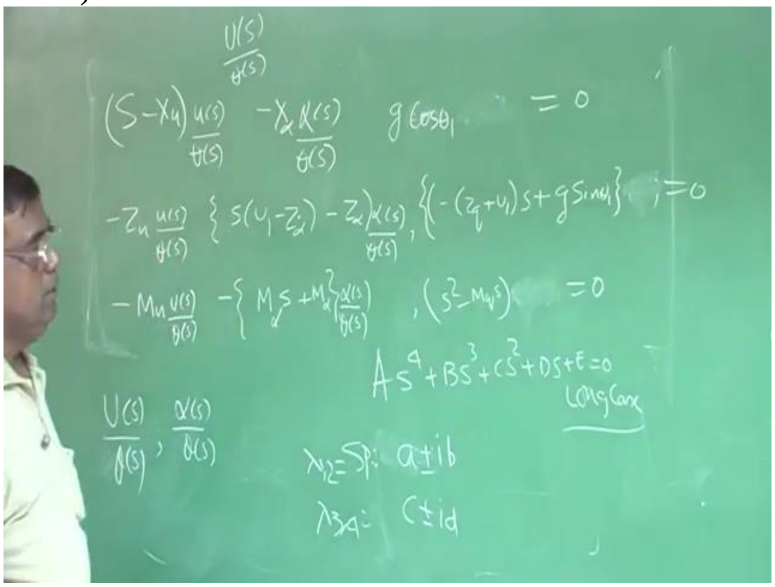

One of the books I follow but today's understanding, these are the equation, we have to find this, these ratios by algebraically manipulating these. You can use Cramer's rule whatever you want to do. By substitution, you can do. And these are for your convenience, I have given this. What is the message now? We are talking about mode shapes.

That means, we want to see the relative change in the magnitude and phase of the motion variables. For longitudinal case, it is alpha and Theta. Remember we have equation, AS4 + BS cube + CS square + DS + E equal to 0 say longitudinal case. Right? It has trouble us so much. Huge equation. But we also know, for short period, we had lambda 1 + -, I say lambda 12.

Or lambda short period we have fruits let us say capital A class IB, + - IB. And lambda 34 for Phugoid, let us say C + - ID. Now if we want to see the mode shapes corresponding to short period, then what you have to do? You have to put the value of X in lambda 1 lambda 2 which you have got from solving this equation.

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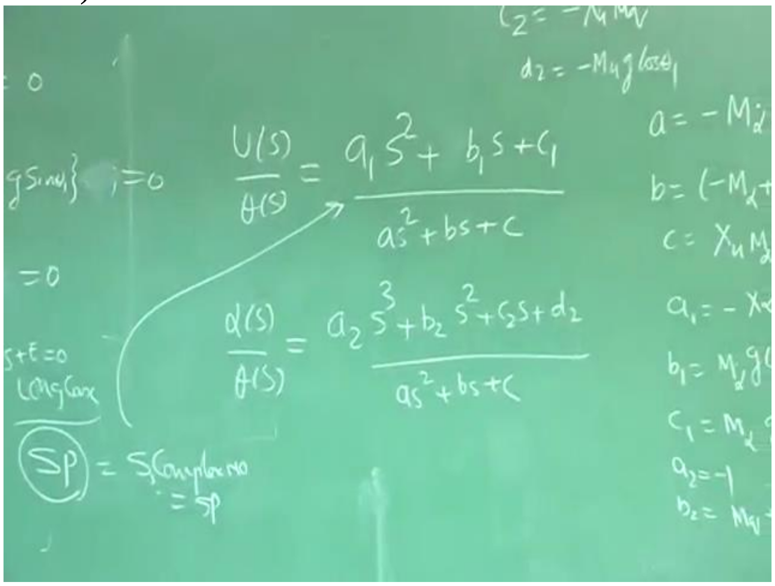

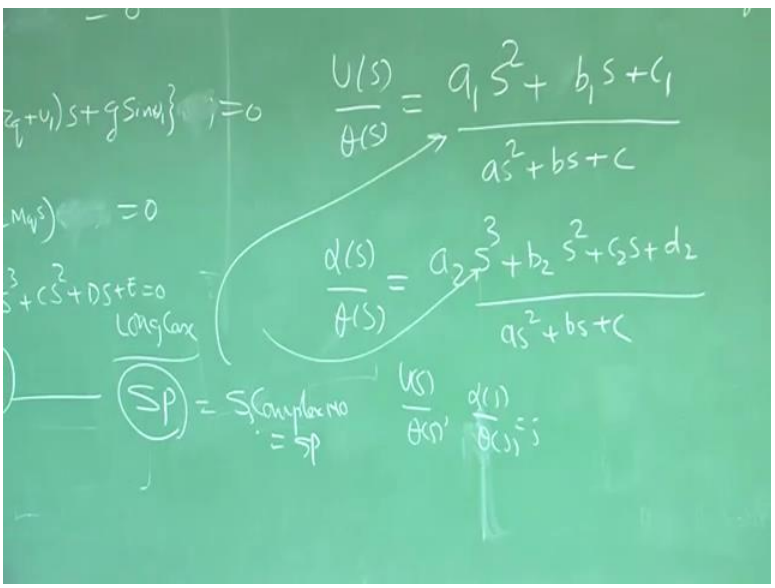

Those values of S which is a complex number corresponding to short period root whatever you have got, that you plug in here. If you want to know alpha of S by Theta of S for short period, again the value of S what you have got plug in here. A1, A2, A, B, C, all these things are known. It depends mostly on dimensional stability derivative. Once you do that, so you know how U of S and Theta of S or Alpha S by Theta of S are varying for each mode, short period or Phugoid. And after all, these are complex numbers. So you know complex number, how to find magnitude and phase. That is our purpose.

Just for completion, let me write the expression so that you can do yourself. I thought I must complete this. Nowadays, you have Matlab and all. You can directly compute these mode shapes once you leave the values.

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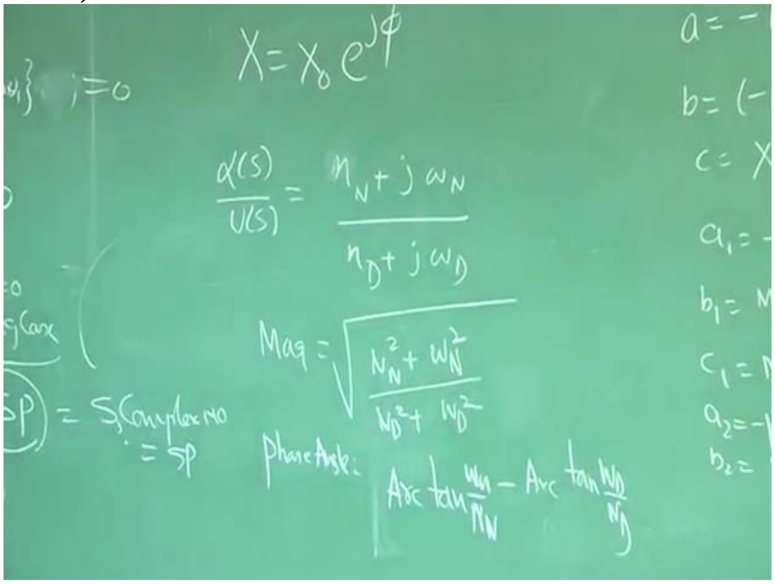

Just to Revise your understanding in complex numbers, complex number is represented by this. And if I have a ratio of alpha of S by U of S as N numerator is J omega numerator by N denominator + J omega denominator. Suppose the complex number is having this sort of a form.

Then I know, the magnitude will be given as N numerator square + omega N numerator, do not say omega, W numerator square by N denominator square + W denominator square. And the phase angle is given as Arc tan WN by NN - Arc Tan WD by ND. So Tan inverse. It is so straightforward. So let us again revisit how do I find this phase difference or magnitude to understand the mode shape or motion variable alpha, Theta and U.

What we do? We take the characteristic equation. Right? Then divide by Theta of S and alpha of Theta of S. Then from here I can find out U of S, expression for U of S by Theta of S and alpha of S by Theta of S which will come as ratio of 2 complex numbers. And you know, if this is a ratio of 2 complex numbers, how do I find magnitude and phase angle? It is as simple as that.

If I want to find out the mode shape for short period mode, so in the expression of alpha S and U of S which had S, we see the earlier expressions were we have to put roots corresponding to short period. For Phugoid, we have to put the value of S corresponding to Phugoid. You will get ratio of 2 complex numbers.

Then you can find magnitude and phase angle. And from there, you could see that for that disturbance, which motion variable is predominantly changing and which is not changing and between the 2, what is their phase difference? Okay? So this is the background for mode shape. Thank you very much.